QFT II, Winter 2009
HW#8 Solutions

Problem 1

Sol'n: Each Feynman amplitude contains I internal propagators, V interaction vertices, and L loop integrations. There is an "i" from each propagator, vertex, and loop integration (Wick rotation!). So the factor of "i" combines

$$(i)^{I+V+L} \sim i\mathcal{M}$$

use $L = I - V + 1 \Rightarrow \mathcal{M} \sim (i)^{2I} = (-1)^I$

Problem 2

$$S = \int d^4x \left(\frac{1}{2}\partial_\mu\phi_0 \partial^\mu\phi_0 - \frac{1}{2}m_0^2\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4 \right)$$

$$\phi_0 = \mu^{-\varepsilon}\sqrt{Z_2}\,\phi\,; \quad m_0 = Z_m m, \quad \lambda_0 = \mu^{2\varepsilon} Z_\lambda \lambda.$$

(a) In d-dim, $[\phi_0] = \frac{d-2}{2} = \frac{(4-2\varepsilon)-2}{2} = 1-\varepsilon$.

$$[\lambda_0] = 4\cdot\frac{d-2}{2} - d = -2\varepsilon.$$

So the above definitions render ϕ, m, λ with canonical dimensions.

(b). The relevant one-loop diagrams have been computed in P&S:

Eq. (10.29), (10.30) $\Rightarrow$

$$-\frac{i}{2}\lambda \frac{\mu^{-d+4}}{(4\pi)^{\frac{d}{2}}} \cdot \frac{\Gamma(1-\frac{d}{2})}{(m^2)^{1-\frac{d}{2}}} + i(p^2\delta_Z - \delta_m)$$

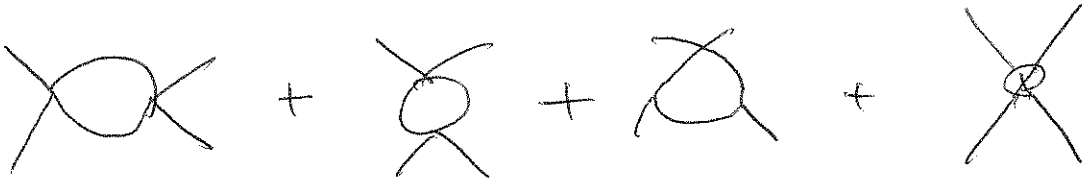
$$= -\frac{i}{2}\lambda \frac{\mu^{+2\varepsilon}}{(m^2)^{\varepsilon}} \cdot \frac{m^2}{16\pi^2} \frac{1}{(4\pi)^{-\varepsilon}} \cdot \frac{\Gamma(\varepsilon)}{-1+\varepsilon} + i(p^2\delta_Z - \delta_m)$$

$$= +\frac{i}{2}\lambda \cdot \frac{m^2}{16\pi^2} \cdot \left(\frac{1}{\varepsilon} - \gamma_E + \log\frac{4\pi\mu^2}{m^2} + \cdots\right) + i(p^2\delta_Z - \delta_m)$$

$$\Rightarrow \quad \delta_Z = 0, \qquad \delta_m = \frac{\lambda}{2}\frac{m^2}{16\pi^2}\cdot\frac{1}{\varepsilon}$$

$$\therefore \quad Z_2 = 1 + \delta_Z = 1.$$

$$Z_m^2 = \frac{m^2 + \delta_m}{m^2 Z_2} = \left(1 + \frac{\lambda}{2}\frac{m^2}{16\pi^2}\frac{1}{\varepsilon}\right) \Rightarrow Z_m = \left(1 + \frac{\lambda}{4}\frac{1}{16\pi^2}\frac{1}{\varepsilon}\right).$$



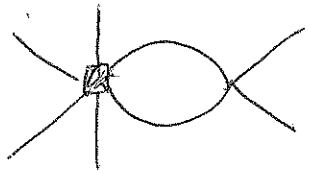
Eq. (10.23) $\Rightarrow$

$$+\frac{i3\lambda^2}{32\pi^2}\int_0^1 dx \left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi + \cdots\right) - i\delta_\lambda.$$

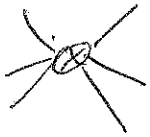
$$\therefore \delta_\lambda = \frac{3\lambda^2}{32\pi^2}\frac{1}{\varepsilon} = Z_\lambda Z_2^2 - 1$$

$$\Rightarrow Z_\lambda = 1 + \delta_\lambda = 1 + \frac{3\lambda^2}{32\pi^2}\frac{1}{\varepsilon}.$$

<u>Problem 3</u> The 1-loop divergent diagrams involving ϕ^6 are of the type



$\rightarrow$ $C_2^6 = 15$ distinctive diagrams

$+\ i$  $-i\delta_{Z_\phi}$.

The amplitude for each diagram is identical to the 4-pt function computed in P&S and <u>Prob 2(b)</u>

$$\Rightarrow \quad C_2^6 \cdot \frac{i\lambda}{32\pi^2}\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi\right) - i\delta_{Z_\phi}$$

$$\Rightarrow \quad \delta_{Z_\phi} = C_2^6 \frac{\lambda}{32\pi^2}\frac{1}{\varepsilon} = \frac{Z_2^2}{Z_{\phi^6}} - 1 = \frac{1}{Z_{\phi^6}} - 1$$

$$\Rightarrow \quad Z_{\phi^6} = 1 - \frac{15\lambda}{32\pi^2}\frac{1}{\varepsilon}.$$

then $$\gamma_{\phi^6} = \mu\frac{d}{d\mu}\log Z_{\phi^6} = \overbrace{\mu\frac{d}{d\mu}\lambda}^{\beta_\lambda = -2\lambda\varepsilon + \cdots} \cdot \frac{\partial}{\partial\lambda}\log Z_{\phi^6}$$

$$= \frac{15\lambda}{32\pi^2} \times 2 = \frac{15\lambda}{16\pi^2}$$