Northwestern University Department of Physics and Astronomy

PHYSICS 428-2 QUANTUM FIELD THEORY II

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Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFTII/QFT_Winter09.html`

ASSIGNMENT #7

Due at 3 PM, February 23rd

(Two page and three problems.)

Reading Assignments:
Sections 6.4, 7.3, 12.4, and 12.5 of Peskin and Schroeder.

Problem 1
Do Problem 12.1 in Peskin and Schroeder.

Problem 2
In class we showed how to compute in QED the renormalization-group coefficients such as β and γ_m in the MS scheme. The result is summarized as follows:

$$\begin{aligned}
\beta(e,\epsilon) &= \mu\frac{d}{d\mu}e(\mu) = -2\epsilon e - 2eZ_g^{(1)} + 2e\frac{d}{de}(eZ_g^{(1)}) \\
\gamma_m(e) &= \frac{1}{m(\mu)}\frac{d}{d\log\mu}m(\mu) = \frac{1}{2}e\frac{d}{de}Z_m^{(1)}
\end{aligned}$$

where

$$\begin{aligned}
e_0 &= \mu^\epsilon Z_g\, e\ , \qquad Z_g\, e = e\left(1+\sum_{n=1}^{\infty}\frac{Z_g^{(n)}(e)}{\epsilon^n}\right) \\
m_0 &= Z_m\, m\ , \qquad Z_m = 1+\sum_{n=1}^{\infty}\frac{Z_m^{(n)}(e)}{\epsilon^n}
\end{aligned}$$

(a) In a different momentum-independent subtraction scheme such as the $\overline{MS}$ scheme,

$$Z_{g,m} = \left(Z_{g,m}^{(0)}(\epsilon,e) + \sum_{n=1}^{\infty}\frac{Z_g^{(n)}(e)+C_{g,m}^{(n)}}{\epsilon^n}\right)$$

where $C_{g,m}^{(n)}$ are some numerical coefficients independent of e. Derive the β and γ_m in this case. Do you get the same coefficients as in the MS scheme?
(b) In a momentum-dependent subtraction scheme, the scaling factors $Z_{g,m} = Z_{g,m}(e, m/\mu)$ will be functions of e and m/μ in general:

$$Z_{g,m} = \left(Z_{g,m}^{(0)}(\epsilon,e,m/\mu) + \sum_{n=1}^{\infty}\frac{Z_g^{(n)}(e,m/\mu)}{\epsilon^n}\right).$$

(You should think of the scale μ as the mass scale M corresponding to the subtraction point, since the μ scale arising from DR doesn't appear in the counter term anymore.) Derive β and γ_m. Do you get the same coefficients as in other schemes?

Problem 3

Consider in QED the conserved vector current $j^\mu = \bar{\psi}\gamma^\mu\psi(x)$. Define the scaling factor $j^\mu = Z_j^{-1}\bar{\psi}_0\gamma^\mu\psi_0 = Z_2 Z_j^{-1}\bar{\psi}\gamma^\mu\psi$.

(a) Draw all six Feynman diagrams contributing to the Green's function $\langle 0|T(\bar{\psi}(y)\psi(0)j^\mu(x))|0\rangle$ up to order $\alpha = e^2/4\pi$.

(b) Verify explicitly that the conserved vector current is not renormalized. In other words, compute the diagrams and show that $Z_j = 1$.