Northwestern University Department of Physics and Astronomy

PHYSICS 428-2 QUANTUM FIELD THEORY II

Ian Low, Winter 2009

Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFTII/QFT_Winter09.html`

ASSIGNMENT #8

Due at 3 PM, March 2nd

(One page and three problems.)

Reading Assignments:
Sections 6.1, 9.1, and 9.2 of Peskin and Schroeder.

Problem 1
When using Feynman diagrams to compute in perturbation theory, a factor of i is assigned to each vertex and each propagator. Prove that each Feynman amplitude is always real unless the denominator of a propagator vanishes.
(Hint: you will need the identity $L = I - V + 1$, where L is the number of loops, I the number of internal lines, and V the number of vertices in a particular diagram.)

Problem 2
Consider the $\lambda\phi^4$ theory

$$S = \int d^4x \left(\frac{1}{2}\partial^\mu \phi_0 \partial_\mu \phi_0 - \frac{1}{2} m_0^2 \phi^2 - \frac{\lambda}{4!}\phi_0^4 \right)$$

Define the following scaling factors in Dimensional Regularization:

$$\phi_0 = \mu^{-\epsilon}\sqrt{Z_2}\phi, \quad m_0 = Z_m\, m, \quad \lambda_0 = \mu^{2\epsilon} Z_\lambda \lambda$$

(a) Verify that the above definitions render the renormalized quantities with the same engineering dimension as they would have at $d = 4$.
(b) Compute all the Z factors in the MS scheme.

Problem 3
Consider the composite operator $\mathcal{O} = \phi^6(x)$ in the $\lambda\phi^4$ theory. Compute the anomalous dimension $\gamma_{\mathcal{O}}$ at one-loop order.