The Muon $(g-2)$, $B^0 - \overline{B^0}$ Mixing and Rare $B_{(d,s)}$ Decays in SUSY Models

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S. Baek, and W.Y Song, hep-ph/0205259, 0208112

- SUSY Flavor Problem

  (Constraints on $(\delta^d_{13})_{AB}$ from $B^0 - \overline{B^0}$ mixing and $B \to X_d\gamma$)

- a_μ (basic review, what in the decoupling scenario)

- $B \to X_s\gamma$, $B \to X_s l^+ l^-$ and $B_s \to \mu^+\mu$

- SUSY breaking mediation mechanisms

- Summary

(Talk at the ANL Theory Institute 2002, Sep. 9-13, 2002)

SUSY Flavor Problem

- In the interaction basis, sfermion mass matrix is not diagonal in general

  $\rightarrow$ Generically Large FCNC,

$$K^0 - \overline{K^0} \text{ mixing} : \frac{A^2}{m_{\tilde{Q}}^2} \left(\frac{\delta m_{\tilde{Q}}^2}{m_{\tilde{Q}}^2} \right)^2 \leq 5 \times 10^{-9} \text{ GeV}^{-2}$$

  $*$ UNLESS Sfermions with the same electric charges are almost degenerate

  (i.e., $\delta m_{\tilde{Q}}^2 = 0$), OR

  $*$ $U = \tilde{U}$ (for which $A^2 = 0$), OR

  $*$ Decoupling of the first two generations of Sfermions (10–100 TeV)

  with the third generation sferion mass below ~ 1 TeV

- Possible Solutions to SUSY FCNC :

  – Universality : mSUGRA, No scale scenario, Gauge mediation, etc. $*$ True only at some messenger scale, RG running induce some flavor violations

  – Alignment : Flavor symmetry for Yukawa (SUSY) and sfermion masses (SUSY breaking) $\rightarrow$ SUSY FCNC problem is medigated, but generically larger FCNC than universality assumption

  – Decoupling : 1st/2nd generation sfermions are very heavy and almost degenerate, only the third sfermion and gauginos are light (below ~ 1 TeV)

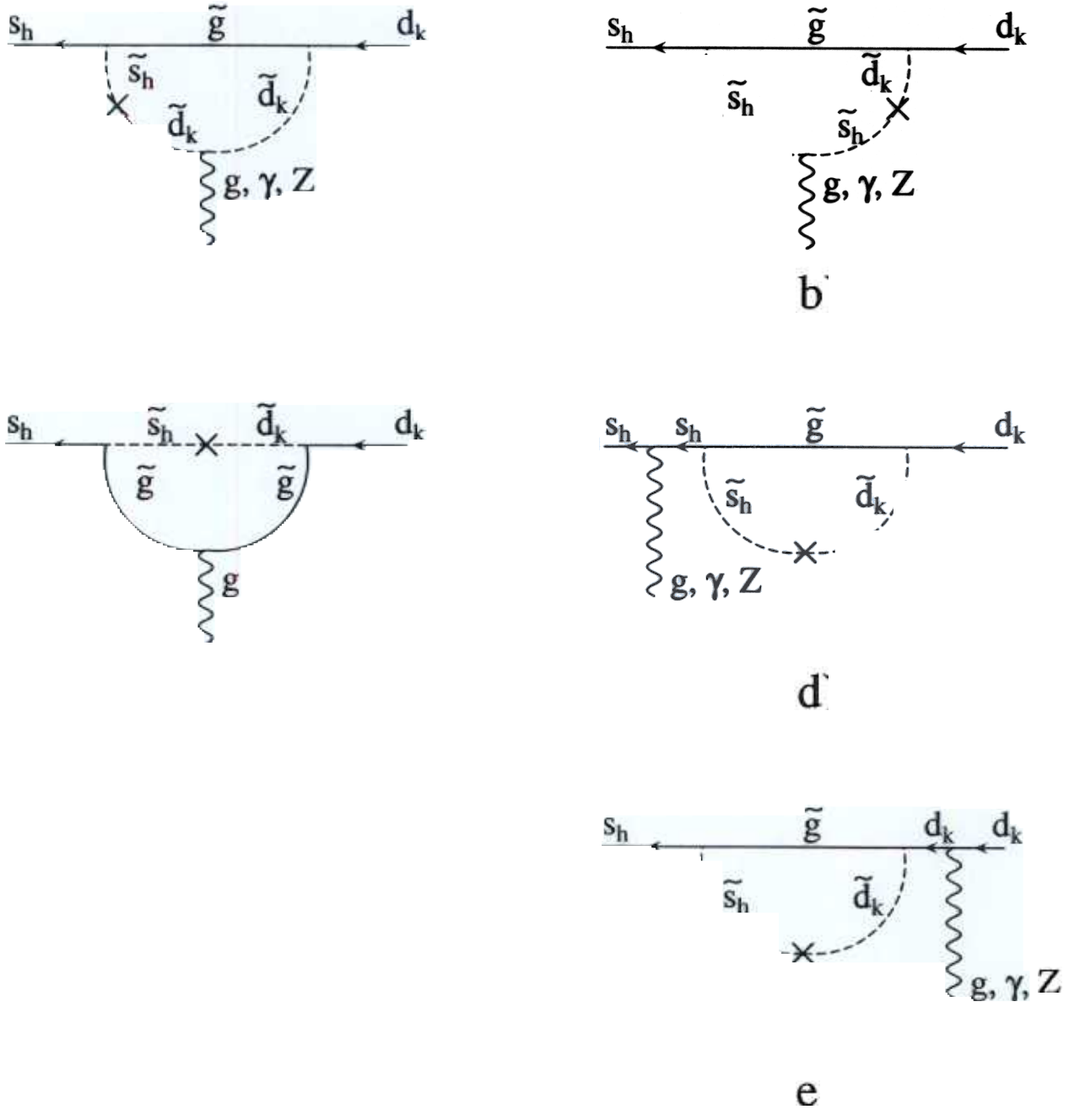


Figure 6: Penguin diagrams for ΔS transitions, with h, $L, R\}$

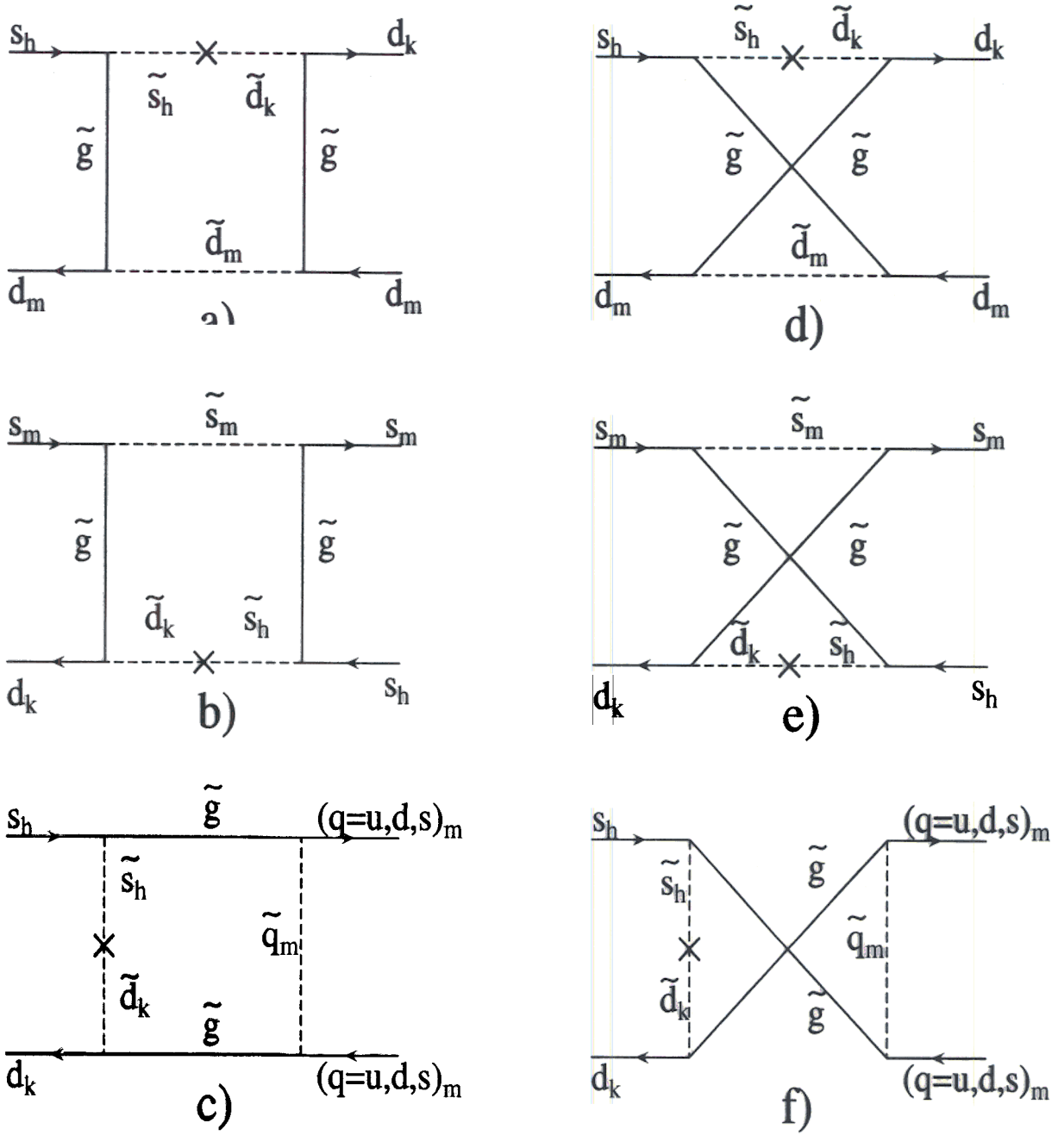


Figure 5: Box diagrams for $\Delta S = 1$ transitions, with $h, k, m = \{L, R\}$.

(From NPB paper by Gabbiani et al.)

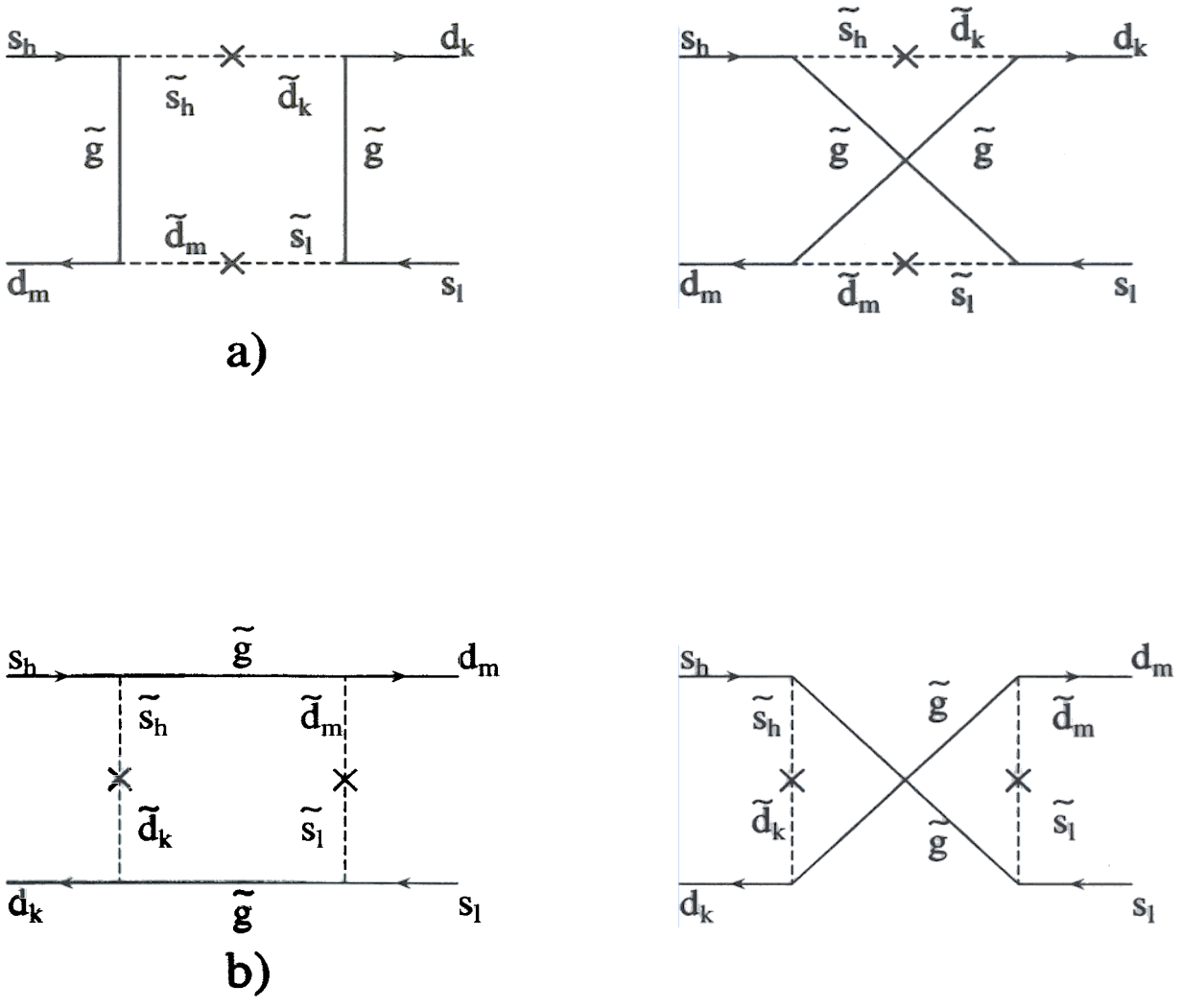


Figure 1: Feynman diagrams for $\Delta S = 2$ transitions, with $h, k, l, m = \{L, R\}$.

Constraints on $(\delta^d_{13})_{AB}$ from $B^0 - \overline{B^0}$ mixing and $B \to X_d\gamma$

(work with G. Kramer and J.H. Park, to appear in EPJC)

– $B^0 - \overline{B^0}$ mixing

The most general effective Hamiltonian for $B^0 - \overline{B^0}$ mixing ($\Delta B = 2$) can be written in the following form:

$$H_{\rm eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i,$$

where the operators Q_i's are defined as

$$\begin{aligned}
Q_1 &= \bar{d}^\alpha_L \gamma_\mu b^\alpha_L \; \bar{d}^\beta_L \gamma^\mu b^\beta_L \\
Q_2 &= \bar{d}^\alpha_R b^\alpha_L \; \bar{d}^\beta_R b^\beta_L \\
Q_3 &= \bar{d}^\alpha_R b^\beta_L \; \bar{d}^\beta_R b^\alpha_L \\
Q_4 &= \bar{d}^\alpha_R b^\alpha_L \; \bar{d}^\beta_L b^\beta_R \\
Q_5 &= \bar{d}^\alpha_R b^\beta_L \; \bar{d}^\beta_L b^\alpha_R
\end{aligned}$$

and the operators $\tilde{Q}_i$ are obtained from Q_i's by the exchange of $L \leftrightarrow R$. α, β are color indices, and $q_{L,R} \equiv (1 \mp \gamma_5)q/2$. The Wilson coefficients C_i's receive contributions from both the SM and the SUSY loops: $C_i \equiv C_i^{\rm SM} + C_i^{\rm SUSY}$

In the SM, the $t - W$ box diagram generates only contribution to the operator Q_1, and the corresponding Wilson coefficient $C_1^{\rm SM}$ at the m_t scale is given by

$$C_1^{\rm SM}(m_t) = \frac{G_F^2}{4\pi^2} M_W^2 (V^*_{td} V_{tb})^2 S_0(x_t),$$

where

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1-x_t)^3},$$

with $x_t \equiv m_t^2/m_W^2$.

The Wilson coefficients C_i's are obtained by calculating the $t - W$ in the SM and $\tilde{g} - \tilde{q}$ box diagrams in the mass insertion approximations in general SUSY models :

$$\begin{aligned}
C_1^{\rm SUSY} &= \frac{\alpha_s^2}{216\tilde{m}^2} \left(24x f_6(x) + 66\tilde{f}_6(x)\right) \left(\delta_{13}^d\right)_{LL}^2 \\
C_2^{\rm SUSY} &= \frac{\alpha_s^2}{216\tilde{m}^2} 204x f_6(x) \left(\delta_{13}^d\right)_{RL}^2 \\
C_3^{\rm SUSY} &= \frac{\alpha_s^2}{216\tilde{m}^2} 36x f_6(x) \left(\delta_{13}^d\right)_{RL}^2 \\
C_4^{\rm SUSY} &= -\frac{\alpha_s^2}{216\tilde{m}^2} \Big[\left(504x f_6(x) - 72\tilde{f}_6(x)\right) \left(\delta_{13}^d\right)_{LL} \left(\delta_{13}^d\right)_{RR} \\
&\quad -132\tilde{f}_6(x) \left(\delta_{13}^d\right)_{LR} \left(\delta_{13}^d\right)_{RL} \Big] \\
C_5^{\rm SUSY} &= -\frac{\alpha_s^2}{216\tilde{m}^2} \Big[\left(24x f_6(x) + 120\tilde{f}_6(x)\right) \left(\delta_{13}^d\right)_{LL} \left(\delta_{13}^d\right)_{RR} \\
&\quad -180\tilde{f}_6(x) \left(\delta_{13}^d\right)_{LR} \left(\delta_{13}^d\right)_{RL} \Big]
\end{aligned}$$

The loop functions $f_6(x)$ and $\tilde{f}_6(x)$, evaluated in terms of $x \equiv m_{\tilde{g}}^2/\tilde{m}^2$, are given by

$$\begin{aligned}
f_6(x) &= \frac{6(1+3x)\ln x + x^3 + 9x^2 - 9x + 17}{6(x-1)^5} \\
\tilde{f}_6(x) &= \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5}
\end{aligned}$$

These Wilson coefficients for SUSY and SM are calculated at $\mu \sim m_{\tilde{g}} \sim \tilde{m}$ and respectively, and the RG running between these two scales will be ignored.

D. Becirevic et al., hep-ph/0112303 for details)

There are also contributions from $(\delta^d_{13})^{\rm ind}_{LR}$, which is the induced LR mixing due to the double mass insertion :

$$(\delta^d_{13})^{\rm ind}_{LR} = (\delta^d_{13})_{LL} \times \frac{m_b(A_b - \mu \tan\beta)}{\tilde{m}^2}$$

We numerically checked that this contribution of $(\delta^d_{13})^{\rm ind}_{LR}$ to M_{12} is smaller than the SUSY contribution we showed above, and we do not show the explicit form here.

$$M_{12}(B^0) \equiv \frac{1}{2m_B} \langle B^0 | H_{\rm eff}^{\Delta B=2} | \overline{B^0} \rangle$$

$\Delta m_{B_d} = 2|M_{12}(B^0_d)|$

The phase of the $B^0 - \overline{B^0}$ mixing amplitude $M_{12}(B^0) \equiv \exp(2i\beta^{'})$ $M_{12}(B^0)|$ appears in the time dependent asymmetry

$$A^{\rm mix}_{\rm CP}(B^0 \to J/\psi K_s) = \sin 2\beta^{'} \ \sin \Delta m_{B_d} t. \tag{7}$$

cf. $\beta^{'} = \beta$ in the SM

Dilepton charge asymmetry : A_{ll}

Finally, the dilepton charge asymmetry A_{ll} is also determined by $M_{12}(B^0)$ albeit a possible long distance contribution to $\Gamma_{SM}(B^0)$. Define

$$|B_{1,2}\rangle \equiv \frac{1}{\sqrt{1+|\eta|^2}} \left[|B^0\rangle \pm \eta |\overline{B^0}\rangle \right]$$

with $\eta \equiv \sqrt{(M_{12}^* - i\Gamma_{12}^*)/(M_{12} - i\Gamma_{12})}$. Then

$$A_{ll} \equiv \frac{N(BB) - N(\bar{B}\bar{B})}{N(BB) + N(\bar{B}\bar{B})} = -\frac{|\eta|^4 - 1}{|\eta|^4 + 1} = \frac{\mathrm{Im}(\Gamma_{12}/M_{12})}{1 + |\Gamma_{12}/M_{12}|^2/4} \approx \mathrm{Im}(\Gamma_{12}/M_{12}).$$

In the SM, the quantity $\Gamma_{12}^{\mathrm{SM}}$ is given by (Buras, 1997)

$$\Gamma_{12}^{\mathrm{SM}} = (-1)\frac{G_F^2\, m_b^2\, M_{B_d}\, B_{B_d}\, f_{B_d}^2}{8\pi} \left[v_t^2 + \frac{8}{3} v_c\, v_t \left(z_c + \frac{1}{4} z_c^2 - \frac{1}{2} z_c^3 \right) + v_c^2 \left\{ \sqrt{1-4z_c} \left(1 - \frac{2}{3} z_c \right) + \frac{8}{3} z_c + \frac{2}{3} z_c^2 - \frac{4}{3} z_c^3 - 1 \right\} \right],$$

where $v_i \equiv V_{ib}\, V_{id}^*$ and $z_c \equiv m_c^2/m_b^2$. For $(54.8 \pm 6.2)^\circ$ and $f_{B_d} = (200 \pm 30)$ MeV, the SM prediction becomes

$$-1.54 \times 10^{-3} \leq A_{ll}^{\mathrm{SM}} \leq -0.16 \times 10^{-3},$$

The current world average of the data is

$$A_{ll}^{\mathrm{exp}} \approx (0.2 \pm 1.4) \times 10^{-2}$$

In the presence of SUSY, M_{12} could be affected strongly by SUSY particles, whereas Γ_{12} is not:

$$M_{12}^{\mathrm{FULL}} = M_{12}^{\mathrm{SM}} + M_{12}^{\mathrm{SUSY}}$$

whereas

$$\Gamma_{12}^{\mathrm{FULL}} \approx \Gamma_{12}^{\mathrm{SM}}$$

$$A_{ll} = \mathrm{Im}\left(\frac{\Gamma_{12}^{\mathrm{SM}}}{M_{12}^{\mathrm{SM}} + M_{12}^{\mathrm{SUSY}}}\right) \tag{10}$$

The possible ranges of A_{ll} in a class of general SUSY models (with alignment mechanisms and decoupling scenarios) can be studied. (Randall and Su, Ko and Park,...)

Relevant operators for $B \to X_d\gamma$ –

$$\mathcal{H}_{\rm eff}(b \to d\gamma) = -\frac{4G_F}{\sqrt{2}} V_{td}^* V_{tb} \sum_{i=2,7,8} C_i(\mu_b) Q_i(\mu_b)$$

with

$$\begin{aligned} & \bar{d}_L \gamma^\mu c_L \; \bar{c}_L \gamma_\mu b_L, \\ & \frac{e}{16\pi^2} m_b \; \bar{d}_L \sigma^{\mu\nu} F_{\mu\nu} b_R, \\ O_{8g} \;=\; & \frac{g_s}{16\pi^2} m_b \; \bar{d}_L \sigma^{\mu\nu} t^a G^a_{\mu\nu} b_R. \end{aligned} \tag{11}$$

$$\frac{B(B \to X_d\gamma)}{B(B \to X_c e\nu)} = \left| \frac{V_{td}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha}{\pi f(z)} \, |C_7(m_b)|^2.$$

$f(z)$ is the phase space factor for the $b \to c$ semilepton decays

Neglecting the RG running between heavy SUSY particles and top quark mass scale, we get the following relations :

$$\begin{aligned} C_7(m_b) \;&\approx\; -0.31 + 0.67\, C_7^{\rm new}(m_W) + 0.09\, C_8^{\rm new}(m_W) \\ C_8(m_b) \;&\approx\; -0.15 + 0.70\, C_8^{\rm new}(m_W) \end{aligned} \tag{12}$$

The new physics contributions to C_2 are negligible so that we use $C_2(m_b) = C_2^{\rm SM}(m_b) \approx 1.11$.

The direct CP asymmetry can be written as

$$A_{\rm CP}^{b\to d\gamma}(\text{in}\%) = \frac{1}{|C_7|^2}\,[10.57\ \mathrm{Im}\,(C_2C_7^*) - 9.40\ \mathrm{Im}\,((1+\epsilon_d)C_2C_7^*)$$
$$9.51\ \mathrm{Im}\,(C_8C_7^*) + 0.12\ \mathrm{Im}\,((1+\epsilon_d)C_2C_8^*)] \qquad (13)$$

where

$$\epsilon_d \equiv \frac{V_{ud}^*V_{ub}}{V_{td}^*V_{tb}} \approx \frac{(\rho - i\eta)}{(1-\rho+i\eta)}.$$

In our model, the Wilson coefficients $C_{7\gamma}$ and C_{8g} are modified in the double mass insertion approximation as follows:

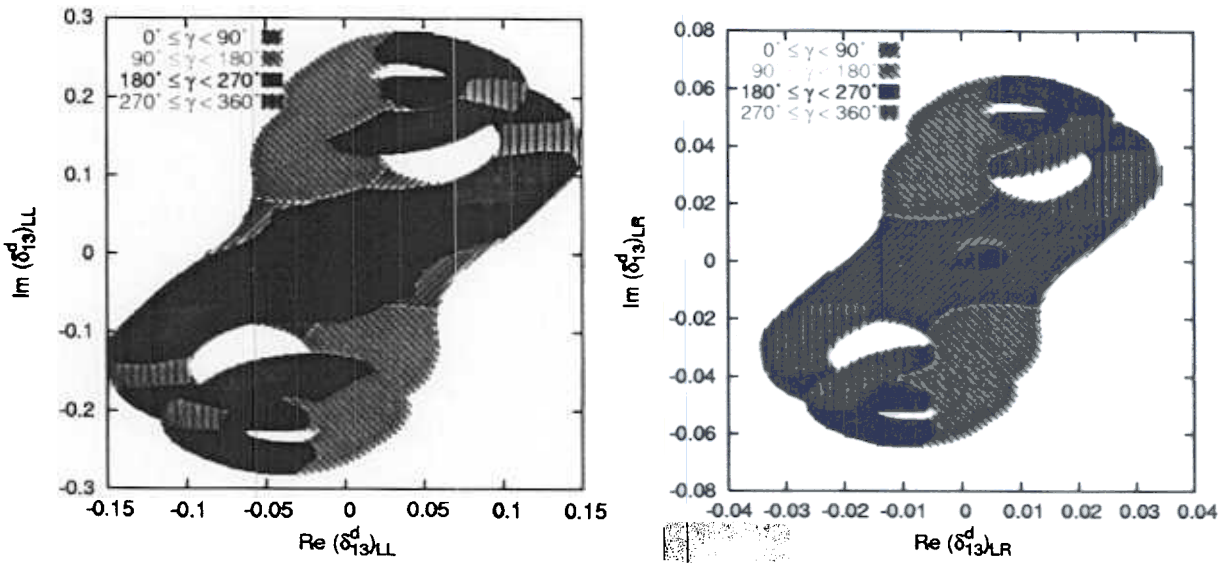
$$C_{7\gamma}^{\rm SUSY}(m_W) = \frac{8\pi Q_b\alpha_s}{3\sqrt{2}G_F\tilde{m}^2V_{td}^*V_{tb}}\Big[(\delta_{13}^d)_{LL}M_4(x)$$
$$-(\delta_{13}^d)_{LR}\left(\frac{\tilde{m}\sqrt{x}}{m_b}\right)4B_1(x) - (\delta_{13}^d)_{LR}^{\rm ind}\left(\frac{\tilde{m}\sqrt{x}}{m_b}\right)M_2(x)\Big]$$
$$C_{8g}^{\rm SUSY}(m_W) = \frac{2\pi\alpha_s}{\sqrt{2}G_F\tilde{m}^2V_{td}^*V_{tb}}\Big[(\delta_{13}^d)_{LL}\left(\frac{3}{2}M_3(x) - \frac{1}{6}M_4(x)\right)$$
$$+(\delta_{13}^d)_{LR}\left(\frac{\tilde{m}\sqrt{x}}{m_b}\right)\frac{1}{6}\left(4B_1(x) - 9x^{-1}B_2(x)\right)$$
$$-(\delta_{13}^d)_{LR}^{\rm ind}\left(\frac{\tilde{m}\sqrt{x}}{m_b}\right)\left(\frac{3}{2}M_1(x) - \frac{1}{6}M_2(x)\right)\Big]$$

- Constraints :

  - $\Delta m_{B_d} = (0.472 \pm 0.017)\ \mathrm{ps}^{-1} = 2|M_{12}(B^0)|$

    $A_{\rm CP}^{\rm mix} = 0.79 \pm 0.10 = \sin(2\beta')$, where $2\beta'$ is the argument of $M_{12}(B^0)$

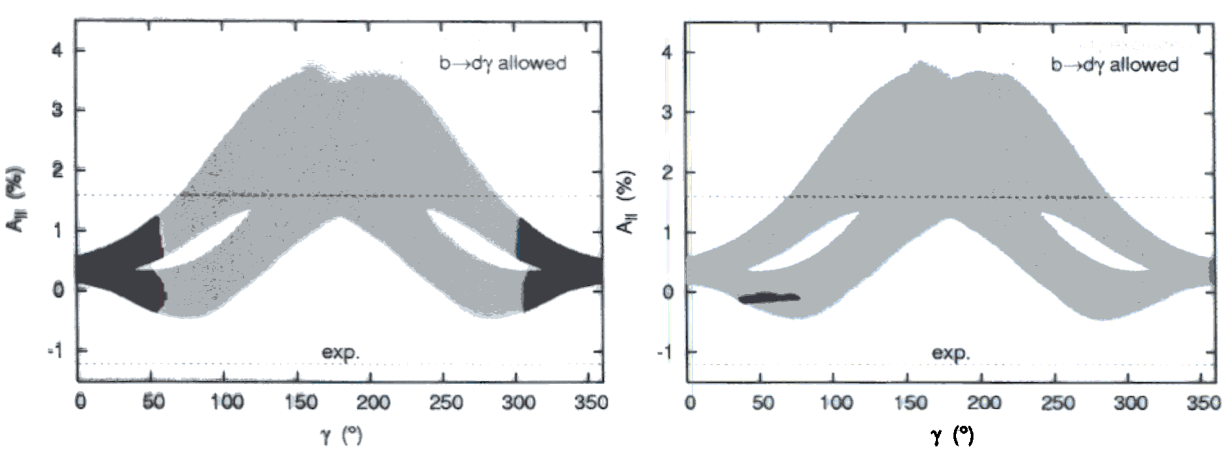
    $Br(B\to X_d\gamma) \lesssim 0.5\times 10^{-4}$ from $B(B\to\rho\gamma) < 2.3\times 10^{-6}$



(a) LL mixing only

(b) LR mixing only

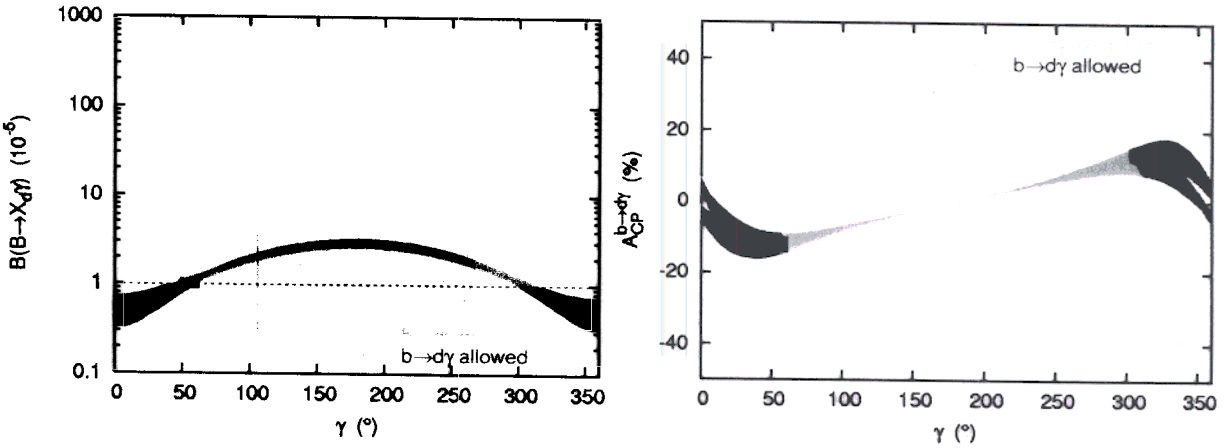
FIG. 1: The allowed ranges in (a) the LL and (b) the LR insertion cases for the parameters $(\mathrm{Re}(\delta^d_{13})_{AB}, \mathrm{Im}(\delta^d_{13})_{AB})$ for different values of the KM angle γ with different color codes: dark (red) for $0° \leq \gamma \leq 90°$, light gray (green) for $90° \leq \gamma \leq 180°$, very dark (blue) for $180° \leq \gamma \leq 270°$ and gray (magenta) for $270° \leq \gamma \leq 360°$. The region leading to a too large branching ratio for $\bar{B}_d \to X_d\gamma$ is colored lightly and covered by parallel lines.



(a) LL mixing only

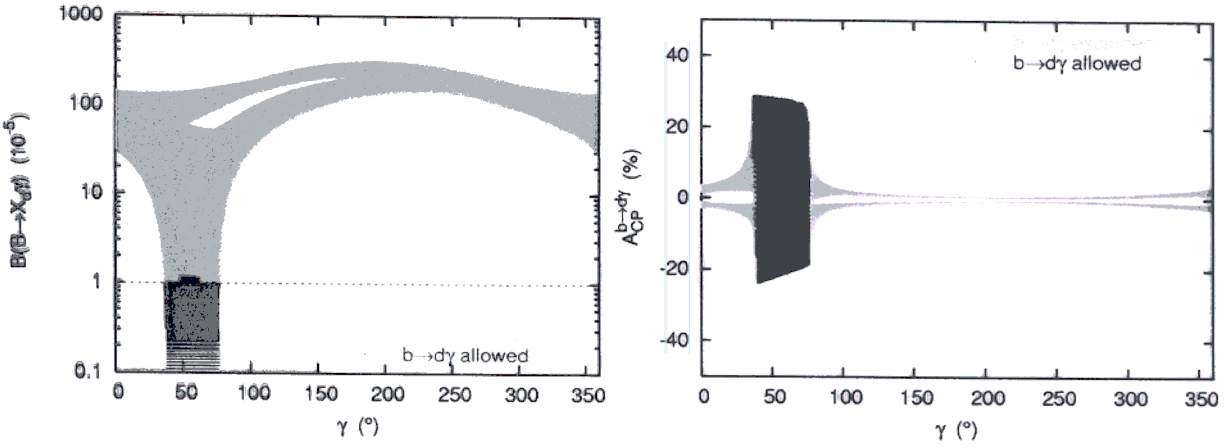
(b) LR mixing only

FIG. 2: The possible ranges of the dilepton charge asymmetry in (a) the LL and (b) the LR cases as functions of the KM angles γ. The black rectangle around $\gamma \simeq 55°$ is the SM prediction. Those parameters which lead to $B(B \to X_d\gamma) > 1 \times 10^{-5}$ are denoted by the the gray (magenta) region, and those for $B(B \to X_d\gamma) < 1 \times 10^{-5}$ by the dark (blue) region. The 1σ range for the world average of $A_{ll}^{\mathrm{exp}} = (0.2 \pm 1.4)\%$ is shown to lie between the short dashed lines.



(a) B $(B \to X_d\gamma)$ (b) $A_{\rm CP}^{b\to d\gamma}$

FIG. 3: The possible ranges of (a) $B(B_d \to X_d\gamma)$ and (b) $A_{\rm CP}^{b\to d\gamma}$ as functions of the KM angle γ in the LL insertion case. The black rectangle around $\gamma \simeq 55°$ is the SM prediction. Those parameters which lead to $B(B \to X_d\gamma) > 1 \times 10^{-5}$ are represented by the gray (magenta) region, and those for $B(B \to X_d\gamma) < 1 \times 10^{-5}$ by the dark (blue) region.



(a) B $(B \to X_d\gamma)$ (b) $A_{\rm CP}^{b\to d\gamma}$

FIG. 4: The possible ranges of (a) $B(B_d \to X_d\gamma)$ and (b) $A_{\rm CP}^{b\to d\gamma}$ as functions of the KM angle γ in the LR insertion case . The black rectangle around $\gamma \simeq 55°$ is the SM prediction. Those parameters which lead to $B(B \to X_d\gamma) > 1 \times 10^{-5}$ are represented by the gray (magenta) region, and those for $B(B \to X_d\gamma) < 1 \times 10^{-5}$ by the dark (blue) region.

SUSY in the Loop

(1) Why do we consider SUSY in the loop ?

- Flavor Conserving sector : Usually hidden under the SM contributions (which are often tree level effects)

  $\rightarrow$ SUSY effects are usually small and decouples quickly

  N.B. : Exception : the muon $(g-2)_\mu$, because of the very accurate measurement is possible

- Flavor Changing Neutral Current Processes

  – SM Contributions : already loop suppressed due to GIM mechanism

  – SUSY Contributions : also loop suppressed, but can compete with SM contributions, if SUSY particles are not too heavy

  $B \rightarrow X_s \gamma$: QCD part is under control due to HQET and Heavy Quark Expansion

  $\rightarrow$ Small theoretical uncertainties

  (Also, $B^0 - \overline{B^0}$ mixing, $B \rightarrow X_s l^+ l^-$ and $B_s \rightarrow \mu^+ \mu^-$, etc.)

- Never can be a clear evidence for SUSY.

  Only a Smoking Gun for New Physics

- Can be complementary for direct search at colliders, especially if SUSY particles are too heavy to be produced directly at colliders

(2) Muon Anomalous Magnetic Moment $(g-2)_\mu$

Magnetic Moment in QFT

- Fundamental Properties of a particle
  - Mass M
  - Spin s : Bosons and Fermions with Diff. Statistics
  - Charge Q : integral multiples of e_p
  - Magnetic Dipole Moment (MDM) μ
- MDM vs. Spin

$$\vec{\mu} = g\frac{e\hbar}{2mc}\,\vec{s}$$

- Pointlike Dirac particle : $g = 2$ (e.g., electron, muon, quark etc.)

  cf. $g = 2.79 \;\; (-1.86)$ for proton (neutron)

  $\rightarrow$ Indicate p and n are composite particles

- Quantum Corrections modifies g at loop levels.

$$a_\mu \equiv \left(\frac{g-2}{2}\right)_\mu$$

$\rightarrow$ Anomalous Magnetic Dipole Moment

Theoretically well understood

(except for the hadronic contibutions)

Measured with high precisions

Indirect Probe of New Physics Beyond the SM

SM P edictions

(A Czarnecki and W Marciano hep-ph/0102122)

- $a_\mu^{\rm SM} \quad a^{\rm QED} + a^{\rm EW} + \mu^{\rm hadronic}$

  $a^{\rm QED}$ calculated upto five loops

  $$a_\mu^{\rm QED} \quad \left(\frac{\alpha}{2\pi}\right) + 0\ 765\ 857\ 376(27)\ \left(\frac{\ }{\pi}\right) + 24.050\ 508\ 98(44)\left(\frac{\ }{\pi}\right)$$
  $$+\ 126\ 07(41\ \left(\frac{\ }{\pi}\right) + 930(170)\left(\frac{\alpha}{\pi}\right.$$

  $(a_e) \quad 137\ 035\ 999\ 58(52)$ from a_e

  cf $\quad (qH \ = \ 37\ 036\ 003\ 00(270)$

  $a_\mu^{\rm QED} \quad 16\ 584\ 705\ 7(2.9) \times 10$

- $a_\mu^{\rm EW}$ upto loops $\quad 95 \times 10 \quad 43(4) \times 10 \quad 52(4) \times 10$

  $a_\mu^{\rm Had} = a_\mu^{\rm Had}$ (vac. pol. $+ a^{\rm Had}$ (light by light)

  $\leftarrow$ Most Uncertain Theoretically

  Adding these altogether

$a_\mu^{\rm exp} \quad a_\mu^{\rm SM} \quad (33.9 \pm \ .2) \times 10 \quad$ (from $\ \ \rightarrow$ hadrons)

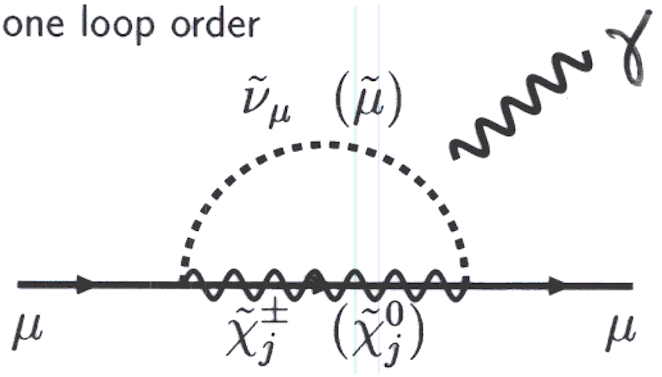
$a_\mu^{\rm exp} \quad a_\mu^{\rm SM} \quad (16 \ \pm 10\ 7) \times 10 \quad$ (from hadronic τ decays)

(Davier Eidelman, Hocker and Zhang, hep-ph/0208177)

$\rightarrow$ Just σ effect and no strong demands for New Physics

SUSY Models –

- Feynman Diagram at one loop order



- Results in general MSSM
  - Schematic Expressions for a_μ :

$$a_\mu^{\rm SUSY} = 14 \times 10^{-10} \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2 \tan\beta$$

  - Relatively light SUSY particles and Relatively large $\tan\beta$

- Minimal SUGRA, String Inspired Models (Dilaton and/or Moduli), Gauge Mediation, Anomaly Mediation, No-scale scenario etc.

- Higgs and light stau mass bounds and $B \to X_s\gamma$ put very strong additional constraints on the parameter space

- LSP – Nucleus Scattering relevant to Neutralino Dark Matter Search Experiments such as DAMA (DArk MAtter) KIMS (Korea Invisible Matter Search), etc.

- BNL data can be easily accommodated in SUSY models

Effective SUSY Models

(with S. Baek and J.H. Park, EJPC(2002) ; see also G.Cho, Haba and Hisano)

- Naive expectation : $a_\mu \approx 0$, since $m_{\tilde{\mu}} \sim O(10)$ TeV
- R parity violation will surely save the effective SUSY model
- Still some contribution from flavor changing neutralino-stau loop
- The Slepton mass matrix in the super-CKM basis:

$$M_{\tilde{l}}^2 = \begin{pmatrix} V_L^E M_L^2 V_L^{E\dagger} + m_l^2 + \frac{\cos 2\beta}{2}(M_Z^2 - 2M_W^2)\mathbf{1} & -m_l(\mu \tan\beta \mathbf{1} + A_l^* \\ -m_l(\mu^* \tan\beta \mathbf{1} + A_l) & V_R^E M_E^{2T} V_R^{E\dagger} + m_l^2 - \cos 2\beta M \end{pmatrix}$$

- Assuming $m_\nu = 0$ and $m_l = 0$,

$$M_{\tilde{\nu}_3}^2 = V_L^\nu M_L^2 V_L^{\nu\dagger} + \frac{\cos 2\beta}{2} M_Z^2 \mathbf{1},$$

$$m_{\tilde{\nu}_3}^2 = \tilde{m}_L^2 + \cos 2\beta M_W^2$$

- $V_L^E V_L^{\nu\dagger} \equiv V_{\rm MNS}$ with large mixing angles

  $\rightarrow$ flavor changing interactions involving neutralinos

- $\tilde{\tau} - \chi^0$ loop : enhanced by m_τ/m_μ relative to other contribution $\rightarrow$ can compensate the small mixing angle

  Nothing but the origin of SUSY flavor problem in the lepton sector

  cf. Similar effects in $b \rightarrow s$ reansition in SUSY due to gluino mediations (Kaplan, Nelson et al; Y.G.Kim, P.Ko and J.S.Lee)

$\to B \to X_s\gamma, X_s l^+ l^-$ etc.

- Important constraint:

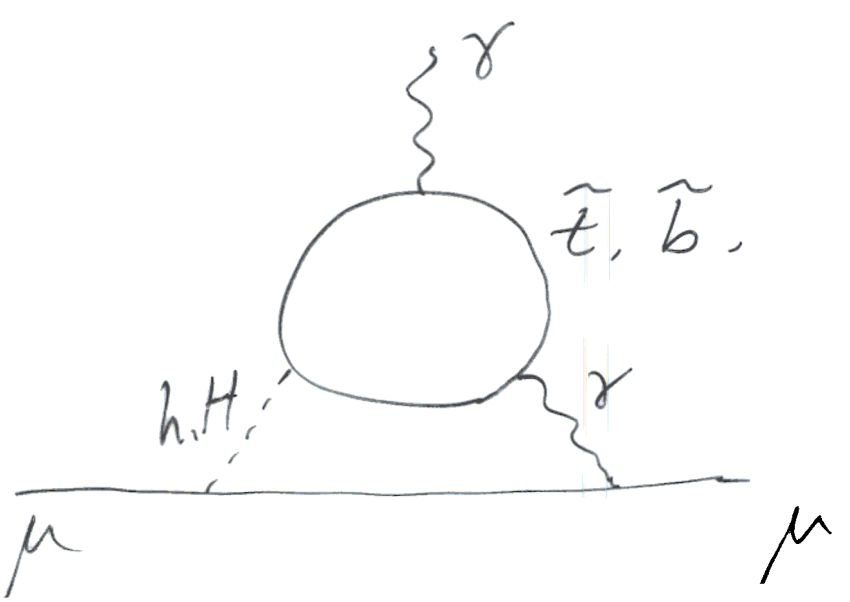
$$B(\tau \to \mu\gamma) < 1.0 \times 10^{-6}$$

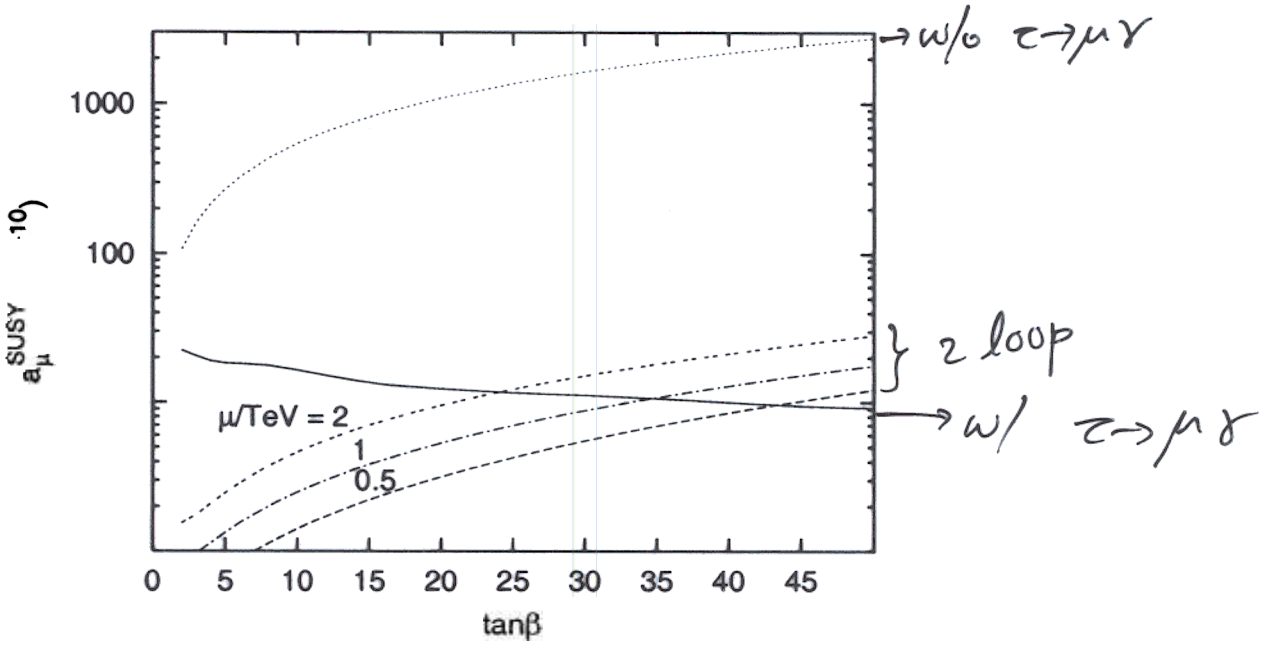
- 2 loop Barr-Zee type contribution can be significant for large $\tan\beta$ (free from $\tau \to \mu\gamma$)

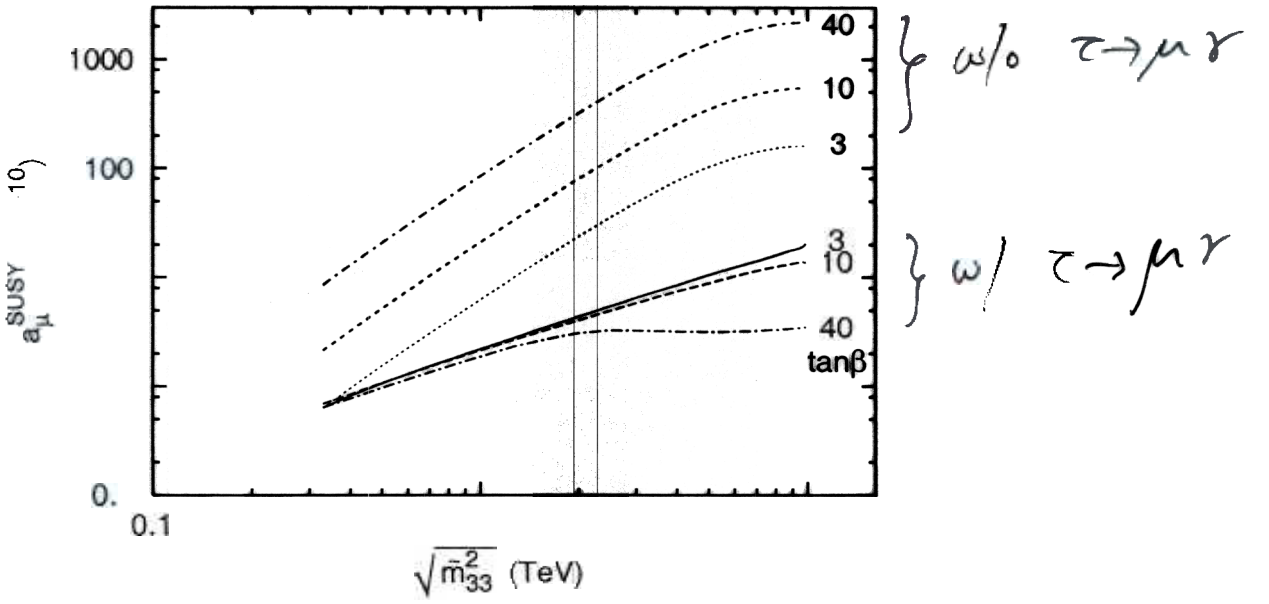
  (A. Ahrib and S.Baek)

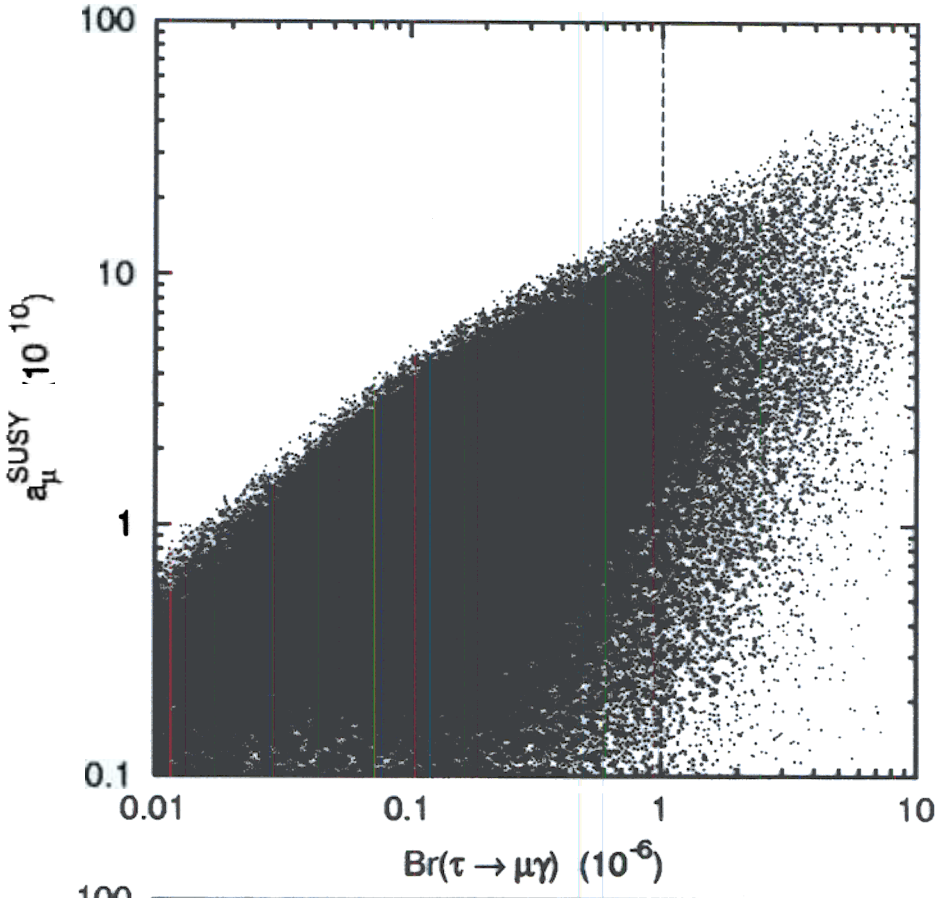
  cf. Analogous to the 2 loop EDM's of e/n (Chang-Keung -Pilaftsis)

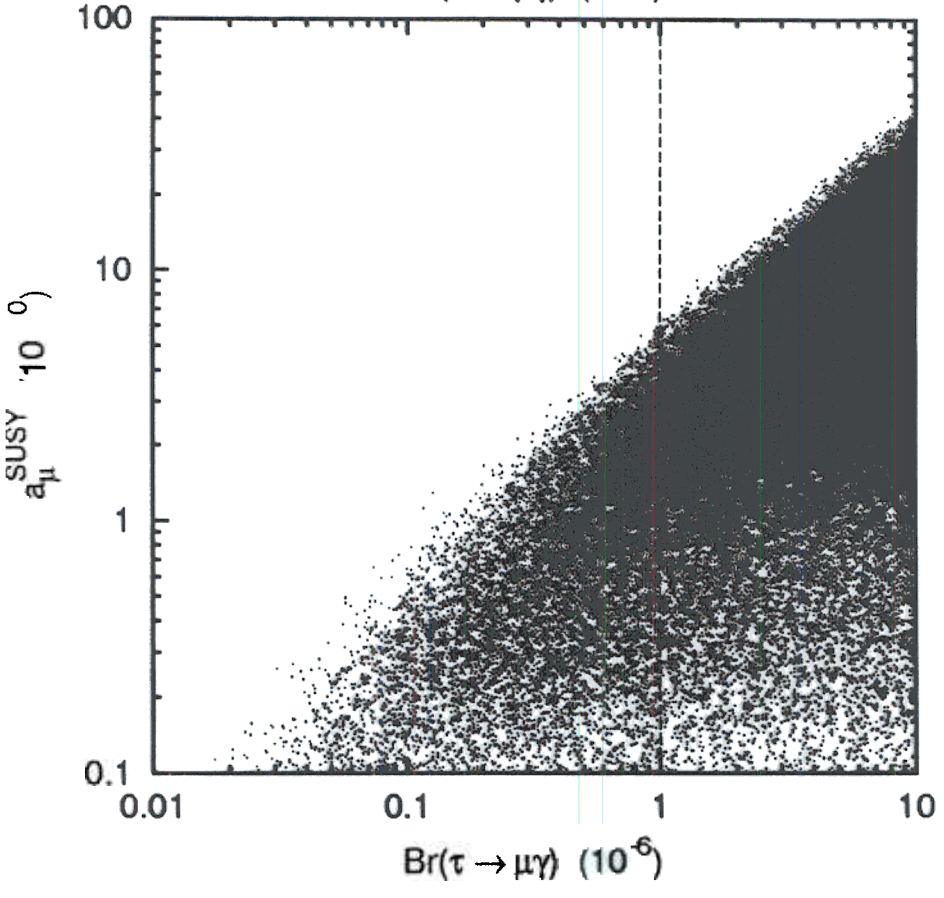
- Our study shows that the $a_\mu^{\rm SUSY}$ can be as large as $\sim 20 \times 10^{-10}$ in the effective SUSY models for all $\tan\beta$ if there is a large mixing between the 2nd and third generation sfermions.




1000
100
a_μ^{SUSY} $\cdot 10^{10}$
μ/TeV = 2
1
0.5
0
5
10
15
20
25
30
35
40
45
tanβ
→ w/o τ→μγ
} 2 loop
→ w/ τ→μγ

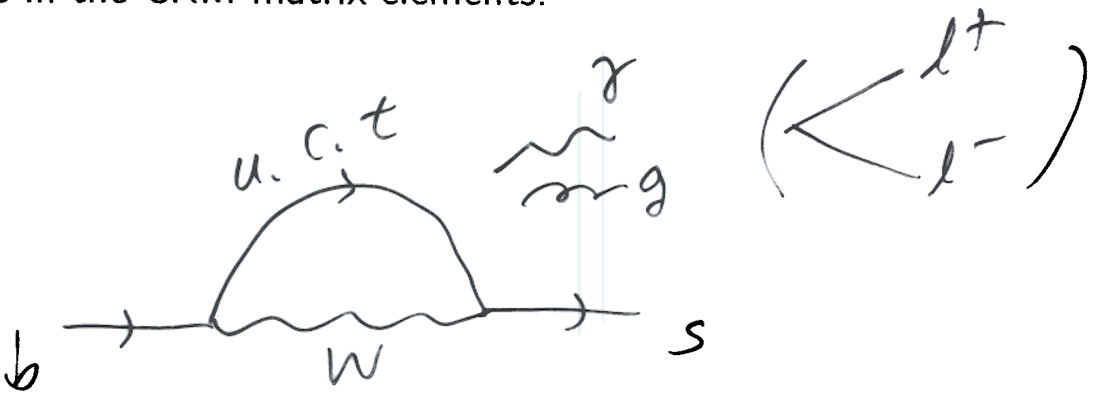

1000
100
0.
a_μ^{SUSY} $\cdot 10^{10}$
40
10
3
3
10
40
tanβ
0.1
$\sqrt{\tilde{m}_{33}^2}$ (TeV)
} w/o τ→μγ
} w/ τ→μγ


100
10
1
0.1
0.01
0.1
1
10
a_μ^{SUSY} (10^{-10})
$Br(\tau \to \mu\gamma)$ (10^{-6})


100
10
1
0.1
0.01
0.1
1
10
a_μ^{SUSY} (10^{-10})
$Br(\tau \to \mu\gamma)$ (10^{-6})

(4) $B \to X_s\gamma$ and $B \to X_s l^+ l^-$

- SM : the leading contribution is the $t - W$ loop, due to the hierarchical structures in the CKM matrix elements.



$$H_{eff}(b \to s\gamma) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=2,7,8} C_i(\mu_b) Q_i(\mu_b)$$

$$\bar{s}_L \gamma^\mu c_L \; \bar{c}_L \gamma_\mu b_L$$
$$\frac{e}{16\pi^2} m_b \; \bar{s}_L \sigma^{\mu\nu} F_{\mu\nu} (1+\gamma_5) b_R$$
$$\frac{g_s}{16\pi^2} m_b \; \bar{s}_L \sigma^{\mu\nu} t^a G^a_{\mu\nu} (1+\gamma_5) b_R$$

$$\frac{B(B \to X_s\gamma)}{B(B \to X_c e\nu)} \qquad \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha}{\pi f(z)} |C_{7,\gamma}(m_b)|^2$$

- CLEO data : $B(B \to X_s\gamma) = (3.21 \pm 0.43 \pm 0.27^{+0.18}_{-0.10}) \times 10^{-4}$

  vs. SM prediction : $(3.57 \pm 0.30) \times 10^{-4}$ (for $E_\gamma > 1.6$ GeV) to NNLO (Buras, Misiak, et al.)

  $\to$ Only a little room for New Physics Contributions to $B \to X_s\gamma$

- SUSY $= (t - W^-)$ loop $+ (t - H^-$ loop$) + (\tilde{t} - \chi^-$ loop$)$
  - $(t - H^-$ loop$)$: alsways additive to the SM contribution
  - $(\tilde{t} - \chi^-$ loop$) \propto A_t\mu \propto M_3\mu$

    Either constructive ($\mu < 0$) or destructive ($\mu > 0$) w.r.t. the SM and charged Higgs contributions
  - $a_\mu^{\rm SUSY}$ is consistent with $B \to X_s\gamma$
- $Br(B \to X_s\gamma)$ only determines $|C_{7,\gamma}|$, but not its sign

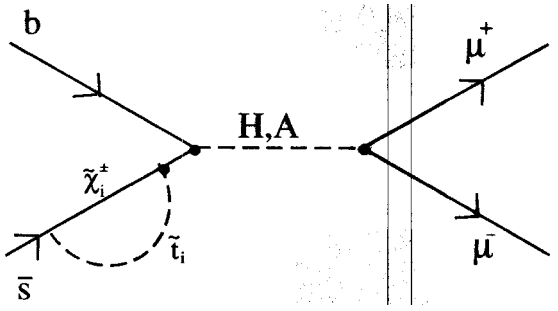
  $\to C_{7,\gamma}^{\rm exp} \approx \pm C_{7,\gamma}^{\rm SM}$
- Sign of $C_{7,\gamma}$ can be probed in $B \to X_s l^+ l^-$ in principle
- $R_{\ell\ell} \equiv \frac{B(B\to X_s \ell^+\ell^-)_{\rm SUSY}}{B(B\to X_s \ell^+\ell^-)_{\rm SM}}$ for $2m_\ell \leq m_{\ell\ell} \leq (m_{J/\psi} - 100\ {\rm MeV})$
- As of 1999, KEK group analysis shows that both signs of $C_{7,\gamma}$ are compatible with the experimental observations for large $\tan\beta$ and light stop and charginos

  $\to R_{\ell\ell}$ could be as large as ~ 1.8
- As of 2002, this is no longer true due to strong constraints from Higgs and SUSY particle search limits

  $\to$ 0.9 $\leq R_{\ell\ell} \leq$ 1.1 in SUSY breaking mediation scenrios we consider
- The FB asymmetry is also sensitive to new physics effects (A. Ali et al.)

(5) $B_s \to \mu^+\mu$

- SM prediction : $(3.7 \pm 1.2) \times 10^{-9}$
- Current upper bound from CDF : $< 2.6 \times 10^{-6}$ (90 % CL)
- CDF aims at sensitivity down to 10^{-8} at during Run II with 15 fb^{-1}
- SUSY contribution : Neutral Higgs exchange amp. can grow like $\tan^3\beta$ (Babu and Kolda, C.S. Huang et al., Bobeth et al., Arnowitt et al., Isidori and Retico, ...)



$\to Br(B_s \to \mu^+\mu^-) \propto \tan^6\beta$ for large $\tan\beta$ (upto $\sim 3 \times 10^{-7}$)

- $$C_S \simeq \frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* \left(\frac{\tan^3\beta}{4\sin^2\theta_W}\right) \left(\frac{m_b m_\mu m_t \mu}{M_W^2 M_A^2}\right) \frac{\sin 2\theta_{\tilde{t}}}{2} f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2).$$

  $\to$ Need a large LR mixing in the stop sector, light stop and H A for larger C_S (Talks by Nierste, Kolda)

- Remember $a_\mu \propto \mu \tan\beta$

  $\to$ Strong correlation between the large a_μ and $Br(B_s \to \mu^+\mu^-) \propto \tan^6\beta$ in mSUGRA (Dedes, Nierste et al.)

- Can cover mSUGRA parameter space with heavy SUSY particles

* How SUSY breaking is communicated to Our World ?

(1) Importance of this question

- MSSM itself cannot be the Ultimate Theory of Nature, either
- 124 parameters of MSSM are arbitrary in principle. BUT!!
- Generic parameter space is already incompatible with various low energy data on FCNC and CP violations

  $\Delta M_K, \quad \Delta M_B, \quad \epsilon_K, \quad e/n$ EDM's, etc.
- The sparticle spectrum is crucially dependent on how SUSY breaking is communicated to our world

  $\rightarrow$ Phenomenologically very important
- We need some nice mechanisms which can solve the so-called SUSY Flavor (FCNC) and SUSY CP problems

  $\leftarrow$ Two of the most serious problems of SUSY models compared to te SM
- Many Scenarios

  Gravity Mediation

  – GaugeMediation

  Anomaly Mediation

  – Gaugino Mediation

  Kaluza-Klein Mode of Bulk Matters,.

- Basic Picture

$$\boxed{\text{Hidden Sector SUSY Breaking}}$$

$$\langle F \rangle \neq 0 \ ,$$

$$\downarrow$$

$$\boxed{\text{Messengers (Flavor Blind)}}$$

$$\downarrow$$

$$\boxed{\text{Visible Sector (Our World : MSSM)}}$$

Gravity Mediation (mSUGRA)

- Gravity is already there
- Matter fields in our world

  Fields in the hidden sector
- We need to know

  * Kähler potential K
  * guage kinetic function f_a
  * superpotential W
- The soft terms are given by

$$m_{soft} \sim \frac{\langle F \rangle}{M_{Pl}}$$

  * $m_{soft} \sim 10^3$ GeV, if $\sqrt{\langle F \rangle} \sim 10^{11}$ GeV (Intermediate Scale)
- With some assumptions, we end up with the following parameters:

$$m_0, \quad M_{1/2}, \quad A_0, \quad \tan\beta, \quad \mathrm{sign}(\mu)$$

(with Seungwon Baek and Wan Young Song, hep-ph/0205259, 0208112)

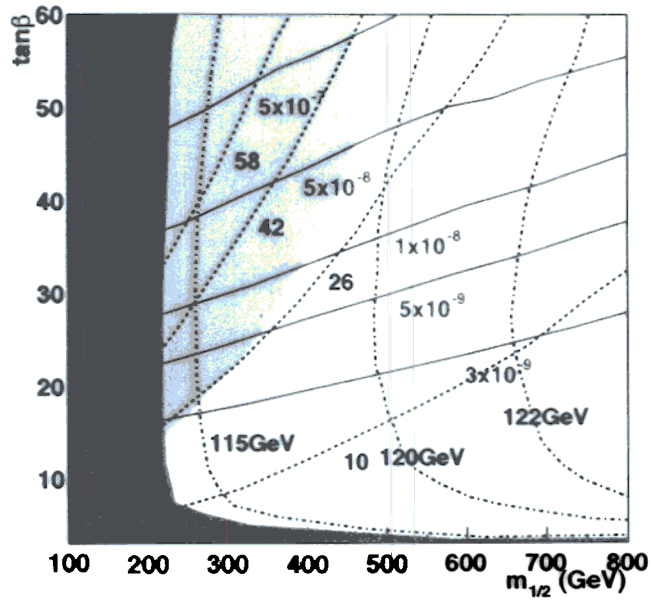


FIG. 1: The contour plots for $a_\mu^{\rm SUSY}$ in unit of 10^{-10} (in the short dashed curves) and the Br $(B_s \to \mu^+\mu^-)$ (in the solid curves) in $(M_{1/2}, \tan\beta)$ plane in the minimal SUGRA model for $m_0 = 300$ GeV and $A_0 = 0$.

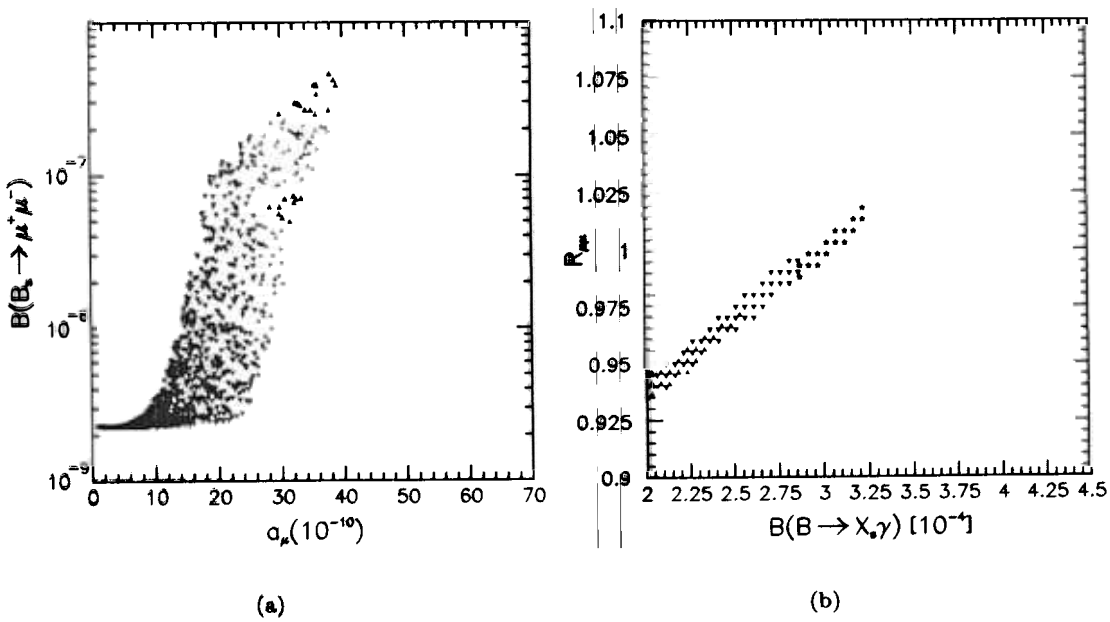


FIG. 2: The correlations between (a) the muon $a_\mu^{\rm SUSY}$ and $B(B_s \to \mu^+\mu^-)$, and (b) $Br(B \to X_s\gamma)$ and $R_{\mu\mu}$ in the mSUGRA model with $A_0 = 0$ and $m_0 = 300$ GeV. The regions $a_\mu^{\rm SUSY} < 10^{-10}$, $10 \times 10^{-10} < a_\mu^{\rm SUSY} < 26 \times 10^{-10}$, $26 \times 10^{-10} < a_\mu^{\rm SUSY} < 42 \times 10^{-10}$, $42 \times 10^{-10} < a_\mu^{\rm SUSY} < 58 \times 10^{-10}$, and $a_\mu^{\rm SUSY} > 58 \times 10^{-10}$ are represented by the stars (black), the inverted triangles (red), the triangles (green), the squares (blue) and the circles (yellow).

Gauge Mediation (GMSB)

In the GMSB models, the SUSY braking scale Λ is around $10 - 100$ TeV, and the soft terms are (to lowest order)

- New Ingredients :
  - Ψ_i, Ψ_i^c : Messenger Fields ($i = 1, 2, ..., N_{\text{mess}}$) in the vectorlike representation of the MSSM gauge group
  - X : Gauge singlet superfield with nonvanishing VEV's in both Scalar and Auxiliary Components ($\langle X \rangle$ and $\langle F_X \rangle$)
  - Superpotential includes

$$W = \lambda_i X \Psi_i \Psi_i^c$$

  - SUSY breaking scale : $\Lambda \approx \langle F_X \rangle / \langle X \rangle$

- Soft Parameters at M_{mess}

$$N_{\text{mess}} \Lambda \; g\left(\frac{\Lambda}{M_{\text{mess}}}\right) \; \alpha_a,$$
$$2 N_{\text{mess}} \Lambda^2 \; f\left(\frac{\Lambda}{M_{\text{mess}}}\right) \; \sum_a \left(\frac{\alpha_a}{4\pi}\right)^2 \; C_a$$
$$0$$

- C_a : the quadratic Casimir invariant of the MSSM matter fields

- m_3 $\Lambda/M_{Pl} \lesssim O$ keV

Thus the LSP is very ght Gravit no $\tilde{G}$

NL: P the ightest neutra nos or even sleptons deca nto $\tilde{G}$ plus M particles such as photon or the lepton

$\chi^0 \to \gamma + \tilde{G}$ AND $\to + \tilde{G}$

These provide characteristic sign ls it colliders

Remember $pp \to e^+e \ \gamma\gamma +$ missingE_T even was interpreted as selectron pair production and thei decays into $+\chi' \to e + \gamma + \tilde{G}$

- Model Parameters

$$M \qquad N_{\rm mess} \quad \Lambda \quad \tan\beta \quad \text{sign}\ \mu$$

In practive trade Λ for Bino mass parameter M and scan over

$$0^4\ \text{GeV} < \Lambda \leq \ \times\ 0^5\ \text{GeV}$$

$$M_{\rm mess} - \quad 0 \qquad 0^5\ \Lambda$$

- Higher M $\to$ Larger A_t and lighter stop mass via RG running

- Larger $N_{\rm mess}$ $\to$ Lighter sfermion mass)^2 the messenger scale

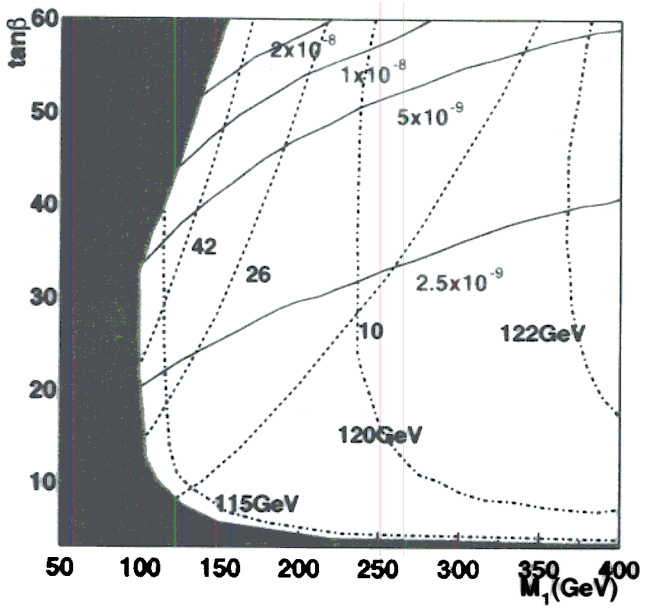
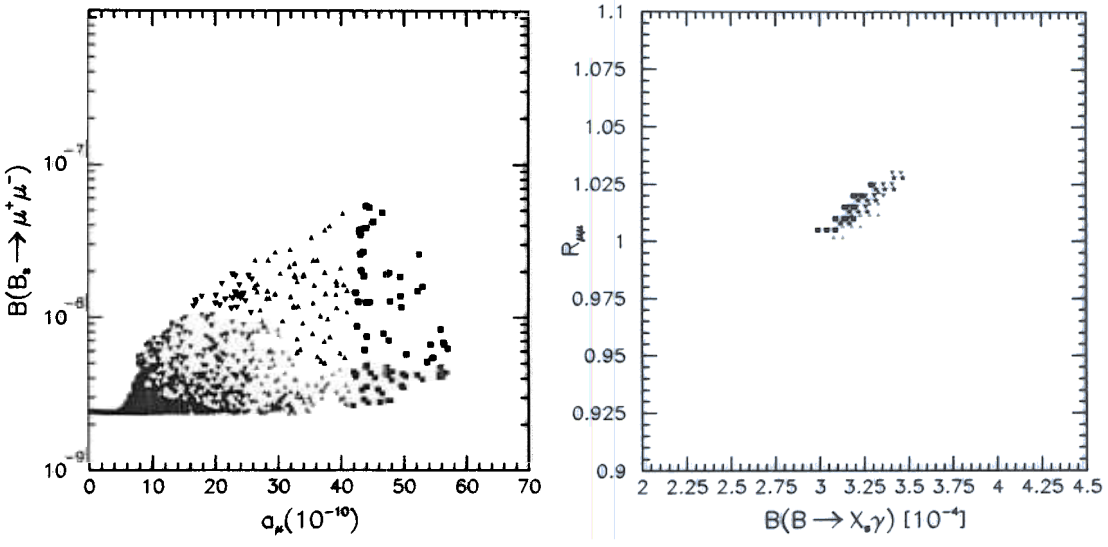


FIG. 3: The contour plots for $a_\mu^{\rm SUSY}$ in unit of 10^{-10} (in the short dashed curves) and the Br $(B_s \to \mu^+\mu^-)$ (in the solid curves) in the $(M_1, \tan\beta)$ plane in the GMSB model with $N_{\rm mess} = 1$ and $M_{\rm mess} = 10^6$ GeV. The light gray region is excluded by the light stau mass bound, and the dark region is excluded by the lower bounds on the masses of Higgs and other SUSY particles.



(a) (b)

FIG. 4: The correlations of the $Br(B \to X_s\gamma)$ with (a) $R_{\mu\mu}$, and (b) $a_\mu^{\rm SUSY}$ in the GMSB scenario with $N_{\rm mess} = 1$ and $M_{\rm mess} = 10^6$ GeV. The legends are the same as Fig. 2.

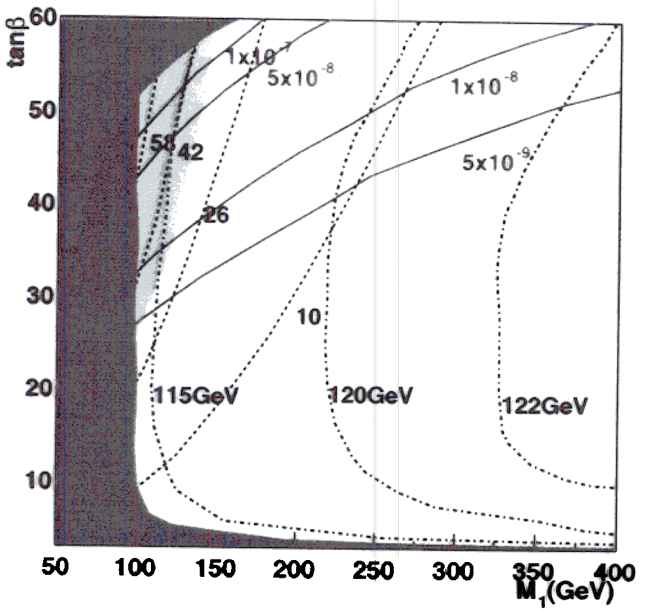
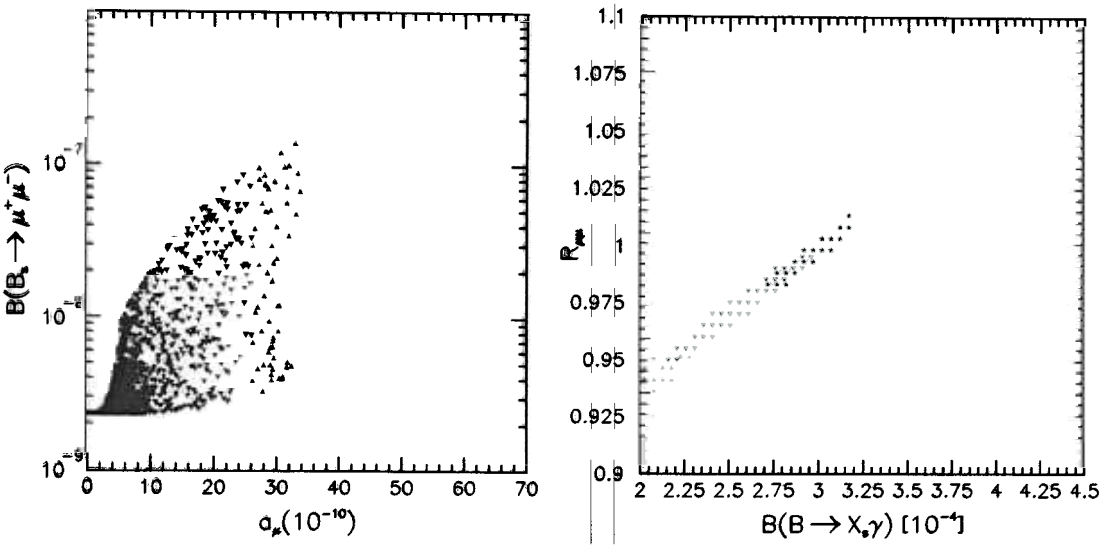


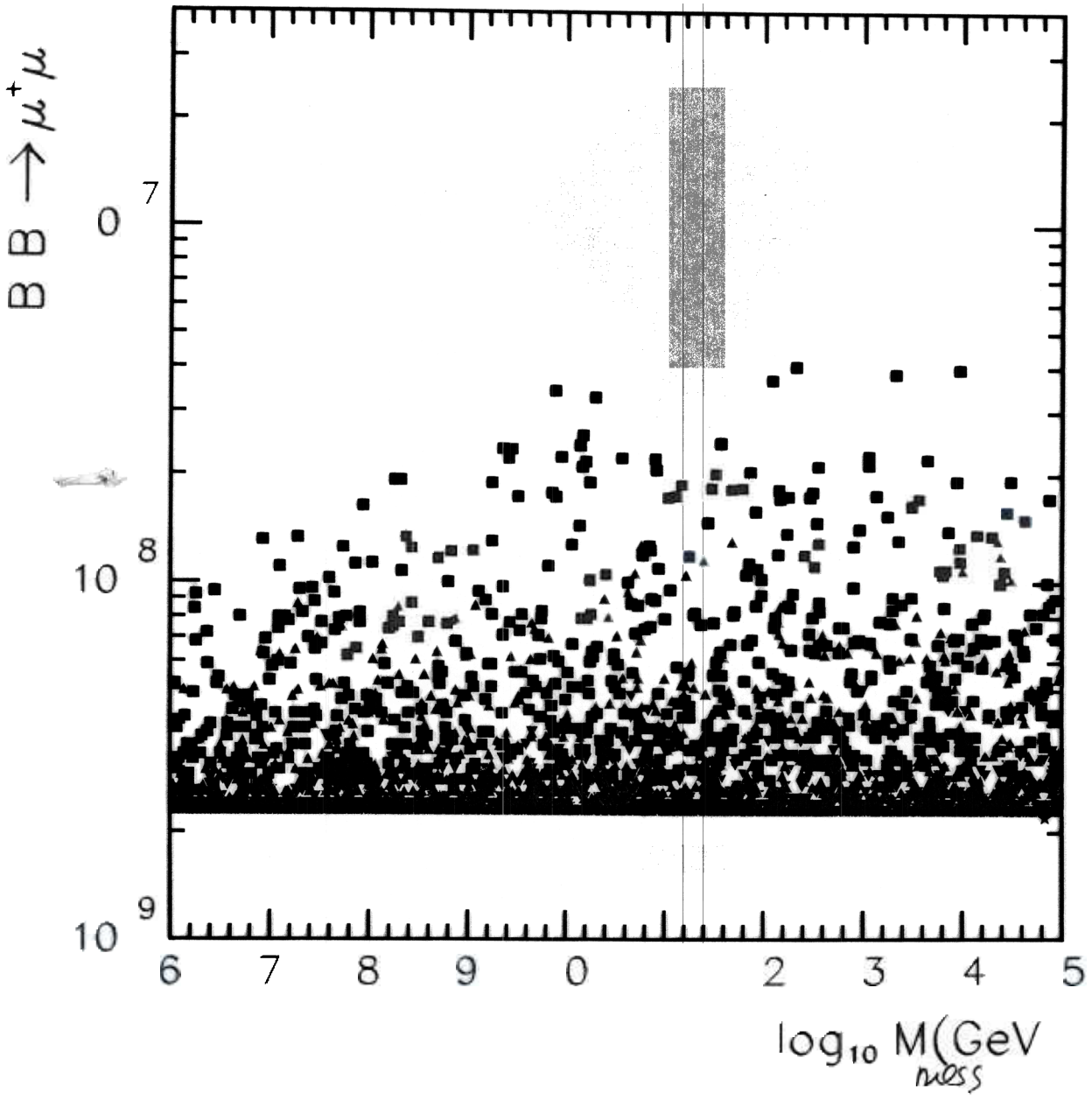
FIG. 5: The contour plots for $a_\mu^{\rm SUSY}$ in unit of 10^{-10} (in the short dashed curves) and the Br $(B_s \to \mu^+\mu^-)$ (in the solid curves) in the $(M_1, \tan\beta)$ plane for the GMSB model with $N_{\rm mess} = 1$ and $M_{\rm mess} = 10^{15}$ GeV. The gray region is excluded by the NLSP mass bound.



(a) (b)

FIG. 6: The correlations of the $Br(B \to X_s\gamma)$ with (a) $R_{\mu\mu}$, and (b) $a_\mu^{\rm SUSY}$ in the GMSB scenario with $N_{\rm mess} = 1$ and $M_{\rm mess} = 10^{15}$ GeV. The legends are the same as Fig. 2.

GMSB w/ N_{mess}



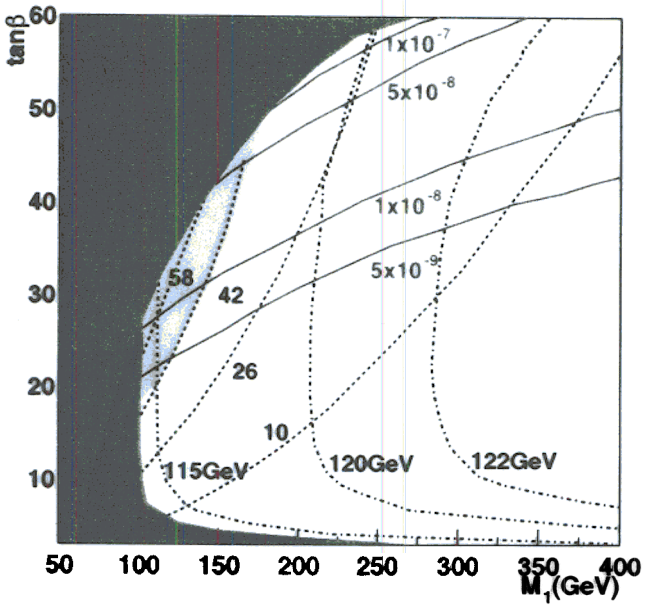


FIG. 7: The contour plots for $a_\mu^{\rm SUSY}$ in unit of 10^{-10} (in the short dashed curves) and the Br $(B_s \to \mu^+\mu^-)$ (in the solid curves) in the $(M_1, \tan\beta)$ plane for the GMSB model with $N_{\rm mess} = 5$ and $M_{\rm mess} = 10^{15}$ GeV. The gray region is excluded by the NLSP mass bound.

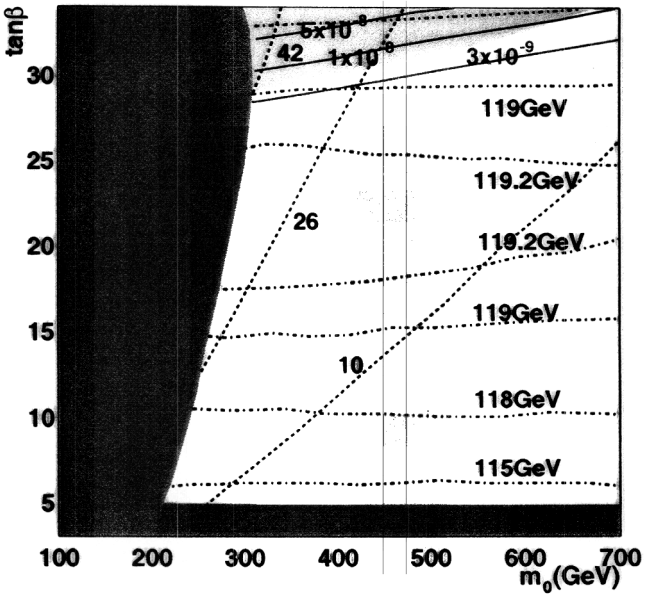


FIG. 8: The contour plots for $a_\mu^{\rm SUSY}$ in unit of 10^{-10} (in the short dashed curves) and the Br $(B_s \to \mu^+\mu^-)$ (in the solid curves) in the $(m_0, \tan\beta)$ plane for the minimal AMSB scenarios with $M_{\rm aux} = 50$ TeV and $\mu > 0$.

Anomaly Mediation (AMSB)

- No Tree Level SUSY breaking
- SUSY breaking due to conformal anomaly (one loop level)

  $\rightarrow$ (the scale dependence of coupling) $\propto \beta$ function

$$-\frac{b_a \alpha_a}{4\pi} M_{\text{aux}}$$

$$\left(-\frac{\dot{\gamma}}{4} + m_0^2\right) \delta_{ij}$$

$$\frac{1}{2}\left(\gamma_i + \gamma_j + \gamma_k\right) M_{\text{aux}}$$

- $\gamma_i \equiv -d \ln Z_i / d \ln \mu$: the anomalous dimension of the field Φ_i
- the "dot" denotes the differentiaton w.r.t. $\ln \mu$
- This is generically present in any SUSY models
- Two main differences from gravity mediation

  $*$ inverted relation between gaugino masses at high energy scale

$$M_1 : M_2 : M_3 = b_1 : b_2 : b_3 = 33/5 : 1 : -3$$

  $*$ $m_{\tilde{l}}^2 < 0$ (tachyonic) for $m_0^2 = 0$ (pure AMSB) $\rightarrow$ serious problem $\rightarrow$ Add universal scalar mass parameters m_0^2, and assume that the above relation holds at high energy scale around M_{GUT}

- Neutralino LSP (almost Wino like)

  * Almost degenerate with NLSP and Hard to Observe NLSP decays

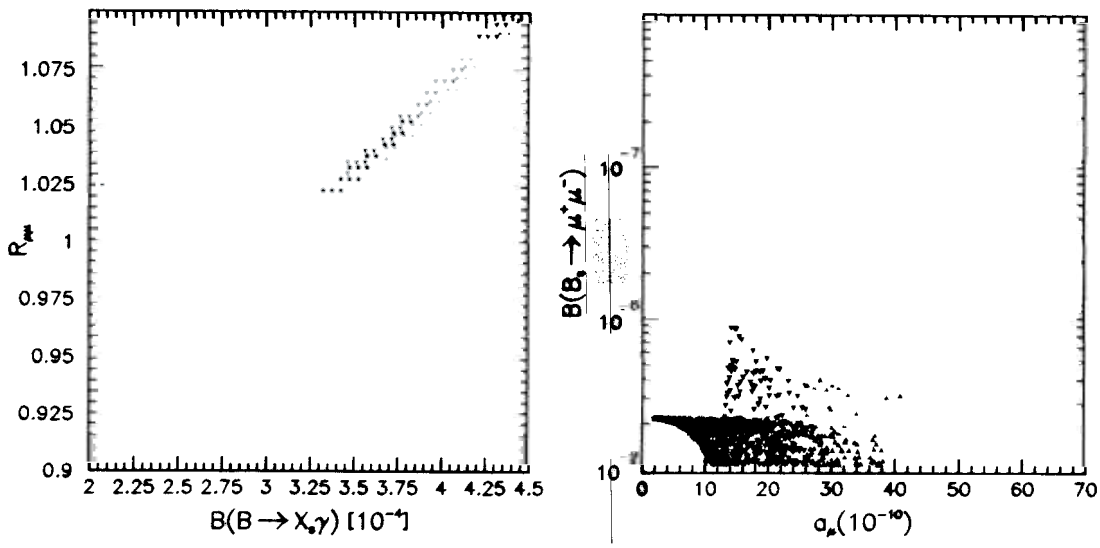
- $\tilde{\nu}_L$ LSP : $\tilde{\chi}^+ \to \tilde{\nu}_L l^+$ $\to$ charged lepton and missing energy signal

- Model Parameters

$$M_{\rm aux}, \quad \tan\beta, \quad m_0, \quad {\rm sign}(\mu)$$

Scan over the following range :

$$\begin{aligned} 20 \text{ TeV} &\leq m_{\rm aux} \leq 100 \text{ TeV} \\ 0 &\leq m_0 \leq 2 \text{ TeV}, \\ 1.5 &\leq \tan\beta \leq 60, \\ {\rm sign}(\mu) \end{aligned}$$



(a) (b)

FIG. 9: The correlations of the $Br(B \to X_s\gamma)$ with (a) $R_{\mu\mu}$, and (b) $a_\mu^{\rm SUSY}$ in the minimal AMSB scenario for $M_{\rm aux} = 50$ TeV. The legends are the same as Fig. 2.

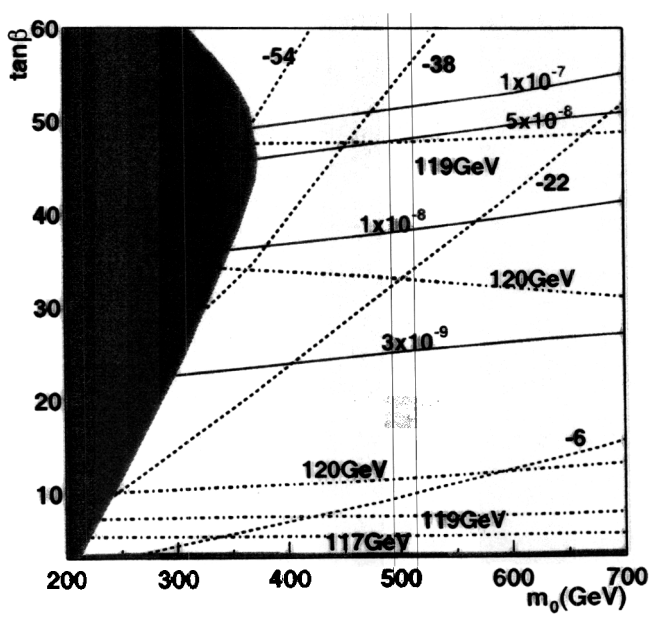


FIG. 10: The contour plots for $a_\mu^{\rm SUSY}$ in unit of 10^{-10} (in the short dashed curves) and the Br $(B_s \to \mu^+\mu^-)$ (in the solid curves) in the $(m_0, \tan\beta)$ plane for the minimal AMSB scenarios with $M_{\rm aux} = 50$ TeV and $\mu < 0$.