Northwestern University Department of Physics and Astronomy

PHYSICS 428-1 QUANTUM FIELD THEORY I

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Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFT/QFT_Fall08.html`

ASSIGNMENT #1

Due at 3:30 PM, October 2nd

(Two pages and three problems in total.)

Reading Assignments:
(a) Read the pages on "Notations and Conventions" in Peskin and Schroeder.
(b) F. Wilczek, "Quantum field theory," Rev. Mod. Phys. **71**, S85 (1999), which can be downloaded at `http://arxiv.org/abs/hep-th/9803075`
It is all right if you don't understand everything he says, but you should try to read it over and over again until you think you have understood "something!"

Problem 1
Explain what is the phenomenon of "dimensional transmutation?"
(Hint: if you have done your reading assignment, you will be able to answer it.)

Problem 2
Here are a few exercises on dimensional analysis and the Natural Unit system $c = \hbar = 1$.
(a) The proton mass in the SI unit is $m_p = 1.62\times10^{-27}$ kg. Convert m_p into the Natural Unit. The Large Hadron Collider (LHC) at CERN in Geneva is a proton-proton collider designed to have a center-of-mass energy of 14 TeV. What is the speed of the proton, expressed in terms of the speed of light c, when the LHC is operating at the designed CM energy?
(b) In SI unit Maxwell's equations contain three dimensionful coupling constants: the electric charge of the electron $e = 1.62 \times 10^{-19}$ C, the permittivity of free space $\epsilon_0 = 8.854 \times 10^{-12}$ F/m, and the permeability of free space $\mu_0 = 1/(\epsilon_0 c^2)$ which can be traded in for the speed of light c. Can you generate a quantity out of e, ϵ_0, c, and $\hbar$ that is *dimensionless*? Is there more than one dimensionless quantity that can be generated?
(c) Divide your answer(s) in (b) by 4π and call it α. What is the numerical value of α? Approximate α by a fraction $\alpha \approx 1/N$ where N is an integer. What is N? Can you recognize that α is a well-known fundamental constant?
(d) The Newton's constant $G_N = 6.674 \times 10^{-11}$ m^{3}kg^{-1}s^{-2}. In the Natural Unit it has the mass dimension of -2, which is used to define a Planck mass $M_p = 1.22\times10^{-19}$ GeV. Convert M_p into the SI unit by expressing it in terms of G_N, $\hbar$, and c.
(e) With a CM energy of 14 TeV, typical energy scales at the LHC will be at around TeV. For a typical quantum mechanical amplitude at the LHC, how large is the correction to the amplitude due to effects of gravity?

Problem 3
During the class we showed that the following amplitude

$$\langle \vec{x}|e^{iHt}|\vec{x}=0\rangle = \int \frac{d^3k}{(2\pi)^3} e^{-ik\cdot(x-y)} \tag{1}$$

is non-vanishing outside of the forward light cone of the particle. However, you may protest that the integration measure d^3k is not invariant under Lorentz transformation and that is why causality is violated. One could, instead, use a relativistically invariant amplitude as follows

$$D(x-y) \equiv \int \frac{d^4k}{(2\pi)^4}(2\pi)\delta(k^2-m^2)\Big|_{k^0>0} e^{-ik\cdot(x-y)}. \tag{2}$$

Use the technique we used in class to show that even this Lorentz-invariant amplitude violates causality in that it still is non-vanishing outside of the forward light cone. There is actually a shortcut to this problem that you are *not* allowed to use:

$$\frac{\partial}{\partial t}D(x-y) = \int \frac{d^3k}{(2\pi)^3} e^{-ik\cdot(x-y)}. \tag{3}$$

However, you can use the above relation to check your answer.