2.5

(a) $$đf = A\,dx + B\,dy$$

If $đf$ is an exact differential, $A = \frac{\partial f}{\partial x}$ $\quad B = \frac{\partial f}{\partial y}$

Since second derivatives are equal, $\frac{\partial A}{\partial y} = \frac{\partial B}{\partial x}$

(b) If $đf$ is exact, $\int_A^B đf = f(B) - f(A) = 0$ if $A = B$

2.7

(a) The particle in a state with energy E does work, $đW$, when the length of the box is changed to $L_x + dL_x$. This work is done at the expense of the energy, i.e., $đW = -dE$. Since $đW = F_x\,dL_x$, it follows that $F_x = -\frac{\partial E}{\partial L_x}$.

(b). The energy levels are given by $E = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right)$

$$p = \frac{F_x}{L_y L_z} = -\frac{1}{L_y L_z}\frac{\partial E}{\partial L_x} = \frac{\pi^2\hbar^2}{m L_x L_y L_z}\left(\frac{n_x^2}{L_x^2}\right)$$

$$\bar{p} = \frac{\pi^2\hbar^2}{mV}\left(\frac{\overline{n_x^2}}{L_x^2}\right) \qquad \text{where} \quad V = L_x L_y L_z$$

If $L_x = L_y = L_z$, by symmetry $\overline{n_x^2} = \overline{n_y^2} = \overline{n_z^2}$, and using the expression for E we have $\bar{p} = \frac{2}{3}\frac{\bar{E}}{V}$

2.11

$\bar{p} = \alpha V^{-5/3}$, α may be found by evaluating this expression at a point on the curve. In units of 10^6 dynes/m^2 and 10^3 cm^3, $\alpha = 32$. For the adiabatic process, $\Delta E = E_B - E_A = -\int_1^8 32 V^{-5/3}\,dV = -36 = -3600$ joules (J)

Since energy is a state function, this is the energy change for all paths.

(a) $$W = 32\int_1^8 dV = 22400 \text{ J}$$

$$Q = \Delta E + W = -3600 + 22400 = 18800 \text{ J}$$

(b) $p = 32 - \frac{31}{7}(V-1)$

$$W = \int_1^8 \left[32 - \frac{31}{7}(V-1)\right] dV = 11550 \text{ J}$$

$$Q = 11550 - 3600 = 7950 \text{ J}$$

(c) $W = \int_1^8 (1)\, dV = 700 \text{ J}$

$$Q = 700 - 3600 = -2900 \text{ J}$$

3.2

From problem 2.4 (b)

(a) $$\ln \Omega(E) = -\frac{1}{2}\left(N - \frac{E}{\mu H}\right) \ln \frac{1}{2}\left(1 - \frac{E}{N\mu H}\right) - \frac{1}{2}\left(N + \frac{E}{\mu H}\right) \ln \frac{1}{2}\left(1 + \frac{E}{N\mu H}\right)$$

where we have neglected the small term $\ln \frac{\delta E}{2\mu H}$.

$$\beta = \frac{\partial}{\partial E} \ln \Omega(E) = \frac{1}{2\mu H} \ln \frac{\frac{1}{2}\left(1 - \frac{E}{N\mu H}\right)}{1 - \frac{1}{2}\left(1 - \frac{E}{N\mu H}\right)}$$

Consequently $E = -N\mu H \tanh \frac{\mu H}{kT}$

(b) $T < 0$ if $E > 0$

(c) $M = \mu(n_1 - n_2) = \mu(2n_1 - N)$ where n_1 is the number of spins alligned parallel to H.

Since $n_1 = \frac{1}{2}\left(N - \frac{E}{\mu H}\right)$ from problem 2.4 (a),

$$M = \mu\left(N - \frac{E}{\mu H} - N\right) = -\frac{E}{H} = N\mu \tanh \frac{\mu H}{kT}$$