QUANTUM FIELD THEORY 3 - SPRING 2009
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Homework 5 - Due Friday, May 22

There are three problems. Please try all of them.

1. **Toy Two Higgs U(1) Model** – Lets us consider a $U(1)$ gauge theory with two charged scalar fields ϕ_1 and ϕ_2, with charges q_1 and q_2.

   (a) Assume that both ϕ_1 and ϕ_2 acquire nonzero vacuum expectation values v_1 and v_2. What is the photon mass in this case as a function of $v_1/\sqrt{2}, v_2/\sqrt{2}$, q_1, q_2 and the gauge coupling g? How many scalar degrees of freedom does this system describe (after the gauge bosons "eat" the goldstone bosons)?

   (b) Assume that the scalar potential is given by (this depends on the relative sizes of q_1 and q_2 and other global symmetries that the system may contain)

$$V(\phi_1, \phi_2) = -m_1^2|\phi_1|^2 - m_2^2|\phi_2|^2 + \lambda_1|\phi_1|^4 + \lambda_2|\phi_2|^4 + 2\lambda_{12}|\phi_1|^2|\phi_2|^2, \quad (1)$$

   where m_1^2, m_2^2 are positive. What are the conditions on the parameters of the potential such that nonzero, finite v_1 and v_2 really are the minima of V? How many massive scalars are there, and what are their masses?

2. **Higgs Triplet** – Imagine that there is a Higgs $SU(2)_L$ triplet, (χ^+, χ^0, χ^-), in the Standard Model and that its electrically neutral component χ^0 develops a vacuum expectation value (vev) ϵ in addition to the ordinary Higgs doublet vev v. Here χ^+ is the antiparticle of the χ^- state, and $(\chi^0)^\dagger = \chi^0$.

   (a) Write down the kinetic energy term for χ, including the gauge interactions. Note that you need to decide what its hypercharge is (you need to know the three dimensional representation of $SU(2)$. It is given by $T_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $T_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$, and $T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$).

   (b) A nonzero ϵ potentially modifies both the Z and W boson masses. In particular it modifies the ration m_W/m_Z. Compute the modified gauge boson masses and m_W^2/m_Z^2. The $\epsilon = 0$ prediction $m_W^2 = m_Z^2 \cos^2\theta_W$ agrees with experiment with an error of ± 0.1 GeV. Use this fact to place an upper bound on ϵ.

3. **Exotic Heavy Leptons** – If both the right and left chiral components of a fermion field have the same transformation properties under $SU(2)_L$, it is allowed to have a mass independent of the Higgs vacuum expectation value. These exotic fermions are constrained to be heavy, given that they couple to the photon and the heavy gauge bosons (and we haven't seen them yet).

   (a) Suppose that there is a singly charged lepton L^+ whose right and left-handed chiral fields are both $SU(2)_L$ singlets. Determine the coupling of L^+ to the Z-boson in terms of the electric charge e and the Weinberg angle θ_W.

   (b) Suppose that there is a new lepton doublet $\mathbf{L} = (L^{++}, L^+)^t$ and that *both* its left and right chiral components transform as $SU(2)$ doublets. What is the Z coupling to L^+ and L^{++}? What about the W coupling?