Northwestern University Department of Physics and Astronomy

PHYSICS 428-2 QUANTUM FIELD THEORY II

Ian Low, Winter 2009

Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFT/QFT_Winter09.html`

ASSIGNMENT #4

Due at 3 PM, February 2nd

(One page and two problems.)

Reading Assignments:
Sections 7.5, 10.2, and 10.3 of Peskin and Schroeder.

Problem 1
Consider the $\lambda\phi^3$ theory in $d = 4$ where the (bare) Lagrangian is of the form:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_0)^2 - \frac{1}{2}m_0^2\phi_0^2 - \frac{\lambda_0}{3!}\phi_0^3$$

(a) Compute the one-loop divergent contribution to the (bare) scalar self-energy $\Sigma^{(0)}(p^2, m_0)$ using dimensional regularization. Write down the counter term you need to introduce in order to remove the divergence in $\Sigma^{(0)}(p^2, m_0)$. Then proceed to renormalize the self-energy by imposing the on-shell condition on the renormalized self-energy:

$$\Sigma^{(R)}(p^2, m)\Big|_{p^2=m^2} = 0.$$

Compute $\Sigma^{(R)}(p^2, m)$ explicitly, even though you do not have to perform the integration over the Feynman parameter.
(b) There is a second divergent diagram at one-loop level, as shown in Fig. 1, which is called the tadpole diagram. Compute the tadpole diagram in dimensional regularization. Determine the counter term which could remove the divergence. Then proceed to compute the counter term by imposing the condition that the tadpole contribution vanishes.

Problem 2
Do Part (b) in the Final Project *"Radiation of Gluon Jets"* of Part I of Peskin and Schroeder.

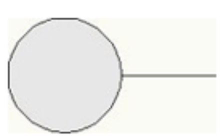

FIG. 1: The tadpole diagram.