Northwestern University Department of Physics and Astronomy

PHYSICS 428-2 QUANTUM FIELD THEORY II

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Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFTII/QFT_Winter09.html`

ASSIGNMENT #5

Due at 3 PM, February 9th

(Two pages and three problems.)

Reading Assignments:

(a) Sections 7.4 and 10.1 of Peskin and Schroeder.

(b) Sidney Coleman's lecture on renormalization from "Aspects of Symmetry." You can download an electronic copy from

`http://www.hep.anl.gov/ian/teaching/QFTII/Coleman_Renormalization.pdf`

Problem 1

Scaleless integrals in DR (Dimensional Regularization) are conventionally set to zero:

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i\epsilon)^n} = 0$$

This is because there is no mass scale on the left-hand side of the equation, the right-hand side cannot depend on any mass scale either and hence is zero. One can make the argument slightly more formal by splitting the integral, using $n = 1$ as an example, into two pieces:

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 + i\epsilon} = \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - M^2 + i\epsilon} - \int \frac{d^d k}{(2\pi)^d} \frac{M^2}{(k^2 + i\epsilon)(k^2 - M^2 + i\epsilon)}$$

where M is an arbitrary mass scale.

(a) Evaluate the two integrals in the right-hand side explicitly. Show that the M^2 dependence indeed cancels and yields a null result.

(b) Determine what ranges of d would render the two terms on the right-hand side convergent, respectively, as well as whether the divergence is in the UV or IR region. There is a value d_0 of d for which both terms on the right are divergent. Take $d \to d_0$ explicitly and show that the $1/\epsilon$ poles cancels among the two terms in the right-hand side:

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 + i\epsilon} = \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} = 0.$$

In other words, DR does not distinguish between UV and IR divergences and scaleless integrals are zero because the IR divergence cancels the UV divergence!

Problem 2

Consider the following bare QED Lagrangian:

$$S = \int d^4 x \left[\bar{\psi}_0 (i\not{\partial} - m_0 - ie_0 \not{A}_0) \psi_0 - \frac{1}{4} (F_0^{\mu\nu})^2 \right].$$

In DR, where $d = 4 - 2\epsilon$, we can introduce the following scaling factors:

$$
\begin{aligned}
\psi_0 &= \mu^{-\epsilon} \sqrt{Z_2}\, \psi \\
A_0^\mu &= \mu^{-\epsilon} \sqrt{Z_3}\, A^\mu \\
m_0 &= Z_m\, m \\
e_0 &= \mu^{\epsilon} Z_g\, e
\end{aligned}
$$

Compute Z_2, Z_3, Z_m and Z_g to order α in the $\overline{MS}$ scheme. Verify the Ward identity $\sqrt{Z_3} Z_g = 1$ or, alternatively, $Z_2 = Z_1$ where $Z_1 = Z_2 \sqrt{Z_3} Z_g$.

Problem 3

Do Problem 7.3 in Peskin and Schroeder.