Northwestern University Department of Physics and Astronomy

PHYSICS 428-1 QUANTUM FIELD THEORY I

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Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFT/QFT_Fall08.html`

ASSIGNMENT #7

Due at 3:30 PM, November 14th

(Two pages and three problems.)

Reading Assignments:
Sections 4.1 and 4.2 of Peskin and Schroeder.

Problem 1
(a) Show that the Dirac Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

is invariant under C, P, and T separately.
(Hint: for the charge conjugation, if you blindly use the table in the end of Chapter 3 in Peskin and Schroeder, you will not get the right answer. Instead, start from the basics.)
(b) Replace the derivative ∂^μ by the covariant derivative $D^\mu = \partial^\mu - igA^\mu$ and show that, if electromagnetism is invariant under charge conjugation, the photon A^μ must be odd under charge conjugation

$$CA^\mu(x)C = -A^\mu(x).$$

Problem 2
Do Problem 3.8 in Peskin and Schroeder.

Problem 3
Consider an interacting theory of four scalar fields $\phi^a, a = 1, 2, 3, 4$ with different masses

$$\mathcal{L}_0 = \sum_{a=1}^{4}\left[\frac{1}{2}(\partial_\mu\phi^a)^2 - \frac{m_a^2}{2}(\phi^a)^2\right] + \frac{g}{4!}\epsilon_{abcd}\ \epsilon^{\mu\nu\lambda\sigma}\ \partial_\mu\phi^a\partial_\nu\phi^b\partial_\lambda\phi^c\partial_\sigma\phi^d$$

(a) If we define the operation of parity as $P_s : (t, \vec{x}) \to (t, -\vec{x})$ and $P_s : \phi^a(t, \vec{x}) \to \phi^a(t, -\vec{x})$, show that the Lagrangian is parity-violating.
(b) The Lagrangian has many discrete internal symmetries. One example is $\phi^a(t, -\vec{x}) \to -\phi^a(t, \vec{x})$ for $a = 1, 2, 3, 4$.. Enumerate all possible internal discrete symmetries.
(c) Show that it is possible to define the parity transformation as $P \equiv P_s P_i$, where P_i is one of the internal discrete symmetries, such that the theory is invariant under the newly defined parity P. Do you get $P^2 = 1$? How many pseudo-scalars are in this "parity-invariant" theory?
(d) Now add to the Lagrangian an additional interaction term

$$\mathcal{L}_1 = -g_3 \sum_{a=1}^{4}(\phi^a)^3$$

Show that no definition of parity leaves the Lagrangian invariant. The theory $\mathcal{L}_0 + \mathcal{L}_1$ now violates parity.
(e) Now coonsider a complex scalar χ interacting with ϕ^a through the Lagrangian

$$\mathcal{L} = \sum_{a=1}^{4} \left[\frac{1}{2} (\partial_\mu \phi^a)^2 - \frac{m_a^2}{2} (\phi^a)^2 \right] + \partial_\mu \chi^* \partial^\mu \chi - m^2 \chi^* \chi - g_3 \sum_{a=1}^{4} (\phi^a)^3$$
$$+ \frac{g}{4!} \epsilon_{abcd} \, \epsilon^{\mu\nu\lambda\sigma} \, \partial_\mu \phi^a \partial_\nu \phi^b \partial_\lambda \phi^c \partial_\sigma \phi^d (\chi^2 + \chi^{*2})$$

Show that there are ways to define the parity P such that $\mathcal{L}$ is parity-invariant. Furthermore, show that you do not get $P^2 = 1$ when the parity acts on some of the scalars in the theory.