

1.5

$$(a) \left(\frac{5}{6}\right)^N$$

(b) (Probability of being shot on the N^{th} trial)

$$= (\text{Probability of surviving } N-1 \text{ trials}) \times (\text{Probability of shooting oneself on the } N^{\text{th}} \text{ trial})$$

$$= \left(\frac{5}{6}\right)^{N-1} \left(\frac{1}{6}\right)$$

(c) Probability of being shot on the N^{th} trial = $\left(\frac{5}{6}\right)^{N-1} \left(\frac{1}{6}\right)$

$$\Rightarrow \text{the mean number of times} = \sum_{N=1}^{\infty} \left(\frac{5}{6}\right)^{N-1} \left(\frac{1}{6}\right) \times N = 6$$

1.7

The probability of n successes out of N trials is given by the sum

$$W(n) = \sum_{i=1}^2 \sum_{j=1}^2 \cdots \sum_{m=1}^2 w_i w_j \cdots w_m$$

with the restriction that the sum is taken only over terms involving w_i n times.

$$\text{Then } W(n) = \sum_{i=1}^2 w_i \sum_{j=1}^2 w_j \cdots \sum_{m=1}^2 w_m$$

If we sum over all $i \cdots m$, each sum contributes $(w_1 + w_2)$ and

$$W'(n) = (w_1 + w_2)^N$$

$$W'(n) = \sum_{n=0}^N \frac{N!}{n!(N-n)!} w_1^n w_2^{N-n} \quad \text{by the binomial theorem.}$$

Applying the restriction that w_i must occur n times we have

$$W(n) = \frac{N!}{n!(N-n)!} w_1^n w_2^{N-n}$$

1.10

$$(a) \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} = e^{-\lambda} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^{-\lambda} e^{\lambda} = 1$$

$$(b) \bar{n} = \sum_{n=0}^{\infty} \frac{n \lambda^n e^{-\lambda}}{n!} = \lambda e^{-\lambda} \frac{\partial}{\partial \lambda} \left(\sum \frac{\lambda^n}{n!} \right) = \lambda e^{-\lambda} e^{\lambda} = \lambda$$

$$(c) \bar{n}^2 = \sum_{n=0}^{\infty} \frac{n^2 \lambda^n}{n!} e^{-\lambda} = e^{-\lambda} \left(\lambda \frac{\partial}{\partial \lambda} \right)^2 \left(\sum \frac{\lambda^n}{n!} \right) = \lambda^2 + \lambda$$

$$\overline{(\Delta n)^2} = \bar{n}^2 - \bar{n}^2 = \lambda$$

1.11

(a) The mean number of misprints per page is 1.

Thus $w(n) = \frac{\lambda^n}{n!} e^{-\lambda} = \frac{e^{-1}}{n!}$

and $w(0) = e^{-1} = 0.37$

(b) $P = 1 - \sum_{n=0}^{\infty} \frac{e^{-1}}{n!} = 0.08$