Spin Correlations and Velocity-Scaling in NRQCD Matrix Elements

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The NRQCD Factorization Approach

(GTB, E. Braaten, G. P. Lepage)

NRQCD

- The effective field theory Nonrelativistic QCD (NRQCD) separates long-distance quarkonium dynamics ($p \lesssim mv$) from short-distance processes ($p \gtrsim m$).
- Physics with $p < \Lambda \sim m$ is reproduced in the effective theory.
- Physics with $p > \Lambda$ is integrated out, but affects the coefficients of local interactions in the effective theory.
- Λ is the UV cutoff of the effective theory.
- Leading terms in the heavy-quark velocity v are just the Schrödinger action.

$$\mathcal{L}_0 = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2m} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2m} \right) \chi$$

$$D_t = \partial_t + igA_0.$$

$$\mathbf{D} = \partial - ig\mathbf{A}.$$

- ψ is the Pauli spinor field that annihilates a heavy quark.
- χ is the Pauli spinor field that creates a heavy antiquark.

- To reproduce QCD completely, we would need an infinite number of interactions. For example, at next-to-leading order in v^2 we have

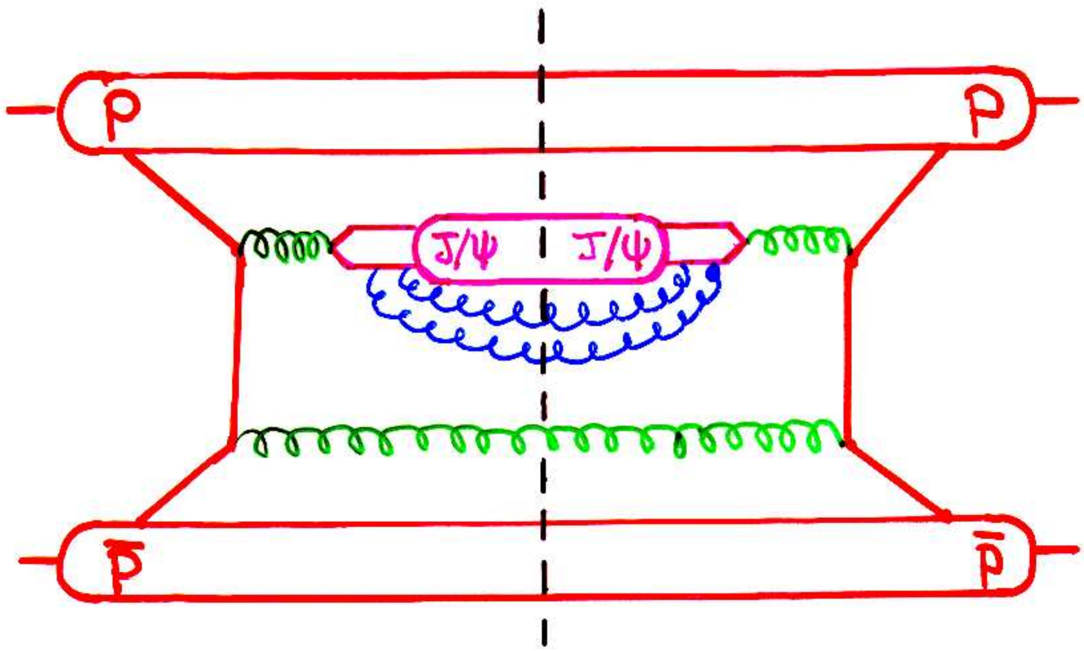
$$
\begin{aligned}
\delta\mathcal{L}_{\mathrm{bilinear}} \;=\; & \frac{c_1}{8m^3}\left[\psi^\dagger(\mathbf{D}^2)^2\psi \;-\; \chi^\dagger(\mathbf{D}^2)^2\chi\right] \\
& + \frac{c_2}{8m^2}\Big[\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi \\
& \qquad\qquad + \chi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\chi\Big] \\
& + \frac{c_3}{8m^2}\Big[\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi \\
& \qquad\qquad + \chi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\Big] \\
& + \frac{c_4}{2m}\left[\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi \;-\; \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right].
\end{aligned}
$$

- In practice, work to a given precision in v.
- NRQCD predicts v-scaling rules for operators and matrix elements.
  - $v^2 \approx 0.3$ for charmonium.
  - $v^2 \approx 0.1$ for bottomonium.

Quarkonium Production in NRQCD

- At large p_T (or p^*), the inclusive quarkonium production cross section can be written as a sum of products of NRQCD matrix elements and "short-distance" coefficients:

$$\sigma(H) = \sum_n \frac{F_n(\Lambda)}{m^{d_n - 4}} \langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle .$$



- The $F_n(\Lambda)$ are short-distance coefficients.
  - Partonic cross sections to make a $Q\bar{Q}$ pair convolved with parton distributions.
  - Calculate as an expansion in α_s.

- Four-fermion operators:

$$\mathcal{O}_n^H = \chi^\dagger \kappa_n \psi \left(\sum_X |H + X\rangle\langle H + X| \right) \psi^\dagger \kappa'_n \chi.$$

  - κ contains Pauli matrices, color matrices, and covariant derivatives.

- The operator matrix elements contain all of the long-distance (nonperturbative physics).

  - Probabilities for a $Q\bar{Q}$ pair to evolve into a heavy-quarkonium.

  - They are universal (process independent).

- The sum over operator matrix elements is an expansion in powers of v.

- A similar factorization formula applies to inclusive quarkonium decays:

$$\Gamma(H \to \mathrm{LH}) = \sum_n \frac{2\ \mathrm{Im}\, f_n(\Lambda)}{m_Q^{d_n-4}} \langle H|\mathcal{O}_n(\Lambda)|H\rangle,$$

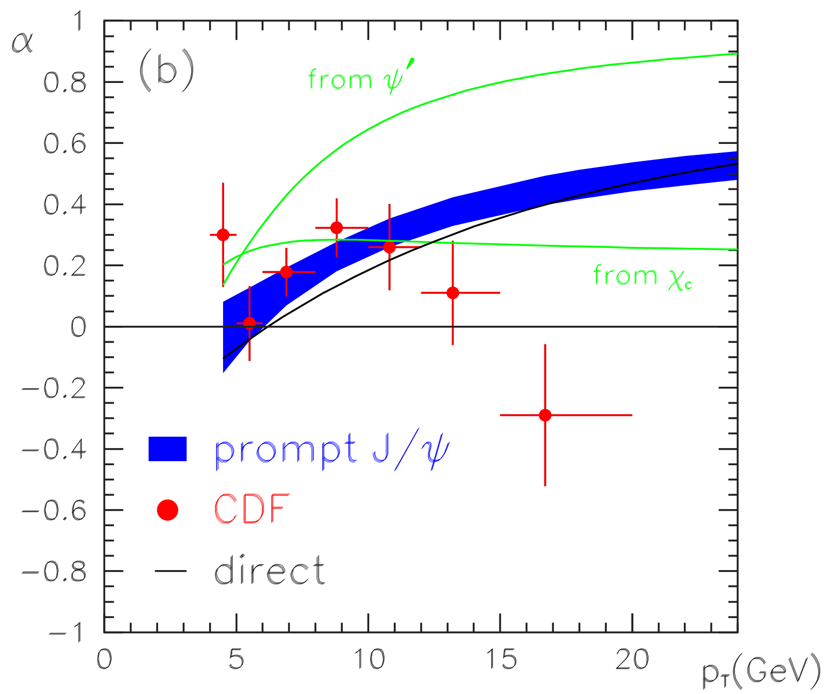
$$\mathcal{O}_n = \psi^\dagger \kappa_n \chi \chi^\dagger \kappa_n' \psi.$$

- The production matrix elements are the crossed versions of quarkonium decay matrix elements.
  - Only the color-singlet production and decay matrix elements are simply related.
- An important feature of NRQCD factorization: Quarkonium decay and production occur through color-octet, as well as color-singlet, $Q\overline{Q}$ states.
- NRQCD factorization relies on
  - NRQCD,
  - hard-scattering factorization.
- Errors of order
  - $\Lambda_{\mathrm{QCD}}^2/p_T^2$ for unpolarized cross sections,
  - $\Lambda_{\mathrm{QCD}}/p_T$ for polarized cross sections.

Polarization of J/ψ's at the Tevatron

- Gluon fragmentation into J/ψ is the dominant production mechanism at large p_T at the Tevatron.
- The gluon produces a $Q\overline{Q}$ pair that evolves nonperturbatively into the J/ψ.
- At high p_T, gluon is nearly on its mass shell.
  - Hence, it is nearly transversely polarized.
  - The gluon transfers its polarization to the $Q\bar{Q}$ pair.
- Nonrelativistic QCD (NRQCD) velocity-scaling rules predict that, in the evolution, the spin-non-flip interactions dominate over spin-flip interactions.
  - Corrections of order $v^2 \approx 0.3$.
- Hence, the J/ψ is predicted to take on most of the transverse polarization of the gluon (Cho, Wise).

- The CDF data for the polarization parameter α lie significantly below the prediction at the largest p_T:



$$d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta,$$
$$-1 \leq \alpha \leq +1.$$

- θ is the angle of the positive lepton in the J/ψ rest frame with respect to the boost vector from the hadronic CM frame to the J/ψ rest frame.

- $\alpha = 1$ corresponds transverse polarization;
  $\alpha = -1$ corresponds to longitudinal polarization.

Lattice Computation of Spin Correlations in NRQCD Color-Octet Matrix Elements

Motivation

- Existing calculations of polarization at the Tevatron assume that v-scaling estimates are valid and neglect spin-flip processes in the NRQCD matrix elements.
- v scaling gives the power behavior as $v \to 0$.
  - Nonperturbative coefficients of the powers of v could be large.
  - In making estimates, it is assumed that the coefficients are of order one.
- It has been suggested that the v-scaling rules may need to be modified for charmonium (Brambilla, Pineda, Soto, Vairo; Fleming, Rothstein, Leibovich).
- We would like to test the validity of the NRQCD v-scaling estimates for the J/ψ production matrix elements.
- But it is not known how to formulate the computation of production matrix elements on a Euclidean lattice.
- Instead, test the v-scaling estimates on the corresponding decay matrix elements.

Lattice NRQCD Action

- Quark action (Lepage, L. Magnea, Nakhleh, U. Magnea, Hornbostel)

$$
\begin{aligned}
S^{(n)} &= a^3 \sum_x \psi^\dagger(x)\psi(x) \\
&\quad -a^3 \sum_x \psi^\dagger(x+a\hat{t}) \left(1 - \frac{aH_0}{2n}\right)^n \left(1 - \frac{a\delta H}{2}\right) \\
&\quad\quad \times U^\dagger_{x,t} \left(1 - \frac{a\delta H}{2}\right) \left(1 - \frac{aH_0}{2n}\right)^n \psi(x).
\end{aligned}
$$

- H_0 is the discrete form of the continuum H_0:

$$H_0 = -\frac{\boldsymbol{\nabla}^{(2)}}{2m}.$$

$$\boldsymbol{\nabla}^{(2)} = \sum_i \nabla_i^{(+)}\nabla_i^{(-)} = \sum_i \nabla_i^{(-)}\nabla_i^{(+)}.$$

$$
\begin{aligned}
a\nabla_\mu^{(+)}\psi(x) &= U_{x,\mu}\psi(x+a\hat{\mu}) - \psi(x). \\
a\nabla_\mu^{(-)}\psi(x) &= \psi(x) - U^\dagger_{x-a\hat{\mu},\mu}\psi(x-a\hat{\mu}).
\end{aligned}
$$

- δH is a discrete form of the action of next-to-leading order in v^2.
  - We use the Lepage *et al.* forms except for $\mathbf{D} \cdot g\mathbf{E} - g\mathbf{E} \cdot \mathbf{D}$.
- There is a similar action for the antiquark.

- n is taken > 1 to avoid a lattice-artifact pole that produces instabilities in the heavy-quark propagator.
  - We use
    $n = 2$ for bottomonium,
    $n = 3$ for charmonium.
- We also include improvements of relative-order a to the lattice action of leading order in v^2.
- Tadpole improvement is implemented by dividing each link in the action by

$$u_0 = \langle \tfrac{1}{3} \mathrm{Tr} U_{\mathrm{plaq}} \rangle^{1/4}.$$

  - Improves the convergence of lattice perturbation theory (more continuum-like).

Matrix-Element Computation

- Create a $Q\bar{Q}$ in a particular spin state and in a singlet color state.
- Propagate forward in Euclidean time according to the lattice NRQCD action in a background gauge field.
- The $Q\bar{Q}$ pair evolves into a quarkonium state whose principal Fock state has the same spin and color.
  - Excited states suppressed by a factor $\exp(-t\Delta E)$.
- Annihilate the $Q\bar{Q}$ pair in a particular spin and color state.
- Multiply by the Hermitian conjugate of this process to obtain the full matrix element.
- Sum over many gauge-field configurations (path-integral over the gauge fields).

Preliminary Results

- Initial computations are on quenched gauge-field configurations (no dynamical fermions).
  - Should reproduce the qualitative features of QCD.
- Preliminary results are based on 400 gauge-field configurations on $12^3 \times 24$ lattices at $\beta = 5.7$.
  - $a = 0.81$ GeV^{-1} for charmonium.
  - $a = 0.73$ GeV^{-1} for bottomonium.
- The quarkonium is well contained in the lattice volume, but the lattice spacing is fairly coarse (requires order-a improvements):
  - $r \approx 1/(mv) \approx 1.2$ GeV^{-1} for charmonium.
  - $r \approx 1/(mv) \approx 0.6$ GeV^{-1} for charmonium.
- Results for the ratio

$$R(S_i, M_i, S_f, M_f) = \frac{\langle {}^{2S_i+1}S_{S_i,M_i}|\mathcal{O}_8\left({}^{2S_f+1}S_{S_f,M_f}\right)|{}^{2S_i+1}S_{S_i,M_i}\rangle}{\langle {}^3S_1|\mathcal{O}_1\left({}^3S_1\right)|{}^3S_1\rangle}.$$

- Average over initial unspecified spins and sum over final unspecified spins.

- Υ at $\beta = 5.7$

Spin Transition	Lattice	v Scaling
singlet $\longrightarrow$ triplet	5.5×10^{-3}	$v^3/(2N_c) \approx 5.3 \times 10^{-3}$
triplet $\longrightarrow$ singlet	1.8×10^{-3}	$v^3/(2N_c) \approx 5.3 \times 10^{-3}$
singlet $\longrightarrow$ singlet	7×10^{-5}	$v^4/(2N_c) \approx 1.7 \times 10^{-3}$
triplet $\longrightarrow$ triplet	8×10^{-5}	$v^4/(2N_c) \approx 1.7 \times 10^{-3}$
triplet up $\longrightarrow$ triplet up	8×10^{-5}	$v^4/(2N_c) \approx 1.7 \times 10^{-3}$
triplet up $\longrightarrow$ triplet long.	noise	$v^6/(2N_c) \approx 1.7 \times 10^{-4}$
triplet up $\longrightarrow$ triplet down	3.1×10^{-6}	$v^6/(2N_c) \approx 1.7 \times 10^{-4}$

- J/ψ at $\beta = 5.7$

Spin Transition	Lattice	v Scaling
singlet $\longrightarrow$ triplet	3.8×10^{-2}	$v^3/(2N_c) \approx 2.7 \times 10^{-2}$
triplet $\longrightarrow$ singlet	1.8×10^{-2}	$v^3/(2N_c) \approx 2.7 \times 10^{-2}$
singlet $\longrightarrow$ singlet	3.9×10^{-4}	$v^4/(2N_c) \approx 1.5 \times 10^{-2}$
triplet $\longrightarrow$ triplet	9×10^{-4}	$v^4/(2N_c) \approx 1.5 \times 10^{-2}$
triplet up $\longrightarrow$ triplet up	5×10^{-4}	$v^4/(2N_c) \approx 1.5 \times 10^{-2}$
triplet up $\longrightarrow$ triplet long.	1.0×10^{-4}	$v^6/(2N_c) \approx 4.5 \times 10^{-3}$
triplet up $\longrightarrow$ triplet down	3×10^{-4}	$v^6/(2N_c) \approx 4.5 \times 10^{-3}$

- $v^2 \approx 0.3$ for J/ψ; $v^2 \approx 0.1$ for Υ.

Discussion

- The hierarchy of v scaling is preserved, but results suggest a smaller expansion parameter ($1/\pi$ for each loop?).
- The triplet $\longrightarrow$ singlet transition rate is large.
  - η_c production rate at the Tevatron may be comparable to the J/ψ production rate.
- The transverse $\longrightarrow$ longitudinal transition
  is small compared with
  the transverse $\longrightarrow$ transverse transition.
  - The prediction of large transverse polarization at large p_T at the Tevatron is supported.
- Phenomenological production matrix elements give
  - $R(\text{triplet-triplet}) = 5.1\text{–}16 \times 10^{-3}$ for the $\Upsilon(1S)$.
  - $R(\text{triplet-triplet}) = 1.9\text{–}24 \times 10^{-3}$ for the J/ψ.
  - The lattice decay matrix elements seem to be smaller.
    But decay $\neq$ production, and lattice $\neq$ continuum.
  - Effects from multiple-gluon radiation may decrease the phenomenological values of the color-octet matrix elements.
    Not yet included in Υ analyses.

- According to the heavy-quark spin symmetry, the ratio $(\text{singlet} \longrightarrow \text{triplet})/(\text{triplet} \longrightarrow \text{singlet})$ should be approximately $3/1$.
  - Corrections are of order v^2.
  - Results agree with a $3/1$ ratio to within order v^2.
- triplet up $\longrightarrow$ triplet down should be suppressed as v^2 relative to triplet up $\longrightarrow$ triplet up.
  - For J/ψ, they are comparable.
- At $\beta = 5.7$, the lattice momentum cutoff is $\Lambda = \pi/a \approx 4.1$ GeV $> m_c$.
  - When $p > m$, NRQCD is no longer valid and the v-scaling rules no longer hold.
  - Spurious power-divergent contributions that go as $[\alpha_s(\Lambda)/\pi]^2(\Lambda/m_c)^4$, $[\alpha_s(\Lambda)/\pi]^2(\Lambda/m_c)^2 \log(\Lambda/m_c)$ may be contaminating the J/ψ matrix elements.
  - Work on controlling them is in progress.
  - Preliminary results suggest that lowering Λ reduces the discrepancies in the v scaling in the J/ψ matrix elements.

Conclusions

- Preliminary lattice calculations support the idea that spin-flip processes are suppressed in charmonium and bottomonium matrix elements.
- The theoretical expectation stands that there should be substantial transverse polarization of J/ψ's at the Tevatron at large p_T.