

# Neutrino Masses in Theories with Dynamical Electroweak Symmetry Breaking

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## Outline

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4. Mass generation for quarks and charged leptons
5. General structure of the neutrino mass matrix
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9. Conclusions

Refs: T. Appelquist and R. Shrock, Phys. Lett. B **548**, 204 (2002);  $\rightarrow$  PRL, in press  
hep-ph/0301180; and  
to appear

GENERAL QUESTION: What do we infer from the discovery of neutrino masses about physics beyond the Standard Model?

Longstanding problem of explaining light neutrino masses in the context of dynamical EWSB models. Common view: light neutrino masses suggest a seesaw involving a (SUSY) GUT mass scale, with  $m_\nu \sim m_D^2/M_{GUT}$ , where  $M_{GUT} \sim 10^{16}$  GeV. But dynamical EWSB models do not have nearly this large a scale; in ETC, typically the largest scale is  $\sim 10^6$  GeV.

So how can one explain neutrino masses in dynamical EWSB framework? Here we will propose an explanation and show how it is realized in explicit ETC models. These models involves a seesaw, but one with mass scales well within the ETC range.

## Some Possible Motivations for Dynamical Electroweak Symmetry Breaking

- This solves the hierarchy problem, i.e., the quadratic sensitivity of the Higgs potential of the SM to high-scale physics and associated fine-tuning (as does SUSY, if one solves the  $\mu$  problem)

It is important to recall that in both of two major previous cases of symmetry breaking in which one used a scalar field as part of a phenomenological model of the phenomenon, the microscopic physics actually involved bilinear fermion condensates:

- Ginzburg-Landau approach to superconductivity using complex scalar field; microscopic BCS theory involved dynamical formation of Cooper pair condensate
- Gell-Mann Levy  $\sigma$  model for spontaneous chiral symmetry breaking, in which the  $S\chi$ SB was manifested via  $\langle\sigma\rangle \neq 0$ ; in the actual underlying QCD theory, the  $S\chi$ SB is due to the dynamical formation of a  $\langle\bar{q}q\rangle$  condensate

Could these previous examples be teaching us something about what we might expect for electroweak symmetry breaking?

## Motivations and Theoretical Framework

Understanding the fermion mass spectrum remains a challenge.

The Standard Model (SM) accommodates quark and charged lepton masses by the mechanism of Yukawa couplings to a postulated Higgs boson, but this does not provide insight into these masses, especially since it requires Yukawa couplings of order  $10^{-6} - 10^{-5}$  for  $e, u, d$ . SM Higgs sector is unstable to large loop corrections.

SM predicts zero neutrino masses and no lepton mixing, and hence must be modified to take account of current experimental evidence

for neutrino masses and mixing.

accel. - K2K

reactor - KamLAND

solar: (CL, Kam., GALLEX, SAGE, Superk, SNO)

atm.: (IMB, Kam., Soudan-2, Superk, MACRO)

Since masses for the quarks, charged leptons, and observed neutrinos break the SM gauge symmetry, an explanation of these masses requires a model for electroweak symmetry breaking (EWSB). Here we assume dynamical EWSB driven by a strongly coupled gauge interaction, associated with an exact gauge symmetry, technicolor (TC) embedded in an extended technicolor (ETC) theory (Weinberg, Susskind, Dimopoulos, Eichten, Lane...).

The TC theory is designed to have "walking" behavior (Holdom, Yamawaki et al., Appelquist et al.), which can produce realistically large quark and charged lepton masses and sufficiently small TC electroweak corrections. Further ingredients are likely needed to explain  $m_t$  and, in some cases, to avoid problems with Nambu-Goldstone bosons.

slowly running gauge coupling over certain range

TC gauge coupling =  $\alpha$

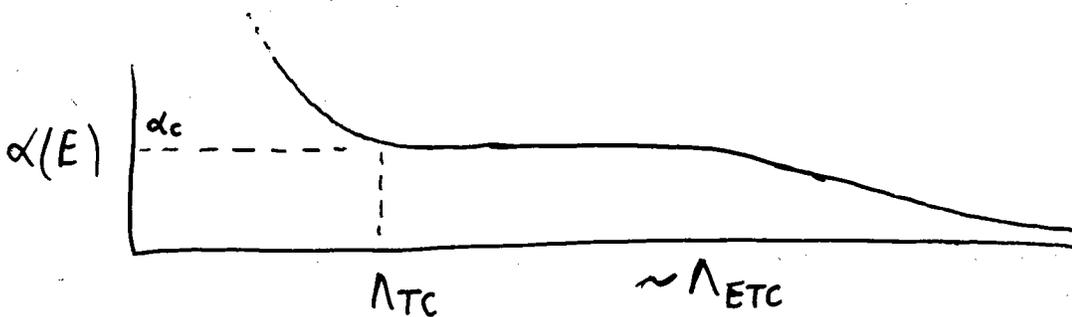
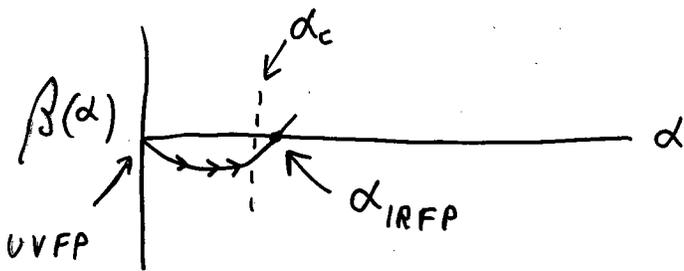
Walking TC:  $\beta(\alpha) = \mu \frac{d\alpha}{d\mu} \propto -\alpha^2 (\beta_0 + \beta_1 \alpha)$

$\beta_0 > 0$ : asymptotic freedom

$\beta_1 < 0 \Rightarrow$

$\exists \alpha_{IRFP} = -\frac{\beta_0}{\beta_1}$

so  $\beta(\alpha_{IRFP}) = 0$



$E = \mu$

As  $E$  decreases,  $\alpha(E)$  increases, but  $|\beta|$  decreases, so  $\alpha$  only evolves slowly for an extended interval in  $E$ . Eventually  $\alpha$  exceeds  $\alpha_c$ , the value for condensation (at  $E \sim \Lambda_{TC}$  here,  $\rightarrow \langle \bar{F}F \rangle$  formation) - then technifermions pick up dynamical masses  $\sim \Lambda_{TC}$ , TC theory confines. Effect of (approx.) IRFP.

## Some General Features of TC, ETC models

We will take the technicolor group to be  $SU(2)_{TC}$ . The technifermions include, as a subset, one family,  $i = 4, 5 : TC$   
 $a : color$

$$Q_L^{i,a} = \begin{pmatrix} U^a \\ D^a \end{pmatrix}_L, \quad L_{TC,L}^i = \begin{pmatrix} N \\ E^i \end{pmatrix}_L, \quad U_R^{i,a}, \quad D_R^{i,a}, \quad N_R^i, \quad E_R$$

transforming according to the fundamental rep. of  $SU(2)_{TC}$  and usual reps. of  $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ . These comprise  $N_f = 2(N_c + 1) = 8$  fermions with vectorial TC couplings.

Choice of  $N_{TC} = 2$  has appeal that with  $N_f \cong 8$ , studies suggest that the TC theory could have an (approximate) infrared fixed point (IRFP) in the confining phase with spontaneous chiral symmetry breaking but near to the phase transition (as a function of  $N_f$ ) beyond which it would go over into a nonabelian Coulomb phase.

Beta function calculation suggests that for  $N_{TC} = 2$ , IRFP exists if  $N_f \gtrsim 5$  or 6 and nonabelian Coulomb phase occurs if  $N_f \gtrsim 8$  (with  $N_f < 11$  for asymptotic freedom). This has two valuable consequences: (i) walking behavior, which enhances condensates and fermion masses, (ii) reduction of TC contributions to  $S$  parameter (Appelquist and Sannino). *Some uncertainty in  $N_f$  for onset of nonabelian Coulomb phase; could be larger.*

$\langle \bar{F}F \rangle$  :  $I = \frac{1}{2}$   $|Y|=1$  operator, same as Higgs vev in SM  
 $\Rightarrow$  usual  $m_W - m_Z$  mass relation -  $m_F = 0$  initially so  $\langle \bar{F}F \rangle \neq 0$   
 breaks associated global technichiral sym.  $\rightarrow$  NGBs.

At a scale  $\Lambda_{TC}$ , the TC coupling gets strong enough to produce a bilinear technifermion condensate  $\langle \bar{F}F \rangle$  and corresponding dynamical masses for the technifermions. Some would-be Nambu-Goldstone bosons become longitudinal modes of  $W$  and  $Z$ , giving them masses

$$m_W^2 = m_Z^2 \cos^2 \theta_W = \frac{g^2}{4} (N_c f_Q^2 + f_L^2) \simeq \frac{g^2}{4} (N_c + 1) f_F^2$$

where it suffices to take the TC pseudoscalar decay constants  $f_L \simeq f_Q \equiv f_F$ . Hence,  $f_F \simeq 130$  GeV. In QCD,  $f_\pi = 93$  MeV and  $\Lambda_{QCD} \sim 170$  MeV, so  $\Lambda_{QCD}/f_\pi \sim 2$ ; hence, we take  $\Lambda_{TC} \sim 260$  GeV.

To generate fermion masses, embed TC in ETC theory; role of ETC gauge bosons is to connect TC-singlet and TC-nonsinglet fermions and communicate EWSB in TC sector to the TC-singlet fermions.

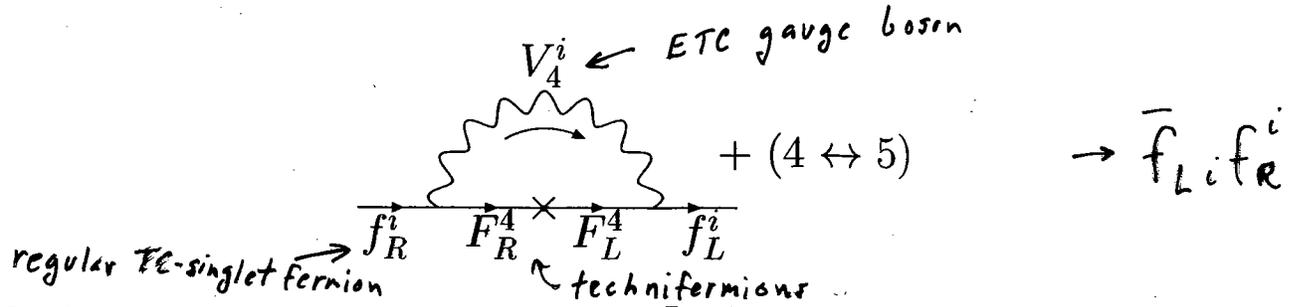
To satisfy constraints on flavor-changing neutral-current processes, ETC vector bosons must have large masses. These can arise from self-breaking of the ETC gauge symmetry, which in turn requires that ETC be a strongly coupled, chiral gauge theory.

The ETC self-breaking occurs in stages, e.g.,  $\Lambda_1 \sim 10^3$  TeV,  $\Lambda_2 \sim 50$  TeV, and  $\Lambda_3 \sim 3$  TeV, leaving as an exact residual invariance group  $SU(N_{TC})$ . This entails the relation

$N_{ETC} = N_{TC} + N_{gen}$ . Hence, with  $N_{gen} = 3$  and  $N_{TC} = 2$ , we have  $N_{ETC} = 5$ .

# Mass Generation for Quarks and Charged Leptons

Recall dynamical ETC mass generation mechanism for quarks and charged leptons. For rough estimate, consider one-loop diagram shown.



One-loop graphs contributing to the mass term  $\bar{f}_{i,L} f_R^i$  where  $1 \leq i \leq 3$ .

This yields

$$m_{f_i} \sim \frac{g_{ETC}^2 \eta_i \Lambda_{TC}^3}{2\pi^2 M_i^2}$$

where

$$M_i \sim g_{ETC} \Lambda_i$$

is the mass of the ETC gauge bosons that gain mass at scale  $\Lambda_i$  and  $g_{ETC}$  is the running ETC gauge coupling at this scale. The factor  $\eta_i$  is a possible enhancement factor incorporating walking:

$$\eta_i = \exp \left[ \int_{f_F}^{\Lambda_i} \frac{d\mu}{\mu} \gamma(\alpha(\mu)) \right],$$

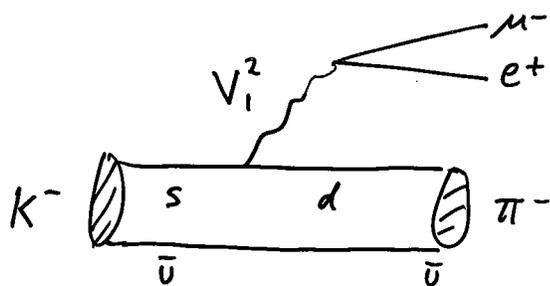
In walking TC, the anomalous dimension  $\gamma \simeq 1$  so

$$\eta_i \simeq \frac{\Lambda_i}{f_F}$$

whence

$$m_{f_i} \sim \frac{\Lambda_{TC}^2}{2\pi^2 \Lambda_i} \quad - \text{ gives larger masses since } \Lambda_{TC} < \Lambda_{ETC}$$

ETC leads, e.g., to FCNC processes such as



$$K^- \rightarrow \pi^- \mu^- e^+$$

$$K^+ \rightarrow \pi^+ \mu^+ e^-$$

Current limit (BNL E865):  $BR(K^+ \rightarrow \pi^+ \mu^+ e^-) < 2.8 \times 10^{-11}$

$$\rightarrow m(V_1^2) \geq 200 \text{ TeV} \quad m(V_1^2) = \Lambda_1$$

Lower bounds on  $\Lambda_i$  - ETC scales  $\Rightarrow$  upper bounds on fermion masses.

QCD-like variants of TC were ruled out by their inability to satisfy the constraints of

- not producing excessively large FCNC processes
- producing large enough fermion masses

The enhancement factor in walking TC played a crucial role in ameliorating this problem, and modern TC models use walking

- still a challenge to get a large enough top quark mass

Constraints from precision electroweak data

$$S \simeq \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2}$$

Naive perturbative evaluation: each extra EW doublet gives contribution

$$\Delta S \simeq \frac{N_{TC}}{6\pi} \left[ 1 - Y \ln \left( \frac{m_{1/2}^2}{m_{-1/2}^2} \right) \right]$$

where subscript is weak  $T_3$ . In one-family TC, have

$$\begin{pmatrix} U^{ia} \\ D^{ia} \end{pmatrix} \quad \begin{pmatrix} N^i \\ E^i \end{pmatrix}$$

where  $\{i\}$  = TC indices,  $a$  = color indices

Nondegeneracy of  $U$ ,  $D$  and of  $N$ ,  $E$  masses can reduce TC contribution to  $S$ , as constrained by  $T$  parameter ( $\rho = \alpha T$ ).

Since technifermions are strongly interacting at scale  $E \sim m_Z$ , one cannot use perturbative estimates, and one does not have reliable methods to calculate TC contributions to  $S$  and  $T$ . Non-QCD-like behavior (walking) means that one cannot reliably try to scale up from QCD.

N.B.: Efforts to perform precision electroweak fits with oblique corrections have been complicated considerably by the NuTeV anomaly

$$N_{\text{gen}} + N_{\text{TC}} = N_{\text{ETC}}$$

$\underbrace{\quad}_3 \quad \quad \quad \underbrace{\quad}_2 \quad \quad \quad \underbrace{\quad}_5$

## General Structure of Neutrino Mass Matrix

In general, (E)TC models have a set of  $n_s$  electroweak-singlet neutrinos, denoted  $\chi_R = (\chi_1, \dots, \chi_{n_s})_R$ , including both TC-singlets and TC-nonsinglets. For our model,  $n_s = 30$  and  $\chi_R = (\psi_{ij}, \zeta^{ij,\alpha})_R$ .

There are three types of contributions to the total neutrino mass matrix; here these are generated by condensates arising at the TC and ETC scales:

1. left-handed Majorana,  $(I = 1, \gamma = 2)$
2. Dirac,  $(I = \frac{1}{2}, \gamma = 1)$
3. right-handed Majorana.  $(I = 0, \gamma = 0)$

The LH Majorana mass terms, which violate total lepton number  $L$  as  $\Delta L = 2$  (and violate the EW gauge symmetry) have the form

$$\sum_{i,j=1}^{N_{ETC}} [\bar{n}_{iL}^T C (M_L)_{ij} n_{jL}] + h.c.$$

where  $n_L = (\{\nu_\ell\}, \{N\})_L$  and  $C = i\gamma_2\gamma_0$ .

Dirac mass terms (which also violate EW symmetry):

$$\sum_{i=1}^{N_{ETC}} \sum_{s=1}^{n_s} \bar{n}_{iL} (M_D)_{is} \chi_{sR} + h.c.$$

Majorana mass terms with SM-singlet (RH) neutrinos:

$$\sum_{s,s'=1}^{n_s} \chi_{sR}^T C (M_R)_{ss'} \chi_{s'R} ,$$

Entries in  $M_L$ ,  $M_D$ , and  $M_R$  that would violate TC are zero since TC is exact; e.g.,  $(M_L)_{ij} = 0$  for  $4 \leq i, j \leq 5$ .

Full neutrino mass term:

$$-\mathcal{L}_m = \frac{1}{2} (\bar{\mathbf{n}}_L \bar{\chi}_L^c) \begin{pmatrix} M_L & M_D \\ (M_D)^T & M_R \end{pmatrix} \begin{pmatrix} n_R^c \\ \chi_R \end{pmatrix} + h.c.$$

The diagonalization of the full  $(N_{ETC} + n_s) \times (N_{ETC} + n_s)$  dimensional neutrino mass matrix yields the neutrino masses and mass eigenstates; combining the corresponding unitary transformation with the unitary transformation diagonalizing the charged lepton mass matrix then yields the observed lepton mixing matrix.

## Specific ETC Models

Our first model has the gauge group

$$G = SU(5)_{ETC} \times SU(2)_{HC} \times \overbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}^{G_{SM}}$$

In addition to <sup>(extended)</sup> technicolor, this has another strongly coupled interaction, hypercolor (HC), which helps to produce the desired symmetry breaking pattern.

The fermions transform according to the representations

$$\begin{aligned} (5, 1, 3, 2)_{1/3, L}, & \quad (5, 1, 3, 1)_{4/3, R}, \quad (5, 1, 3, 1)_{-2/3, R} \\ (5, 1, 1, 2)_{-1, L}, & \quad (5, 1, 1, 1)_{-2, R}, \\ (\overline{10}, 1, 1, 1)_{0, R} = \psi_{ij, R}, & \quad (10, 2, 1, 1)_{0, R} = \zeta_R^{ij, \alpha} \end{aligned}$$

where  $1 \leq i, j \leq 5$  are  $SU(5)_{ETC}$  indices and  $\alpha = 1, 2$  is a  $SU(2)_{HC}$  index. Line 1 contains quarks and techniquarks; line 2 contains LH leptons, technileptons and RH charged leptons and technileptons; line 3 contains SM-singlet fields with quantum numbers of (electroweak-singlet) neutrinos, taken by convention to be right-handed. Explicitly,

$$\begin{aligned} (5, 1, 3, 2)_{1/3, L} &= \overbrace{\begin{pmatrix} u^{1,a} & u^{2,a} & u^{3,a} & u^{4,a} & u^{5,a} \\ d^{1,a} & d^{2,a} & d^{3,a} & d^{4,a} & d^{5,a} \end{pmatrix}}^{N_{g_{TC}} = 3} \overbrace{L}^{N_{TC} = 2} \\ &= \begin{pmatrix} u^a & c^a & t^a & U^{4,a} & U^{5,a} \\ d^a & s^a & b^a & D^{4,a} & D^{5,a} \end{pmatrix}_L \end{aligned}$$

$$\begin{aligned} N_{g_{SM}} + N_{TC} &= N_{ETC} \\ 3 + 2 &= 5 \end{aligned}$$

$$\begin{aligned}
(5, 1, 1, 2)_{-1,L} &= \begin{pmatrix} n^1, & n^2, & n^3, & n^4, & n^5 \\ \ell^1, & \ell^2, & \ell^3, & \ell^4, & \ell^5 \end{pmatrix}_L \\
&= \begin{pmatrix} \nu_e, & \nu_\mu, & \nu_\tau, & N^4, & N^5 \\ e, & \mu, & \tau, & E^4, & E^5 \end{pmatrix}_L
\end{aligned}$$

$$(5, 1, 3, 1)_{4/3,R} = (u^{1,a}, u^{2,a}, u^{3,a}, u^{4,a}, u^{5,a})_R = (u^a, c^a, t^a, U^{4,a}, U^{5,a})_R$$

$$(5, 1, 3, 1)_{-2/3,R} = (d^{1,a}, d^{2,a}, d^{3,a}, d^{4,a}, d^{5,a})_R = (d^a, s^a, b^a, D^{4,a}, D^{5,a})_R$$

$$(5, 1, 1, 1)_{-2,R} = (\ell^1, \ell^2, \ell^3, \ell^4, \ell^5)_R = (e, \mu, \tau, E^4, E^5)_R$$

where  $a = 1, 2, 3$  is color index, and  $1 \leq i \leq 5$  is the ETC index, with  $i = 1, 2, 3$  indexing the three generations of technisinglet fermions and  $i = 4, 5$  indexing the technidoublet fermions.

The  $\psi_{ij,R}$  and  $\zeta_R^{ij,\alpha}$  are antisymmetric rank-2 tensor representations of  $SU(5)_{ETC}$ . Assign lepton number  $L = 1$  to  $\psi_{ij,R}$  so that, as usual, the Dirac terms  $\bar{n}_{i,L}\psi_{jk,R}$  conserve  $L$ . ( $L$  left arbitrary for  $\zeta_R^{ij,\alpha}$  since it has no Dirac mass terms with EW-doublet neutrinos.)

The  $SU(5)_{ETC}$  group thus has vectorial couplings to (techni)quarks and charged (techni)leptons, but the SM-singlet fermion (i.e., electroweak-singlet neutrino) content makes the full  $SU(5)_{ETC}$  a chiral gauge theory.

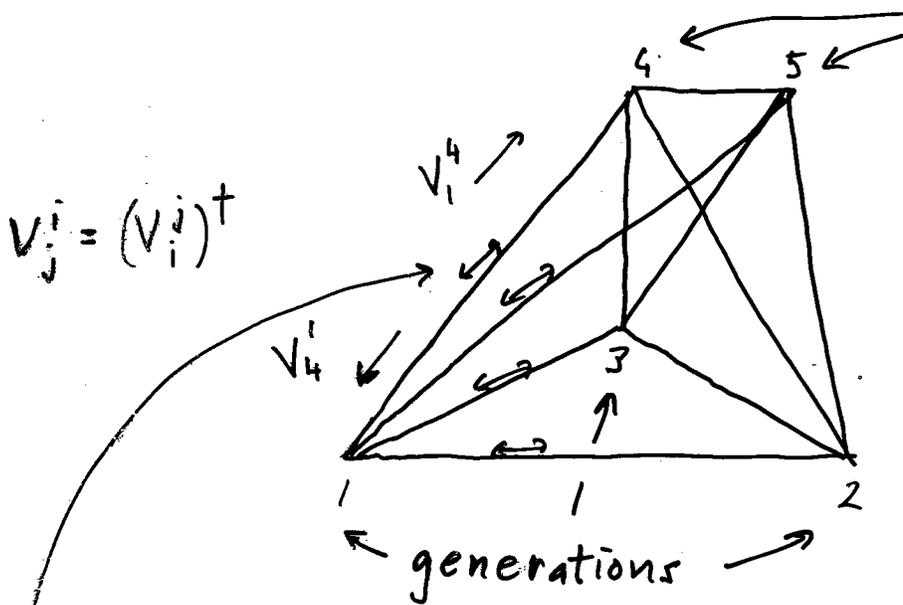
There are no bilinear fermion operators invariant under  $G$  and hence there are no bare fermion mass terms.

Each of the nonabelian factor groups in  $G$  is asymptotically free and  $G$  has no gauge or global anomalies.

The  $SU(2)_{HC}$  and  $SU(2)_{TC}$  subsectors of  $SU(5)_{ETC}$  are vectorial.

This model has some features in common with a previous  $SU(5)$  ETC model (Appelquist-Terning, 1994), but has different total gauge group and fermion content.

$SU(5)_{ETC}$  structure :



$SU(2)_{TC}$  residual unbroken subgroup of  $SU(5)_{ETC}$

9 ETC gauge bosons in coset  $SU(5)/SU(4)$  get masses  $\sim \Lambda_1$  (4 shift operators, their adjoints, and a diagonal op. in Cartan subalgebra,  $T_{24}$ )

etc. for subsequent stages of ETC symmetry breaking

In addition to

$$G = \text{SU}(5)_{ETC} \times \text{SU}(2)_{HC} \times G_{SM}$$

we shall consider two other gauge groups involving strong-electroweak gauge symmetries extended beyond those of the SM:

$$G = \text{SU}(5)_{ETC} \times \text{SU}(2)_{HC} \times G_{LR}$$

where

$$G_{LR} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$$

and

$$G = \text{SU}(5)_{ETC} \times \text{SU}(2)_{HC} \times G_{422}$$

where

$$G_{422} = \text{SU}(4)_{PS} \times \text{SU}(2)_L \times \text{SU}(2)_R$$

Here  $\text{SU}(4)_{PS}$  is the Pati-Salam group, which combines  $\text{U}(1)_{B-L}$  and color  $\text{SU}(3)$  in a maximal subgroup of  $\text{SU}(4)_{PS}$ .

We proceed to discuss our proposal for explaining neutrino masses in theories with dynamical symmetry breaking using the first of these models, with  $G_{SM}$ ; we then go on to consider the other two with extended strong-electroweak gauge groups.

## Dynamical Symmetry Breaking

Next consider the dynamical symmetry breaking in this model. To identify plausible channels for formation of bilinear fermion condensates, we use a generalized most attractive channel (GMAC) analysis that takes account of the attractiveness due to each strong gauge interaction and the cost incurred in producing vector boson masses when gauge symmetries are broken.

Recall use of GMAC (in the special case of vacuum alignment in the presence of additional perturbative gauge interactions) explains why TC produces  $\langle \bar{F}F \rangle$  condensates, where  $F = U^a, D^a, E, N$ , but not, e.g.,  $\langle \bar{U}_a E \rangle$ ,  $\langle \bar{D}_a E \rangle$ ,  $\langle \bar{U}_a N \rangle$ ,  $\langle \bar{D}_a N \rangle$ , which would break color and electric charge and give masses to gluons and the photon.

An approximate measure of the attractiveness of a condensation channel  $R_1 \times R_2 \rightarrow R_{cond.}$  is

*e.g. for  $\langle \bar{q} q \rangle$  in QCD*

$$C_2 = 4/3; \quad \Delta C_2 = \frac{8}{3}$$

$$\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_{cond.})$$

where  $R_j$  denotes the fermion representation under a relevant gauge interaction and  $C_2(R)$  is the quadratic Casimir,

$$\sum_{a=1}^{o(G)} [D_R(T_a)]_j^i [D_R(T_a)]_k^j = C_2(R) \delta_k^i$$

with  $C_2(R) = C_2(R^*)$ . For  $SU(N)$ ,  $C_2(\psi^i) = (N^2 - 1)/(2N)$ ,

$$C_2(\psi^{[ij]}) = \frac{(N-2)(N+1)}{N} \quad \text{etc.}$$

We envision that as the energy  $E$  decreases from high values,  $\alpha_{ETC}$  and  $\alpha_{HC}$  get large; at  $E \sim \Lambda_1 \sim 10^3$  TeV,  $\alpha_{ETC}$  is large enough to produce the condensation

$$(\overline{10}, 1, 1, 1)_{0,R} \times (\overline{10}, 1, 1, 1)_{0,R} \rightarrow (5, 1, 1, 1)_0$$

with  $\Delta C_2 = 24/5$ , breaking  $SU(5)_{ETC} \rightarrow SU(4)_{ETC}$ .

With no loss of generality, take breaking direction in  $SU(5)_{ETC}$  as  $i = 1$ ; this entails the separation of the first generation from the ETC fermions with  $2 \leq i \leq 5$ .

With respect to the unbroken  $SU(4)_{ETC}$ , we have the decomposition

$$(\overline{10}, 1, 1, 1)_{0,R} = (\overline{4}, 1, 1, 1)_{0,R} + (\overline{6}, 1, 1, 1)_{0,R}$$

Denote

$$\begin{aligned} (\overline{4}, 1, 1, 1)_{0,R} &= \psi_{1i,R} \equiv \alpha_{1iR} \\ (\overline{6}, 1, 1, 1)_{0,R} &= \psi_{ij,R} \equiv \xi_{ij,R}, \quad 2 \leq i, j \leq 5 \end{aligned}$$

The associated  $SU(5)_{ETC}$ -breaking,  $SU(4)_{ETC}$ -invariant condensate is

$$\langle \epsilon^{ijkl} \xi_{ij,R}^T C \xi_{kl,R} \rangle = 8 \langle \xi_{23,R}^T C \xi_{45,R} - \xi_{24,R}^T C \xi_{35,R} + \xi_{25,R}^T C \xi_{34,R} \rangle$$

*The six fields  $\xi_{ij,R}$ ,  $2 \leq i, j \leq 5$  gain dynamical masses  $\sim \Lambda_1$ .*

With our lepton number assignments, this and the resultant dynamical Majorana mass terms for  $\xi$  violate  $L$  as  $|\Delta L| = 2$ .

At lower scales, depending on relative strengths of gauge couplings, different symmetry-breaking sequences can occur. We have studied two such sequences and discuss one here (denoted  $G_b$  in the PLB):

As  $E$  decreases to  $\Lambda_{BHC} \lesssim \Lambda_1$  (BHC = broken HC), the  $SU(4)_{ETC}$  interaction produces a condensation

$$(6, 2, 1, 1)_{0,R} \times (6, 2, 1, 1)_{0,R} \rightarrow (1, 3, 1, 1)_0$$

With respect to ETC, this channel has  $\Delta C_2 = 5$ ; it occurs at a lower scale than  $\Lambda_1$  because it is repulsive with respect to HC ( $\Delta C_2 = -1/4$ ). The condensate is given by

$$\langle \epsilon_{ijkl} \zeta_R^{ij,1} {}^T C \zeta_R^{kl,2} \rangle + (1 \leftrightarrow 2) .$$

This is an adjoint rep. of hypercolor and breaks  $SU(2)_{HC} \rightarrow U(1)_{HC}$ . Let  $\alpha = 1, 2$  correspond to  $Q_{HC} = \pm 1$  under the  $U(1)_{HC}$ . The twelve  $\zeta_R^{ij,\alpha}$  fields involved gain dynamical masses  $\sim \Lambda_{BHC}$ .  $\leftarrow 2 \leq i, j \leq 5$

At the lower scale,  $\Lambda_{23}$ , the  $SU(4)_{ETC}$  and  $U(1)_{HC}$  interactions produce the condensation  $4 \times 4 \rightarrow 6$  with  $\Delta C_2 = 5/4$  and condensate

$$\langle \epsilon_{\alpha\beta} \zeta_R^{12,\alpha} {}^T C \zeta_R^{13,\beta} \rangle \quad SU(2)_{ETC} \equiv SU(2)_{TC}$$

which breaks  $SU(4)_{ETC} \rightarrow SU(2)_{ETC}$  and is  $U(1)_{HC}$ -invariant. We take  $\Lambda_{23} \sim 10$  TeV. The  $U(1)_{HC}$  interaction does not couple directly to SM particles. *Consider other condensates also.*

Finally, at  $E \sim \Lambda_{TC}$ , technifermion condensation occurs, breaking  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ .

## Calculations and Results

The full neutrino mass matrix  $M$  is  $35 \times 35$  dimensional, since  $N_{ETC} = 5$  and the number of electroweak-singlet neutrinos is  $n_s = 30$ . Because the hypercolored fields do not form bilinear condensates and resultant mass terms with hypercolor singlets,  $M$  is block-diagonal,

$$\chi = \left( \underbrace{\alpha_{ijR}}_4, \underbrace{F_{ijR}}_6, \underbrace{\xi_{kl,\alpha}}_{20} \right) \quad M = \begin{pmatrix} M_{HCS} & 0 \\ 0 & M_{HC} \end{pmatrix}$$

where the  $15 \times 15$  block  $M_{HCS}$  involves hypercolor-singlet neutrinos and the  $20 \times 20$  block  $M_{HC}$  involves the hypercolored fermions.

The nonzero entries of  $M$  arise in two different ways: (i) directly, as dynamical masses associated with various condensates, and (ii) via loop diagrams involving dynamical mass insertions on internal fermion lines and, in most cases, also mixings among ETC gauge bosons on internal lines.

$M_{HC}$  involves dynamical masses for the  $\zeta_R^{ij,\alpha}$  resulting from HC condensates.

$M_{HCS}$  is defined by the operator product

$$-\mathcal{L}_{HCS} = \frac{1}{2} (\bar{n}_L, \bar{\alpha}_L^c, \bar{\xi}_L^c) M_{HCS} \begin{pmatrix} n_R^c \\ \alpha_R \\ \xi_R \end{pmatrix} + h.c.$$

so that

$$M_{HCS} = \begin{pmatrix} \overbrace{M_L}^5 & \overbrace{(M_D)_{\bar{n}\alpha}}^4 & \overbrace{(M_D)_{\bar{n}\xi}}^6 \\ \hline \underbrace{(M_D)_{\bar{n}\alpha}^T} & \underbrace{(M_R)_{\alpha\alpha}} & \underbrace{(M_R)_{\alpha\xi}} \\ \underbrace{(M_D)_{\bar{n}\xi}^T} & \underbrace{(M_R)_{\alpha\xi}^T} & \underbrace{(M_R)_{\xi\xi}} \end{pmatrix}$$

The states  $\alpha_{1j,R}$  with  $j = 2, 3$  play the role of the RH EW-singlet neutrinos that get induced Dirac neutrino mass terms connecting with  $(n^1, n^2, n^3)_L = (\nu_e, \nu_\mu, \nu_\tau)_L$ . Because these  $\alpha_{1j,R}$  transform as part of a  $\bar{4}$  rather than a 4 of  $SU(4)_{ETC}$ , the resultant masses cannot be generated by the usual one-loop ETC graph that produces quark and charged lepton masses and are strongly suppressed, as in AT94. (similar to Sikivie, Shifman, Voloshin, Zakharov mechanism for  $m_D$  suppression)  
 The Dirac submatrix  $(M_D)_{\bar{n}\alpha}$  is defined by the operator product

$$\bar{n}_{i,L} [(M_D)_{\bar{n}\alpha}]_{ij} \alpha_{1j,R}$$

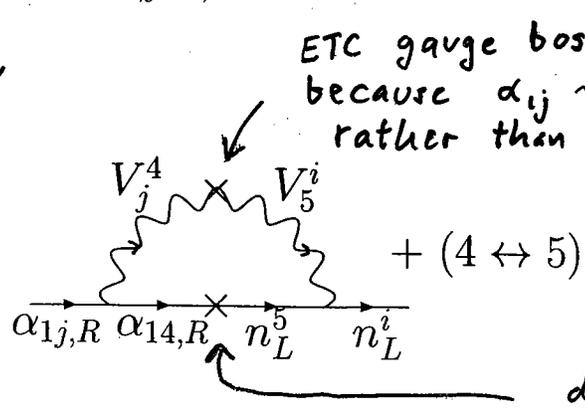
and has the form

$$(M_D)_{\bar{n}\alpha} = \begin{array}{c} \bar{n}_{1L} \\ \bar{n}_{2L} \\ \bar{n}_{3L} \\ \bar{n}_{4L} \\ \bar{n}_{5L} \end{array} \begin{array}{c|cc} \alpha_{12R} & \alpha_{13R} & \alpha_{14R} & \alpha_{15R} \\ \hline b_{12} & b_{13} & 0 & 0 \\ b_{22} & b_{23} & 0 & 0 \\ b_{32} & b_{33} & 0 & 0 \\ \hline 0 & 0 & 0 & c_1 \\ 0 & 0 & -c_1 & 0 \end{array}$$

where the 0's are exact and due to exact TC gauge invariance.

The entry  $c_1$  has magnitude  $|c_1| \sim \Lambda_{TC}$  and represents a TC dynamical mass term  $\sum_{i,j=4,5} \epsilon^{ij} \bar{n}_{i,L} \alpha_{1j,R}$ .

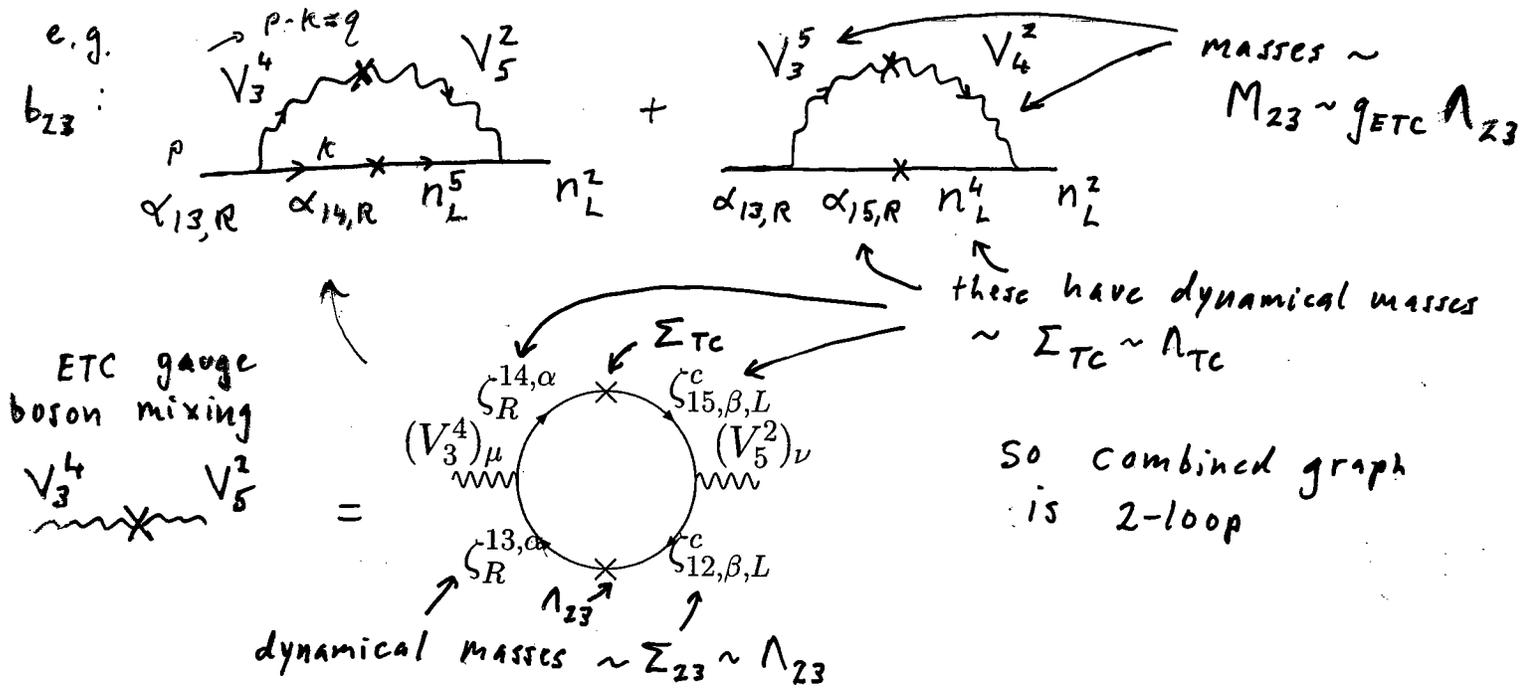
$b_{ij}$ 's produced by lowest-order graphs  $\rightarrow$



Graphs generating  $\bar{n}_{i,L} b_{ij} \alpha_{1j,R}$  for  $i = 1, 2, 3$  and  $j = 2, 3$

NB: The  $\epsilon^{ij}$  TC contraction contrasts with the usual  $\delta_i^j$  TC contraction in  $\langle FF \rangle$  and relies on use of  $SU(2)$  for TC group

dynamical TC mass  $\sum_{TC} (k) \sim c_i$  corresponding to TC condensate  $\langle \sum_{i,j=4,5} \epsilon^{ij} \bar{n}_{iL} \alpha_{1jR} \rangle$



One-loop graph contributing to the gauge boson mixing  $V_3^4 \leftrightarrow V_5^2$ .

We show graphs that contribute to the  $b_{ij}$ 's. The necessary ETC gauge boson mixings occur to leading (one-loop) order for  $b_{23}$  and  $b_{32}$ , which involve  $V_3^4 \leftrightarrow V_5^2$  and  $V_3^5 \leftrightarrow V_4^2$ . The one-loop graph that produces this ETC gauge boson mixing is shown above. Other  $b_{ij}$ 's can be produced by higher-loop diagrams.

We next estimate the leading  $b_{ij}$  entries. Denote the ETC gauge boson 2-point function as

$${}^k\Pi_j^i(q)_{\mu\lambda} = \int \frac{d^4x}{(2\pi)^4} e^{iq \cdot x} \langle T [(V_n^k)_\mu(x/2)(V_j^i)_\lambda(-x/2)] \rangle_0.$$

The above graph yields, after Wick rotation, with  $q = p - k$

$$g_{ETC}^2 [\bar{n}_{i,L}(p) \gamma_\mu \gamma_\lambda \alpha_{j,R}(p)] \int \frac{d^4k}{(2\pi)^4} \frac{k^2 \Sigma_{TC}(k) [{}^i\Pi_j^4((p-k)^2)]^{\mu\lambda}}{(k^2 + \Sigma_{TC}(k)^2)^2 [(p-k)^2 + M_j^2][(p-k)^2 + M_i^2]}$$

where  $\Sigma_{TC}(k)$  is the dynamical technicolor mass associated with the transition  $\alpha_{14,R} \rightarrow n_L^5$ , behaving as

here  $M_i, M_j \sim M_{23}$

$$\Sigma_{TC}(k) \sim \Lambda_{TC} \quad \text{for } k^2 \ll \Lambda_{TC}^2$$

and, for walking TC,

$$\Sigma_{TC}(k) \sim \frac{\Lambda_{TC}^2}{k} \quad \text{for } k^2 \gg \Lambda_{TC}^2$$

We need  ${}^k_n\Pi_j^i((p-k)^2)_{\mu\lambda}$  only for  $(p-k)^2/\Lambda_1^2 \ll 1$ , since the loop momenta in the graph are cut off far below  $\Lambda_1$ . Here we estimate (at  $\Lambda_{23}$ , because of the  $\Sigma_{23}$  mass in the ETC gauge boson mixing loop)

$$[{}^2_5\Pi_3^4(q)]_{\mu\lambda} \sim [{}^2_4\Pi_3^5(q)]_{\mu\lambda} \sim \frac{g_{ETC}^2 \Lambda_{TC}^2}{(2\pi^2)} g_{\mu\lambda}$$

For  $i, j = 2, 3$  and  $3, 2$ , adding the other graph with  $4 \leftrightarrow 5$ , we get

$$|b_{23}| = |b_{32}| \sim \frac{g_{ETC}^4 \Lambda_{TC}^4 \Lambda_{23}}{2\pi^4 M_{23}^4} \sim \frac{\Lambda_{TC}^4}{2\pi^4 \Lambda_{23}^3}$$

Numerically,  $|b_{23}| = |b_{32}| \sim O(1)$  KeV, with other  $b_{ij}$ 's generated at smaller levels. (These are rough estimates; owing to the strongly coupled nature of the ETC and TC theories.)

This shows the heavy suppression of Dirac neutrino masses, by a factor of  $\sim 10^5 - 10^6$  relative to usual 2nd and 3rd generation lepton masses. Although specific results for the various  $b_{ij}$  are dependent on the symmetry breaking pattern, this suppression is a general feature of this type of ETC model.

The other Dirac submatrix,  $(M_D)_{\bar{n}\xi}$ , associated with the operator product

$$\bar{n}_{i,L} [(M_D)_{\bar{n}\xi}]_{i,kn} \xi_{kn,R}$$

can be analyzed in a similar manner; it does not play as important a role as  $(M_D)_{\bar{n}\alpha}$ .

In  $(M_R)_{HCS}$  the  $6 \times 6$  submatrix  $(M_R)_{\xi\xi}$  contains entries  $\sim \Lambda_1$  from the highest-scale condensation. The  $4 \times 4$  submatrix  $(M_R)_{\alpha\alpha}$  associated with the operator product

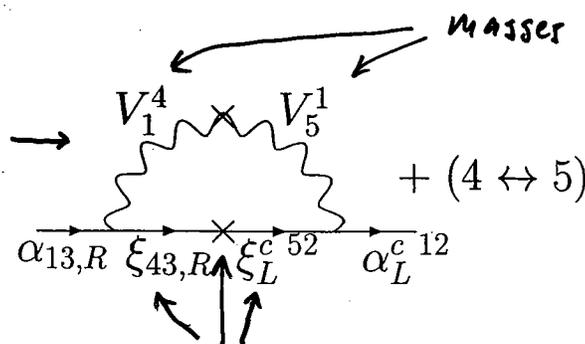
$$\bar{\alpha}_{1i,L}^c r_{ij} \alpha_{1j,R} = \alpha_{1i,R}^T C r_{ij} \alpha_{1j,R} .$$

has the form

$$(M_R)_{\alpha\alpha} = \begin{array}{c} \alpha_{12} \quad \alpha_{13} \quad \alpha_{14} \quad \alpha_{15} \\ \left( \begin{array}{cc|cc} r_{22} & r_{23} & 0 & 0 \\ r_{23} & r_{33} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

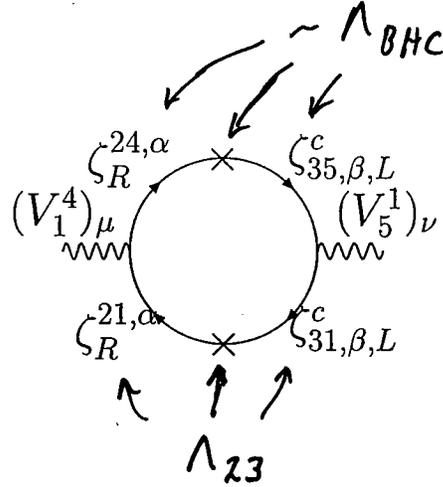
where again the 0's are exact and follow from TC invariance. If the  $2 \times 2$   $r_{ij}$  submatrix has maximal rank, this can provide a seesaw which, in conjunction with the suppression of the Dirac entries  $b_{ij}$  discussed above, can yield adequate suppression of neutrino masses. The submatrix  $r_{ij}$ ,  $2 \leq i, j \leq 3$ , produces this seesaw because  $\alpha_{12,R}$  and  $\alpha_{13,R}$  are the electroweak-singlet techni-singlet neutrinos that remain as part of the low-energy effective field theory at and below the electroweak scale.

This again requires ETC gauge boson mixing



Graphs for  $\alpha_{12,R}^T C r_{23} \alpha_{13,R}$ .

dynamical mass  $\Sigma_1 \sim \Lambda_1$  corresponding to  $\langle \epsilon^{ijkl} \xi_{ij,R}^T C \xi_{kl,R} \rangle$  condensate



again, combined  
graph for  $r_{23}$   
is 2-loop

One-loop graph for the ETC gauge boson mixing  $V_1^4 \leftrightarrow V_5^1$ . The graph with indices 2 and 3 interchanged on the internal  $\zeta$  lines also contributes to  $V_1^4 \leftrightarrow V_5^1$ .

We calculate

$$r_{23} \sim \frac{\Lambda_{BHC}^2 \Lambda_{23}^2}{2\pi^4 \Lambda_1^3}$$

Numerically,  $|r_{23}| \sim O(0.1)$  GeV. The entries  $r_{22}$  and  $r_{33}$  can also be generated. The submatrix  $(M_R)_{\alpha\xi}$  can be analyzed in a similar manner but does not play as important a role. In addition to  $M_R$ , we also take account of terms generated in  $M_L$ .

Carrying out the diagonalization of  $M$ , we obtain the following results. The EW-nonsinglet neutrinos are, to very good approximation, linear combinations of three mass eigenstates with normal hierarchy and  $\nu_3$  mass

$$m_{\nu_3} \sim \frac{|b_{23}b_{32}|}{|r_{23}|} \sim \frac{\Lambda_{TC}^8 \Lambda_1^3}{2\pi^4 \Lambda_{23}^8 \Lambda_{BHC}^2}$$

Since  $|r_{23}| \gg |b_{23}|, |b_{32}|$ , this is a seesaw, but quite different from the SUSY GUT seesaw.

With the above-mentioned numerical values and  $\Lambda_{BHC} \simeq 0.3\Lambda_1$ , we find  $m_{\nu_3} \simeq 0.05$  eV, in agreement with experimental indications; atmospheric neutrino data yields  $|\Delta m_{32}^2| \sim 3 \times 10^{-3}$  eV<sup>2</sup> and with a hierarchical neutrino mass spectrum, this implies  $m_{\nu_3} = 0.05$  eV.

The model naturally yields large  $\nu_\mu - \nu_\tau$  mixing because of the leading off-diagonal structure of the  $b_{ij}$  and  $r_{ij}$  with  $ij = 23$  and  $32$ . The value of  $|\Delta m_{32}^2|$  depends on details of the model but is on the low side of the experimental range. The lightest neutrino mass,  $m(\nu_1)$ , arises from the subdominant terms in  $M_L$  and is therefore predicted to be considerably smaller than  $m(\nu_i)$ ,  $i = 2, 3$ . These are Majorana mass eigenstates.

The model also yields the following mass eigenvalues and corresponding eigenvectors for the other neutrino-like states:

- linear combinations (LC's) the  $\xi_{ij,R}$  with  $2 \leq i, j \leq 5$  get masses  $\sim \Lambda_1$
- LC's of the  $\zeta_R^{ij,\alpha}$  with  $2 \leq i, j \leq 5$  get masses  $\sim \Lambda_{BHC}$
- LC's of the  $\zeta_R^{1j,\alpha}$  with  $j = 2, 3$  get masses  $\sim \Lambda_{23}$
- LC's of the  $\zeta_R^{1j,\alpha}$  with  $j = 4, 5$  and LC's of  $n_{i,R}^c$  and  $\alpha_{1i,R}$  with  $i = 4, 5$  get masses  $\sim \Lambda_{TC}$
- LC's of  $\alpha_{1i,R}$  with  $i = 2, 3$  get masses  $\sim r_{23}$

The  $r_{ij}$  entries in  $M_R$  responsible for the seesaw are not superheavy masses and are actually much smaller than the ETC scales  $\Lambda_i$ . The resultant EW-singlet neutrinos with masses  $\sim r_{23}$  are unstable and appear to be consistent with astrophysical and cosmological

# Neutrino Masses in Models with Extended Gauge Symmetries

We have succeeded in constructing similar models explaining light neutrino masses in theories with dynamical symmetry breaking of extended strong-electroweak gauge groups that have appealing features going beyond those of the SM. The first such group is

$$G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

in which the usual fermions of each generation transform as

$$\begin{aligned} (3, 2, 1)_{1/3, L} , & \quad (3, 1, 2)_{1/3, R} \\ (1, 2, 1)_{-1, L} , & \quad (1, 1, 2)_{-1, R} \end{aligned}$$

The gauge couplings are defined via the covariant derivative

$$D_\mu = \partial_\mu - ig_3 \mathbf{T}_c \cdot \mathbf{A}_{c, \mu} - ig_{2L} \mathbf{T}_L \cdot \mathbf{A}_{L, \mu} - ig_{2R} \mathbf{T}_R \cdot \mathbf{A}_{R, \mu} - i(g_U/2)(B-L)\mathbf{U}_\mu$$

Here the electric charge is given by the elegant relation

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}$$

where  $B$  = baryon no.,  $L$  = lepton number. Given experimental limits on right-handed charged currents and an associated  $W_R$ , and on extra  $Z$ 's,  $SU(2)_R$  must be broken at a scale  $\Lambda_{LR}$  well above the electroweak scale. Similarly for  $U(1)_{B-L}$ ;

$$SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y \text{ at } \Lambda_{LR}$$

The second extended gauge group is

$$G_{422} = \text{SU}(4)_{PS} \times \text{SU}(2)_L \times \text{SU}(2)_R$$

(Pati, Salam, Mohapatra, Senjanovic..)

with the usual fermions transforming as

$$(4, 2, 1)_L, \quad (4, 1, 2)_R$$

$G_{422}$  provides a higher degree of unification since

- It unifies quarks and leptons in the  $(4, 2, 1)_L$  and  $(4, 1, 2)_R$  representations for each generation; e.g., for the first-generation, these are

$$\begin{pmatrix} u^a & \nu_e \\ d^a & e \end{pmatrix}_{L,R}$$

- It combines  $\text{U}(1)_{B-L}$  and  $\text{SU}(3)_c$  (in a maximal subgroup) in the Pati-Salam group  $\text{SU}(4)_{PS}$  and hence relates  $g_U$  and  $g_3$ . Denoting the generators of  $\text{SU}(4)_{PS}$  as  $T_{PS,i}$ ,  $1 \leq i \leq 15$ , with

$$T_{PS,15} = \frac{1}{2\sqrt{6}} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$$

and setting  $U_\mu = A_{PS,15,\mu}$ , one has  $(B - L)/2 = \sqrt{2/3}T_{PS,15}$  and hence

$$\frac{g_U^2}{g_{PS}^2} = \frac{3}{2}$$

- It quantizes electric charge:

$$Q = T_{3L} + T_{3R} + \sqrt{2/3}T_{PS,15} = T_{3L} + T_{3R} + (1/6)\text{diag}(1, 1, 1, -3)$$

The quark-lepton unification in the  $G_{422}$  theory does not lead to proton decay, but it does lead to the decays such as  $K_L \rightarrow \mu^\pm e^\mp$  and  $K^+ \rightarrow \pi^+ \mu^+ e^-$ ; consistency with experimental upper limits on these decays implies that the scale  $\Lambda_{PS}$  at which  $SU(4)_{PS}$  breaks is  $\Lambda_{PS} \gtrsim 300$  TeV.

The conventional approach to the gauge symmetry breaking in these models employs elementary Higgs fields and arranges for a hierarchy of breaking scales by making the vacuum expectation values (vev's) of the Higgs that break  $G_{LR}$  or  $G_{422}$  to  $G_{SM}$  much larger than the Higgs vev's that break  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ .

Questions:

- Can one construct theories with the  $G_{LR}$  and/or  $G_{422}$  groups that exhibit dynamical breaking of all the gauge symmetries other than  $SU(3)_c$  and  $U(1)_{em}$ ?
- If so, can these theories naturally explain why the  $SU(2)_R \times U(1)_{B-L}$  symmetry of the  $G_{LR}$  model breaks to  $U(1)_Y$ , and why the  $SU(4)_{PS} \times SU(2)_R$  part of the  $G_{422}$  model breaks to  $SU(3)_c \times U(1)_Y$ , and why the associated breaking scales  $\Lambda_{LR}$  and  $\Lambda_{PS}$  are large wrt. the electroweak scale?
- Given that one has answered these two questions affirmatively, can one explain light neutrino masses in such dynamical symmetry breaking theories?

Here we present models in which the answers to these three questions are yes.

Note that technicolor itself could not provide a mechanism for all the breaking, because it is too weak at the the scales  $\Lambda_{LR}$  or  $\Lambda_{PS}$  and because the technifermion condensate  $\langle \bar{F}F \rangle = \langle \bar{F}_L F_R \rangle + \langle \bar{F}_R F_L \rangle$  would break both  $SU(2)_L$  and  $SU(2)_R$  at the same scale (to the diagonal (vector) group  $SU(2)_V$ ). Of course, TC must be enlarged anyway (here to ETC) in order to explain quark and lepton masses.

Our models are ETC-type theories, with the breaking of  $G_{LR}$  and  $G_{422}$  to  $G_{SM}$  being driven by the same interactions that break the ETC group and generate quark and lepton masses.

*Different approach to  $G_{LR}$  - NJL 4-fermion ops. - Akhmedov, Lindner, Schnapka, Valle*

## Dynamical Symmetry Breaking of $G_{LR}$

Our DSB model for the theory with  $G_{LR}$  uses the gauge group

$$G = SU(5)_{ETC} \times SU(2)_{HC} \times G_{LR}$$

with fermion content

$$\begin{aligned} & (5, 1, 3, 2, 1)_{1/3, L}, \quad (5, 1, 3, 1, 2)_{1/3, R} \\ & (5, 1, 1, 2, 1)_{-1, L}, \quad (5, 1, 1, 1, 2)_{-1, R} \\ & (\bar{5}, 1, 1, 1, 1)_{0, R}, \quad (\overline{10}, 1, 1, 1, 1)_{0, R}, \quad (10, 2, 1, 1, 1)_{0, R} \end{aligned}$$

$$\mathcal{N}_{iR} \quad \Psi_{ij,R} = \left\{ \begin{array}{l} \alpha_{ij,R}, \xi_{ij} \\ \alpha_{i1,R} \end{array} \right\} \quad \left\{ \begin{array}{l} ij, \alpha \\ R \end{array} \right.$$

$$2 \leq i, j \leq 5$$

The  $SU(5)_{ETC}$  theory is an anomaly-free, chiral gauge theory and, like the TC and HC theories, is asymptotically free. There are no bilinear fermion operators invariant under  $G$ , and hence there are no bare fermion mass terms. The  $SU(2)_{HC}$  and  $SU(2)_{TC}$  subsectors of  $SU(5)_{ETC}$  are vectorial.

As the energy decreases from some high value, the  $SU(5)_{ETC}$  and  $SU(2)_{HC}$  couplings increase. We envision that at  $E \sim \Lambda_{LR} \gtrsim 10^3$  TeV,  $\alpha_{ETC}$  is sufficiently strong to produce condensation in the channel

$$(5, 1, 1, 1, 2)_{-1,R} \times (\bar{5}, 1, 1, 1, 1)_{0,R} \rightarrow (1, 1, 1, 1, 2)_{-1}$$

with  $\Delta C_2 = 24/5$ , breaking  $G_{LR}$  to  $SU(3)_c \times SU(2)_L \times U(1)_Y$ . The associated condensate is  $\langle n_R^i T C \mathcal{N}_{i,R} \rangle$ . The  $n_R^i$  and  $\mathcal{N}_{i,R}$  thus gain dynamical masses  $\sim \Lambda_{LR}$ .

*part of  $SU(2)_R$  doublet,  $T_{3R} = \frac{1}{2}$   
 $L=1$  so  $(B-L)$ -nonsinglet  
 so breaks  $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$   
 ( $Y=0$  for  $n_R, \mathcal{N}_R$ )*

This condensation generates masses

$$m_{W_R} = \frac{g_{2R}}{2} \Lambda_{LR} \quad m_{Z'} = \frac{g_{2u}}{2} \Lambda_{LR}$$

where  $g_{2u} \equiv \sqrt{g_{2R}^2 + g_U^2}$ , for the  $W_{R,\mu}^\pm = A_{R,\mu}^\pm$  gauge bosons and the linear combination

$$Z'_\mu = \frac{g_{2R} A_{3,R,\mu} - g_U U_\mu}{g_{2u}}$$

This leaves the orthogonal combination

$$B_\mu = \frac{g_U A_{3,R,\mu} + g_{2R} U_\mu}{g_{2u}}$$

as the weak hypercharge  $U(1)_Y$  gauge boson, which is massless at this stage. The hypercharge coupling is then

$$g' = \frac{g_{2R} g_U}{g_{2u}}$$

so that, with  $e^{-2} = g_{2L}^{-2} + (g')^{-2} = g_{2L}^{-2} + g_{2R}^{-2} + g_U^{-2}$ , the weak mixing angle is given by

$$\sin^2 \theta_W = \left[ 1 + \left( \frac{g_{2L}}{g_{2R}} \right)^2 + \left( \frac{g_{2L}}{g_U} \right)^2 \right]^{-1}$$

at the scale  $\Lambda_{LR}$ . The experimental value of  $\sin^2 \theta_W$  at  $M_Z$  can be accommodated easily.

For  $E < \Lambda_{LR}$ , the effective theory has gauge symmetry  $SU(5)_{ETC} \times SU(2)_{HC} \times G_{SM}$  and fermion content

$$\begin{aligned} & (5, 1, 3, 2)_{1/3,L} , \quad (5, 1, 3, 1)_{4/3,R} , \quad (5, 1, 3, 1)_{-2/3,R} \\ & (5, 1, 1, 2)_{-1,L} , \quad (5, 1, 1, 1)_{-2,R} \\ & (\overline{10}, 1, 1, 1)_{0,R} , \quad (10, 2, 1, 1)_{0,R} \end{aligned}$$

which is the same content as in our first ETC model. Hence, our discussion for that model can essentially be taken over and applied here.

In a model in which  $L$  is not gauged, it is a convention how one assigns the lepton number  $L$  to the SM-singlet fields. Here, since  $L$  is gauged, this assignment is not a convention;  $L = 0$  for the fields that are singlets under  $G_{LR}$  or  $G_{422}$ , since they are singlets under  $U(1)_{B-L}$  and have  $B = 0$ .

Hence, the condensate  $\langle n_R^{iT} C N_{i,R} \rangle$  and induced Dirac neutrino mass terms like  $\bar{n}_{iL} b_{ij} \alpha_{1j,R}$  violate  $L$  as  $\Delta L = 1$ , while the  $\langle \xi_R^T C \xi_R \rangle$  and  $\langle \zeta_R^T C \zeta_R \rangle$  condensates do not violate  $L$ . The resultant physical left-handed Majorana neutrino bilinears generated by the seesaw violate  $L$  as  $\Delta L = 2$  operators, as before. Because  $L$  is gauged, there is no majoron.  $m_L \nu_L^T C \nu_L$  :  $m_L \sim \frac{m_D^2}{m_R}$  model 1  $\Delta L = 0$  here  $(\Delta L = 1)^2$   $\Delta L = 0$

## Dynamical Symmetry Breaking of $G_{422}$

Our model uses the gauge group

$$G = SU(5)_{ETC} \times SU(2)_{HC} \times G_{422}$$

with fermion content

$$\begin{aligned} & (5, 1, 4, 2, 1)_L, \quad (5, 1, 4, 1, 2)_R \\ & (\bar{5}, 1, 1, 1, 1)_R, \quad (\bar{10}, 1, 1, 1, 1)_R, \quad (10, 2, 1, 1, 1)_R \\ & \quad \mathcal{N}_{i,R} \quad \quad \psi_{ij,R} \quad \quad \zeta_{ij,R}^{\alpha} \end{aligned}$$

As  $E$  decreases through the scale  $\Lambda_{PS}$ , the  $SU(5)_{ETC}$  interaction is large enough to produce condensation in the channel

$$(5, 1, 4, 1, 2)_R \times (\bar{5}, 1, 1, 1, 1)_R \rightarrow (1, 1, 4, 1, 2)$$

This breaks  $SU(4)_{PS} \times SU(2)_R$  directly to  $SU(3)_c \times U(1)_Y$ . The associated condensate is again  $\langle n_R^{iT} C \mathcal{N}_{i,R} \rangle$ , and the  $n_R^i$  and  $\mathcal{N}_{i,R}$  gain masses  $\sim \Lambda_{PS}$ . The above results for  $m_{W_R}$ ,  $m_{Z'}$ , and  $\sin^2 \theta_W$  apply with  $g_U^2/g_{PS}^2 = 3/2$  at  $\Lambda_{PS}$ .

Further breaking at lower scales proceeds as in the first ETC model and in the  $G_{LR}$  model.

The experimental value of  $\sin^2 \theta_W$  can again be accommodated. We use  $\alpha_3(m_Z) = 0.118$ ,  $\alpha_{em}(m_Z)^{-1} = 129$ , and  $(\sin^2 \theta_W)_{\overline{MS}}(m_Z) = 0.231$ . With  $\Lambda_{PS} = 10^6$  GeV, we calculate the values  $\alpha_3 = 0.064$ ,  $\alpha_{2L} = 0.032$ , and  $\alpha_1 = 0.012$  at  $\Lambda_{PS}$ , and hence to fit  $\sin^2 \theta_W$  in this model, we find  $\alpha_{2R}(\Lambda_{PS}) \simeq 0.013$  so  $g_{2R}/g_{2L} \simeq 0.64$  at  $\Lambda_{PS}$ .

Given that the effective low-energy theories in the ETC models with strong-electroweak  $G_{LR}$  and  $G_{422}$  are the same as in our first ETC model with the standard strong-electroweak group, it follows that our explanation for the generation of light neutrino masses also applies to these theories with extended gauge symmetries.

Some topics that we are investigating at present:

- The additional ingredients that are necessary to produce a sufficiently heavy top quark (e.g., top-color...)
- Sources for quark CKM mixing and further lepton mixing
- Contributions to  $S$  parameter in this type of walking theory
- Global symmetries and resultant spectrum of Nambu-Goldstone bosons
- This work provides a motivation for searching with greater sensitivity for possible neutrinos with masses  $\sim O(10^2)$  MeV emitted via lepton mixing in  $K_{\mu 2}^+$  and  $K_{e 2}^+$  decays. This can be done parasitically using present data from BNL E787/949.
- Dark matter candidate - SM-singlet technibaryon  
 $\left. \begin{array}{l} p^{14, \alpha} \\ \searrow R \end{array} \right\} \left. \begin{array}{l} p^{15, \beta} \\ \searrow R \end{array} \right\}$  (possible cusp problem, as for other possible dark matter candidates like LSP in SUSY)  
 $\alpha, \beta = (1, 2)$

## Conclusions

We have addressed the question of what one infers about physics beyond the standard model from the discovery of neutrino masses. As an alternative to the usual SUSY GUT seesaw, we have explored a different approach based on theories with dynamical electroweak symmetry breaking. We have shown that it is possible to get light neutrino masses in this class of theories, thereby providing a plausible answer to a longstanding question of whether this could be done.

Our mechanism uses a seesaw but one with strong suppression of both Dirac and Majorana mass entries, which does not involve any superheavy mass scales such as GUT scales. We have shown the robustness of the mechanism by demonstrating that it works with strong-EW groups  $G_{SM}$ ,  $G_{LR}$ , and  $G_{422}$ .

The model still has a number of phenomenological challenges characteristic of dynamical EWSB theories, but we believe that it motivates further work with this approach.