Northwestern University Department of Physics and Astronomy

PHYSICS 428-1 QUANTUM FIELD THEORY I

Ian Low, Fall 2008

Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFT/QFT_Fall08.html`

ASSIGNMENT #9

Due at 3:30 PM, December 1st

(One page and two problem.)

Reading Assignments:
Sections 4.8 and 5.1 of Peskin and Schroeder.

Problem 1
Do Problem 4.3 in Peskin and Schroder.

Problem 2
Consider a complex scalar field ϕ coupling to the muon (μ^-) field μ and the neutrino (ν) field ν_L through the following interaction

$$\mathcal{L}_I = C_F(\partial_\mu\phi)(\bar{\mu}\gamma^\mu\nu_L) + (\text{h.c.}),$$

where we have assumed the neutrino is massless and exists only in the left-handed component $\nu_L = (1/2)(1-\gamma_5)\nu$. The complex scalar ϕ has a mass m_ϕ and the muon has a mass m_μ.
(a) Compute the differential cross-secion $d\sigma/d\Omega$ for the scattering process $\bar{\nu} + \nu \to \mu^+ + \mu^-$ to leading order in C_F in the centre-of-mass frame.
(b) Compute the total decay width Γ of the complex scalar to leading order in C_F.
(c) We can think of the complex scalar ϕ as the charged pion π^-. In the standard model the coefficient of the interaction is

$$C_F = 2\cos\theta_c\, G_F\, f_\pi$$

where $\cos\theta_c = 0.974$ is the cosine of the Cabbibo angle, $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi constant, and f_π is the pion decay constant. In addition, the pion mass is $m_\pi = 139.6$ MeV and the muon mass $m_\mu = 105.7$ MeV. The measured value of the charged pion lifetime is $2.603 \times 10^{-8}s$. Determine the value of f_π in MeV. By what percentage is your result too big or too small?