Northwestern University Department of Physics and Astronomy

PHYSICS 428-3 QUANTUM FIELD THEORY III

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Course Webpage: `http://www.hep.anl.gov/ian/teaching/QFTIII/QFT_Spring09.html`

ASSIGNMENT #1

Due at 11 AM, April 6th

(One page and two problems in total.)

Reading Assignments:
Chapters 15 and 16 of Peskin and Schroeder.

Problem 1
The Lagrangian for an $SU(N_c)$ gauge theory with one flavor of Dirac fermion in the fundamental representation is given in Eqs. (16.86) and (16.87) in Peskin and Schroeder, which uses the Feynman-'t Hooft gauge $\xi = 1$.
(a) Use Dimensional Regularization to compute all the counter terms to one-loop order in the *minimal subtraction* (MS) scheme. (That is adjust your counter terms so that the $1/\epsilon$ pole is removed from the renormalized quantities.) Draw all the relevant Feynman diagrams in your calculation.
(b) Define the scaling factors as

$$\psi_0 = \sqrt{Z_\psi}\psi, \quad A_0^{a\,\mu} = \sqrt{Z_A}A^{a\,\mu}, \quad g_0 = \mu^\epsilon Z_g g, \quad m_0 = Z_m m, \quad c_0^a = \sqrt{Z_c}c^a.$$

Compute all the Z factors in the MS scheme. Since Z_g can be obtained from four different counter terms in (a), explicitly verify that all four counter terms result in identical Z_g. Furthermore, verify the Taylor-Slavnov identities

$$\frac{Z_1}{Z_\psi} = \frac{Z_1^{3g}}{Z_A} = \frac{Z_1^c}{Z_c} = \left(\frac{Z_1^{4g}}{Z_A}\right)^{\frac{1}{2}}$$

(c) Use your results in (a) and (b) to compute the one-loop beta function $\beta(\epsilon, g)$ with n_f flavors of Dirac fermion. Be sure to keep the ϵ dependence in the $\beta(\epsilon, g)$, and verify that in the limit $\lim_{\epsilon\to 0}\beta(\epsilon, g) = \beta(0, g)$ agrees with Eq. (16.85) in Peskin and Schroeder.

Problem 2
Do Problem 16.2 in Peskin and Schroeder.