2.3

(a) The probability of displacement x is the probability that φ assumes the required value in the expression $x = A\cos(\omega t + \varphi)$.

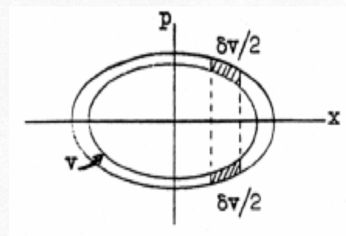
$$w(\varphi)\,d\varphi = \frac{d\varphi}{2\pi} \qquad \text{and} \qquad P(x)\,dx = 2w(\varphi)\,d\varphi = \frac{d\varphi}{\pi}$$

where the factor of 2 is introduced because two values of φ give the same x.

Since $\quad \left|\frac{d\varphi}{dx}\right| = \frac{1}{A\sin(\omega t+\varphi)} = \frac{1}{\sqrt{A^2-x^2}}$

we have $\quad P(x)\,dx = \frac{dx}{\pi\sqrt{A^2-x^2}}$

(b) We take the ratio of the occupied volume, δV, in phase space when the oscillator is in the range x to $x+dx$ and E to $E+\delta E$, to the total accessible volume, V.



The area, A_e, of the ellipse is $\pi p_{max} x_{max}$.

Since $\quad E = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} \quad (1)$

we have $\quad A_e = \pi\sqrt{2mE}\sqrt{\frac{2E}{m\omega^2}} = \frac{2\pi E}{\omega}$, and $V = \frac{2\pi}{\omega}\delta E$.

Hence $\quad P(x)\,dx = \frac{\delta V}{V} = \frac{2\,\delta p\,\delta x}{\frac{2\pi}{\omega}\delta E}$

For any x, $\quad p\frac{\delta p}{m} = \delta E$

$$P(x)\,dx = \frac{m\omega}{\pi}\frac{dx}{p} = \frac{m\omega}{\pi}\frac{dx}{\sqrt{2mE - m^2\omega^2x^2}}$$

where we have used (1). From (1) and $x = A\cos(\omega t+\varphi)$, it follows that $E = \frac{1}{2}\omega^2 m A^2$.

Thus $\quad P(x)\,dx = \frac{1}{\pi}\frac{dx}{\sqrt{A^2-x^2}}$

3.3

(a) For system A, we have from 2.4 (c) $\ln\Omega(E) = -\frac{E^2}{2\mu^2H^2N} + \frac{\ln 2^N}{\sqrt{\pi N/2}} \quad (1)$

where we have neglected the term $\ln\frac{\delta E}{2\mu H}$

$$\beta = \frac{\partial}{\partial E}\ln\Omega(E) = -\frac{E}{\mu^2H^2N}, \quad \beta' = -\frac{E'}{\mu'^2H^2N'}$$

In equilibrium, $\beta = \beta'$. $\quad$ Hence $\quad \frac{\tilde{E}}{\mu^2 N} = \frac{\tilde{E}'}{\mu'^2 N'}$

(b) Since energy is conserved, $\tilde{E} + \tilde{E}' = bN\mu H + b'N'\mu' H$

Substituting from (a) we find $\tilde{E} = \dfrac{\mu^2 N (bN\mu H + b'N'\mu' H)}{\mu^2 N + \mu'^2 N'}$ (2)

(c) $$Q = \tilde{E} - bN\mu H = \frac{NN'H(b'\mu'\mu^2 - b\mu'^2\mu)}{\mu^2 N + \mu'^2 N'} \quad \text{from (2)}$$

(d) $P(E) \propto \Omega(E)\Omega'(E_0 - E)$ where E_0 is the total energy, $E_0 = bN\mu H + b'N'\mu' H$.

From (1) $$P(E) \propto \exp\left[-\frac{E^2}{2\mu^2 H^2 N}\right] \exp\left[-\frac{(E_0 - E)^2}{2\mu'^2 H^2 N'}\right] = c' \exp\left[-\frac{(E-\tilde{E})^2}{2\sigma^2}\right]$$

$$P(E)dE = C \exp\left[-\frac{(E-\tilde{E})^2}{2\sigma^2}\right] dE$$

where $$\sigma^2 = \frac{\mu^2 \mu'^2 H^2 N N'}{\mu^2 N + \mu'^2 N'}$$

The constant C is determined by the normalization requirement

$$\int_{-\infty}^{\infty} C \exp\left[-\frac{(E-\tilde{E})^2}{2\sigma^2}\right] dE = 1$$

Thus $P(E)dE = \dfrac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\dfrac{(E-\tilde{E})^2}{2\sigma^2}\right] dE$, where σ is given above.

(e) $$\overline{(\Delta^* E)^2} = \sigma^2 = \frac{\mu^2 \mu'^2 H^2 N N'}{\mu^2 N + \mu'^2 N'} \quad (3)$$

(f) From equation (3), $\Delta^* E \approx \mu H \sqrt{N}$ $\quad N' \gg N$

$$\tilde{E} = \frac{\mu^2 N (bN\mu H + b'N'\mu' H)}{\mu^2 N + \mu'^2 N'} \approx b' \frac{\mu^2 N H}{\mu'} \quad N' \gg N$$

Hence $$\left|\frac{\Delta^* E}{\tilde{E}}\right| = \frac{\mu'}{\mu b' \sqrt{N}}$$

3.5

(a) In section 2.5 it is shown that $\Omega(E) \propto V^N \chi(E)$, where V is the volume and $\chi(E)$ is independent of volume. Then for two non-interacting species with total energy E_0

$$\Omega(E) = C\Omega_1(E)\Omega_2(E_0 - E) = CV^{N_1+N_2}\chi_1(E)\chi_2(E)$$

(b) $\ln \Omega(E) = (N_1 + N_2)\ln V + \ln C + \ln \chi_1(E)\chi_2(E)$

$$\bar{p} = \frac{1}{\beta}\frac{\partial}{\partial V} \ln \Omega(E) = \frac{1}{\beta}\frac{(N_1+N_2)}{V}, \quad \text{or} \quad \bar{p}V = (N_1+N_2)kT$$

3.6 2 atmospheres.

There is 1 atm helium and 1 atm air inside the bulb, because the particular glass of which the bulb is made is quite permeable only to helium.