Northwestern University Department of Physics and Astronomy

PHYSICS 428-1 QUANTUM FIELD THEORY I

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Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFT/QFT_Fall08.html`

ASSIGNMENT #8

Due at 3:30 PM, November 21st

(Two pages and one problem.)

Reading Assignments:
Sections 4.3, 4.4, 4.6, and 4.7 of Peskin and Schroeder.

Problem 1

Consider the scalar Yukawa theory we discussed in class

$$\mathcal{L} = \int d^4x \; \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi) - \frac{1}{2}m_\phi^2\phi^2 + \partial_\mu\psi^\dagger\partial^\mu\psi - m_\psi^2\psi^\dagger\psi - g\psi^\dagger\psi\phi$$

In the interaction picture, the quantum field is written in the same way as the free field

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_k}}\left(a_k e^{-ik\cdot x} + a_k^\dagger e^{ik\cdot x}\right), \quad \psi(x) = \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_k}}\left(b_k e^{-ik\cdot x} + c_k^\dagger e^{ik\cdot x}\right).$$

Furthermore, define the amplitude $\mathcal{A}$ of a particular process as

$$\langle f|S-1|i\rangle = i\mathcal{A}\;(2\pi)^4\delta^{(4)}\left(\sum_i k_i - \sum_f k_f\right).$$

(a) Derive the following Wick contractions for the complex scalar ψ:

$$\contraction{}{\psi^\dagger}{(x)}{\psi^\dagger}\psi^\dagger(x)\psi^\dagger(y) = 0, \quad \contraction{}{\psi}{(x)}{\psi}\psi(x)\psi(y) = 0, \quad \contraction{}{\psi^\dagger}{(x)}{\psi}\psi^\dagger(x)\psi(y) = \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m_\psi^2 + i\epsilon}e^{-ik\cdot(x-y)}.$$

(b) Compute $i\mathcal{A}$ in the centre-of-mass frame for the scattering process $\psi(k_1) + \psi^\dagger(k_2) \to \psi(p_1) + \psi^\dagger(p_2)$ using Dyson's formula. (Do not use any Feynman rules here!)
(c) Draw the relevant Feynman diagram(s) for the process in (b). Then use Feynman rules to re-derive your answer in (b).
(d) Compute $i\mathcal{A}$ in the centre-of-mass frame for the scattering process $\psi(k_1) + \phi(k_2) \to \psi(p_1) + \phi(p_2)$ using Dyson's formula. (Do not use any Feynman rules here!)
(e) Draw the relevant Feynman diagram(s) for the process in (d). Then use Feynman rules to re-derive your answer in (d).
(f) Compute $i\mathcal{A}$ in the centre-of-mass frame for the scattering process $\psi(k_1) + \psi^\dagger(k_2) \to \phi(p_1) + \phi(p_2)$ using Dyson's formula. (Do not use any Feynman rules here!)
(g) Draw the relevant Feynman diagram(s) for the process in (f). Then use Feynman rules to re-derive your answer in (f).
(h) Assuming $m_\psi < m_\phi$, what is the minimal velocity of ψ in the centre-of-mass frame in

order for the process in (f) to occur?
(i) Compute $i\mathcal{A}$ in the centre-of-mass frame for the decay process $\phi(k) \to \psi(p_1) + \psi^\dagger(p_2)$ using Dyson's formula. (Do not use any Feynman rules here!)
(j) Draw the relevant Feynman diagram(s) for the process in (i). Then use Feynman rules to re-derive your answer in (i).
(k) Can the process in (i) occur for arbitrary masses m_ϕ and m_ψ?
(l) Can the process $\psi(k_1) + \psi(k_2) \to \psi^\dagger(p_1) + \psi^\dagger(p_2)$ occur? Why or why not?
(m) Compute $i\mathcal{A}$ for the scattering process $\phi(k_1) + \phi(k_2) \to \phi(p_1) + \phi(p_2)$ using Dyson's formula. You don't have to evaluate any leftover d^4k integral in the amplitude. (Do not use any Feynman rules here!)
(n) Draw the relevant Feynman diagram(s) for the process in (m). Then use Feynman rules to re-derive your answer in (m).