7.6

Since the total energy is additive,

$$E_i = \epsilon_i(p_i) + U(q_1 \cdots q_n) = \frac{p^2}{2m} + U(q_1 \cdots q_n)$$

Where U is the energy of interaction, the equipartition theorem still applies and

$$\bar{\epsilon} = \frac{3}{2}kT$$

7.7

If the gas is ideal, its mean energy per particle is $\bar{\epsilon} = \frac{\overline{p_x^2}}{2m} + \frac{\overline{p_y^2}}{2m} = kT$

and the mean energy per mole becomes $\bar{E} = N_A kT$

Thus $$C = \frac{\partial \bar{E}}{\partial T} = N_A k = R$$

7.10

(a) Let the restoring force be $-\alpha x$. Then the mean energy of N particles is

$$\bar{E} = N\left(\frac{1}{2}m\overline{\dot{x}^2} + \frac{1}{2}\alpha\overline{x^2}\right)$$

By equipartition $\bar{E} = N\left(\frac{1}{2}kT + \frac{1}{2}kT\right) = NkT$

Thus $$C = \frac{\partial \bar{E}}{\partial T} = Nk$$

(b) If the restoring force is $-\alpha x^3$, the mean energy per particle is

$$\bar{\epsilon} = \frac{1}{2}m\overline{\dot{x}^2} + \frac{1}{4}\alpha\overline{x^4}$$

The kinetic energy term contributes $\frac{1}{2}kT$ by equipartition. The mean of the potential energy is found from

$$\overline{\frac{\alpha x^4}{4}} = \frac{\int_{-\infty}^{\infty} \exp\left[-\frac{\beta\alpha x^4}{4}\right]\frac{\alpha x^4}{4}\,dx}{\int_{-\infty}^{\infty} \exp\left[-\frac{\beta\alpha x^4}{4}\right]dx}$$

$$= -\frac{\partial}{\partial\beta}\ln\int_{-\infty}^{\infty} e^{-\beta(\alpha x^4/4)}\,dx = -\frac{\partial}{\partial\beta}\ln\int_{-\infty}^{\infty} e^{-\alpha y^4/4}\,\beta^{-1/4}\,dy$$

where we have made the substitution, $y = \beta^{1/4}x$.

Thus $$\overline{\frac{\alpha x^4}{4}} = -\frac{\partial}{\partial\beta}\left(-\frac{1}{4}\ln\beta + \ln\int_{-\infty}^{\infty} e^{-\alpha y^4/4}\,dy\right) = \frac{1}{4\beta} = \frac{1}{4}kT$$

Hence $$\bar{\epsilon} = \frac{1}{2}m\overline{\dot{x}^2} + \frac{1}{4}\alpha\overline{x^4} = \frac{1}{2}kT + \frac{1}{4}kT = \frac{3}{4}kT$$

Then the total energy is $\bar{E} = N\bar{\epsilon}$ and $C = \frac{\partial \bar{E}}{\partial T} = \frac{3}{4}Nk$

8.2

(a) For the solid, $\ln P = 23.03 - \frac{3754}{T}$ and for the liquid $\ln P = 19.49 - \frac{3063}{T}$. At the triple point, the pressures of the solid and liquid are equal.

$$23.03 - \frac{3754}{T} = 19.49 - \frac{3063}{T}$$

hence $$T = \frac{691}{3.54} = 195\,K$$

(b) Since $\ln P = -\frac{l}{RT} + \text{constant}$ where l is the latent heat, we have

$$l_{\text{sublimation}} = 3754\,R = 31220 \text{ joules/mole}$$

$$l_{\text{vaporization}} = 3063\,R = 25480 \text{ joules/mole}$$

(c) $$l_{\text{melting}} = l_{\text{sublimation}} - l_{\text{vaporization}}$$

$$= 31220 - 25480 = 5740 \text{ joules/mole}$$