Northwestern University Department of Physics and Astronomy

PHYSICS 428-1 QUANTUM FIELD THEORY I

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Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFT/QFT_Fall08.html`

ASSIGNMENT #10
Due at 3 PM, December 8th
(One page and two problem.)

Reading Assignments:
The subsection on "Mandelstam Variables" in Section 5.4 and the subsection on "Photon Polarization Sums" in Section 5.5 of Peskin and Schroeder.

Problem 1
Do Problem 5.2 in Peskin and Schroeder.

Problem 2
Consider the following Lagrangian for the *scalar* Quantum Electrodynamics:

$$\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi) - m_\phi^2 |\phi|^2 = (\partial_\mu + ieA_\mu)\phi^* \, (\partial^\mu - ieA^\mu)\phi - m_\phi^2 \phi^* \phi.$$

(a) Write down the Feynman rule for the interaction vertices in this theory. Be careful to explain and take into account symmetry factors, if any. (There is actually a subtlety involving the one-derivative term in the interactions, but you can get the correct answer by deriving the Feynman rules in the *naive* way.)
(b) Compute the amplitude for the scattering process $\phi(k_1) + \phi^*(k_2) \to \phi(p_1) + \phi^*(p_2)$ to the lowest order in e using the following massless propagator for the photon:

$$\frac{-ig_{\mu\nu}}{k^2 + i\epsilon}.$$

(c) Compute the amplitude for the scattering process $\phi(k_1) + \gamma(k_2) \to \phi(p_1) + \gamma(p_2)$ to the lowest order in e using Feynman rules/diagrams. Be sure to include all possible diagrams.
(d) Now assume the photon is massive and has the following propagator

$$\frac{-i\,(g_{\mu\nu} - k_\mu k_\nu / m_A^2)}{k^2 - m_A^2 + i\epsilon}.$$

Repeat (b) and show that the amplitude has a smooth limit in taking $m_A \to 0$. (In other words, the $k_\mu k_\nu / m_A^2$ piece in the massive propagator doesn't contribute to the amplitude, just like in the ordinary QED with fermions.)
(e) Write the amplitudes in (c) as $\epsilon_\mu^{(i)} \epsilon_\nu^{(f)} \mathcal{M}^{\mu\nu}$. Show that $k_2^\mu \mathcal{M}_{\mu\nu} = p_2^\nu \mathcal{M}_{\mu\nu} = 0$, again like in the ordinary QED with fermions.
(f) Show that the Coulomb potential resulting from the process in (b) is attractive, whereas it is repulsive for the $\phi + \phi \to \phi + \phi$ scattering.