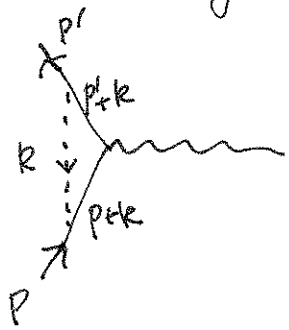
QFT II. Winter 2009.

Solution for HW2.

Problem 6.3 in <u>P&S.</u>

(a). The Feynman diagram is



$$ie\,\bar{u}(p')\,\delta\Gamma_h\,u(p) = \int \frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m_h^2+i\varepsilon}\,\bar{u}(p')\left(\frac{-i\lambda}{\sqrt{2}}\right)\frac{i}{(\not{p}'+\not{k})-m_e+i\varepsilon}\,(ie\gamma^\mu)$$

$$\times\ \frac{i}{\not{p}+\not{k}-m_e+i\varepsilon}\left(\frac{-i\lambda}{\sqrt{2}}\right)u(p)$$

$$\delta\Gamma_h = \int dx\,dy\,dz\,\delta(x+y+z-1)\int\frac{d^4\ell}{(2\pi)^4}\,\frac{i\lambda^2\,\boxed{Num}}{(\ell^2-\Delta+i\varepsilon)^3}$$

$$\ell = k + xp + yp'. \qquad \Delta = z m_h^2 + (1-z)^2 m_e^2 - xyq^2$$

$$\boxed{Num} = (\not{p}'+\not{k}+m_e)\gamma^\mu(\not{p}+\not{k}+m_e)$$

$$\longrightarrow \gamma^\mu\left[(1+z)^2 m_e^2 + xyq^2 - \frac{1}{2}\ell^2\right]$$

$$+\ 2m_e^2(1-z^2)\,\frac{i\sigma^{\mu\nu}q_\nu}{2m_e}.$$

The contribution to the anomalous magnetic moment comes from δF_2

$$\delta_h F_2 = \int dx\,dy\,dz\,\delta(x+y+z-1)\cdot 2m_e^2\lambda^2(1-z^2)\int\frac{d^4\ell}{(2\pi)^4}\frac{i}{(\ell^2-\Delta+i\varepsilon)^3}$$

$$= \frac{\lambda^2}{16\pi^2}\int dx\,dy\,dz\,\delta(x+y+z-1)\,\frac{m_e^2(1-z^2)}{zm_h^2+(1-z)^2m_e^2-xyq^2}$$

$$\delta_h F_2(q^2=0) = \frac{\lambda^2}{16\pi^2}\int_0^1 dz\,\frac{(1-z)(1-z^2)m_e^2}{zm_h^2+(1-z)^2m_e^2}$$

$$\approx \frac{\lambda^2 m_e^2}{16\pi^2 m_h^2}\left[\log\frac{m_h^2}{m_e^2}-\frac{7}{6}+O\left(\frac{m_e^2}{m_h^2}\right)\right]$$

$$\left(=\frac{g-2}{2}\ !\right).$$

(b) $m_e = 0.5\ \text{MeV} = 5\times10^{-4}\ \text{GeV}$.

then for $\lambda \sim 3\times10^{-6}$, $m_h \sim 60$ GeV.

$$\delta_h F_2(q^2=0) \approx 8.8\times10^{-23} \ll 3\times10^{-8}$$

So the bound is <u>NOT</u> very strong!!

For the muon case, let's use present bound on the Higgs mass $m_h \gtrsim 114$ GeV

$$\lambda_\mu = \left(\frac{m_\mu}{m_e}\right) \times \lambda_e \sim 6\times 10^{-4}$$

$$\delta_n F_2(q^2=0)_\mu \sim 2.5\times 10^{-14} \ll 3\times 10^{-8} \;!!$$

(c) Only the numerator is different from (a):

$$\boxed{\text{Num}} = -\gamma^5(\not{p}+\not{k}+m_e)\gamma^\mu(\not{p}+\not{k}+m_e)\gamma^5$$

$$\longrightarrow \gamma^\mu\left[(1-z)^2 m_e^2 + xyq^2 - \tfrac{1}{2}l^2\right] - 2m_e^2\underbrace{(1-z)^2}_{\text{It was } z^2 \text{ in (a)!}}\frac{i\sigma^{\mu\nu}q_\nu}{2m_e}$$

$$\therefore \delta_n F_2(q^2=0) = \frac{\lambda^2}{16\pi^2}\int_0^1 dz\,\frac{(1-z)^3}{(1-z)^2 + z\left(m_a^2/m_e^2\right)}$$

This integral can be done using Mathematica.

For $\frac{m_a^2}{m_e^2} \ll 1 \longrightarrow \frac{\lambda^2}{16\pi^2}\left[\frac{m_a^2}{m_e^2}\left(1+\log\frac{m_a}{m_e}\right)+\frac{1}{2}\right]$

$\frac{m_a^2}{m_e^2} \gg 1 \longrightarrow \frac{\lambda^2}{16\pi^2}\left[\frac{m_e^2}{m_a^2}\left(-\frac{11}{6}+2\log\frac{m_a}{m_e}\right)\right]$.

~~In the 1st case, $\lambda \lesssim 4\times 10^{-5}$~~

~~2nd case,~~

Just do the Plot to get the bound.