QUANTUM FIELD THEORY 3 - SPRING 2009
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Homework 6 - Due Friday, June 5

There are three problems. Please try all of them.

1. **Higgs Decays** – We discussed in class that the Higgs boson (h) likes to decay into heavy gauge bosons and fermions.

   (a) Compute $\Gamma(h \to \bar{f}f)$ for $f = \tau, c, b, t$. Remember that the quarks come in three colors. Note that, as long as you don't neglect the final state fermion mass in your computation, you only need to compute one diagram.

   (b) Compute $\Gamma(h \to W^+W^-)$ and $\Gamma(h \to Z^0Z^0)$, remember that the Z^0 is a neutral, self-conjugate boson (this is relevant when you compute the phase-space integral for the Z^0Z^0 final state). Be sure to keep the gauge boson mass in your computation!

   (c) Plot the branching ratio of the Higgs boson into τ's, c's, b's, t's, W's and Z's as a function of the Higgs mass, assuming that the partial widths you computed above are all the allowed Higgs decay modes. Also plot the Higgs boson total decay width as a function of the Higgs mass.

2. **Higgs Boson Scattering in the Unitary Gauge** – Assume a $U(1)$ gauge theory with one charge=+1 scalar field, Φ, described by the following Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu\Phi)^\dagger D^\mu\Phi + \mu^2|\Phi|^2 - \lambda|\Phi|^4, \tag{1}$$

   where $D_\mu = \partial_\mu - ieA_\mu$ and $\mu^2 > 0$, such that the $U(1)$ gauge invariance is spontaneously broken.

   (a) Rewrite the Lagrangian in terms of v and h, defined as $\Phi = e^{i\theta}(v+h)/\sqrt{2}$, after the θ field is "gauged away."

   (b) Draw the Feynman diagram(s) for $h + h \to A + A$, where A is the quantum created and destroyed by A_μ (the massive gauge boson), and write down the matrix element (you don't have to compute the cross-section or even the matrix element squared!)

3. **Higgs Boson Scattering in the R_ξ Gauge**

   (a) Rewrite the Lagrangian in Eq. (1) in terms of ϕ_1 ϕ_2 and v, defined by $\Phi = (v+\phi_1+i\phi_2)/\sqrt{2}$, after adding the gauge-fixing Lagrangian $\mathcal{L}_{\text{gauge fix}} = -(\partial^\mu A_\mu + \xi ev\phi_2)^2/2\xi$.

   (b) Draw the Feynman diagram(s) for $\phi_1 + \phi_1 \to A + A$, and write down the matrix element in an arbitrary R_ξ gauge (This means you are not allowed to choose, say, $\xi = 0$. Again, you don't have to compute the cross-section or the matrix element squared!). We wrote down the propagator for the gauge field and the scalars in class, for arbitrary ξ. Show that the matrix element does not depend on ξ, even if independent diagrams do. How does your answer compare with the answer you got for 2(b)?