Northwestern University Department of Physics and Astronomy

PHYSICS 428-2 QUANTUM FIELD THEORY II

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Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFT/QFT_Winter09.html`

ASSIGNMENT #3

Due at 3 PM, January 26th

(One page and two problems.)

Reading Assignments:
Sections 7.1 and 7.2 of Peskin and Schroeder.

Problem 1
In class we derived the Källen-Lehmann Representation of the time-ordered product of a real scalar field. In this problem we will consider the vacuum expectation value of the commutator of a *complex* scalar field. All the fields below are *bare* fields.
(a) Using the same procedure as shown in the lecture, derive the spectral representation for $\langle\Omega|[\phi(x),\phi^\dagger(y)]|\Omega\rangle$ and show that it involves the same spectral density $\rho(\mu^2)$ as in the time-ordered product $\langle\Omega|T\phi(x)\phi(y)]|\Omega\rangle$.
(Caution: because now it is a complex scalar, one could in principle define two spectral density functions. You will need to use the causality requirement to show these two are identical.)
(b) Use the spectral representation you derived in (a), in conjunction with the canonical equal-time commutation relations, show that the spectral function must satisfy

$$\int_0^\infty \rho(\mu^2)\, d\mu^2 = 1.$$

Deduce from the above that the wave function renormalizaton $Z \leq 1$.

Problem 2
Do Part (a) in the Final Project *"Radiation of Gluon Jets"* of Part I of Peskin and Schroeder.