

1.7.



1) Mercury thermometer: calculation example:

My thermometer has a cylindrical bulb with diameter as 3 mm and 10 mm long. When temperature increase 1°C , the mercury increase volume by:

$$\Delta V = \beta \cdot \Delta T \cdot V = 1.81 \times 10^{-4} \text{ K}^{-1} \cdot (1 \text{ K}) \cdot \left(\pi \left(\frac{3 \text{ mm}}{2}\right)^2 \cdot 10 \text{ mm}\right)$$
$$= 0.0128 \text{ mm}^3$$

The expansion in the narrow tube is about $\ell = 8 \text{ mm}$ length. Assuming this narrow tube is cylindrical, the radius should be:

$$r = \sqrt{\frac{\Delta V}{\pi \ell}} = \sqrt{\frac{0.0128 \text{ mm}^3}{\pi \cdot 8 \text{ mm}}} = 0.026 \text{ mm} \quad \text{diameter} = 2r = 0.052 \text{ mm}$$

2) When lake is freezing, warm water is ~~denser~~, more dense, and it will sink to the bottom. The water between $0^\circ\text{C} - 4^\circ\text{C}$ will rise to the top. So the water will begin to freeze from the top. Since ice is even less dense, ice will float on the surface. Meanwhile, bottom water will be insulated from the cold weather by the ice, so only a layer of lake freeze ~~is~~ during the winter.

If the thermal expansion coeff is ^{always} positive, the coldest water in lakes will always sink to bottom, so ice would form from the bottom.

1.17 (12 points)

a) $PV = nRT \left(1 + \frac{B(T)}{V/n}\right)$

One way to do it is solving the equation $P\left(\frac{V}{n}\right) = RT \left(1 + \frac{B(T)}{V/n}\right)$ with $\left(\frac{V}{n}\right)$ as the variable. But here we want to introduce an estimation method which will always be useful in the future.

Since the second term $\frac{B(T)}{V/n}$ is small enough, we first assume that $PV_0 = nRT$ to solve $\frac{V_0}{n} = \frac{RT}{P}$ (e.g. $P = 1 \text{ atm}$, $T = 100 \text{ K}$, $\frac{V}{n} = \frac{(8.315 \text{ J/mol} \cdot \text{K}) 100 \text{ K}}{1.013 \times 10^5 \text{ Pa}} = 0.00823 \text{ m}^3/\text{mol}$)

Then we plug in V_0/n into $\frac{B(T)}{V_0/n}$ get the second correction term. (In this way we can solve $PV_1 = nRT \left(1 + \frac{B(T)}{V_0/n}\right)$. for the first-order corrected V .)

MATLAB solve $P\left(\frac{V}{n}\right) = RT \left(1 + \frac{B(T)}{V/n}\right)$ results:

$T \text{ (K)}$	$B/(V/n)$
100	-0.0195
200	-0.0021
300	-0.0001706
400	+0.0002741
500	+0.0004118
600	+0.0004325

Estimation Method Results:

$T \text{ (K)}$	$B/(V/n)$
100	-0.0195
200	-0.0021
300	-0.00017056
400	+0.00027411
500	+0.00041178
600	+0.00043249

b) At low temperature, molecule moves slow, the attractive force dominates so that decrease the pressure. ($B(T)$ is negative). At high temperature, molecules move very fast ^{so} that they have more "touching"

c) Rewrite van der Waals equation:

$$PV = nRT \left(1 - \frac{nb}{V}\right)^{-1} - \frac{an^2}{V}$$

Remind ourselves the hint:

$$\left(1 - \frac{nb}{V}\right)^{-1} \approx 1 + \frac{nb}{V} + \frac{n^2 b^2}{V^2}$$

Plug it into the above equation:

$$\begin{aligned} PV &= nRT \left(1 + \frac{nb}{V} + \frac{n^2 b^2}{V^2}\right) - \frac{an^2}{V} \\ &= nRT \left[1 + \frac{b}{V/n} - \frac{a}{RT(V/n)} + \frac{b^2}{(V/n)^2}\right] \end{aligned}$$

$$B(T) = b - \frac{a}{RT} \quad C(T) = b^2$$

d) I use the "polyfit" in Matlab. get $a = 0.1823$ $b = 0.000642$

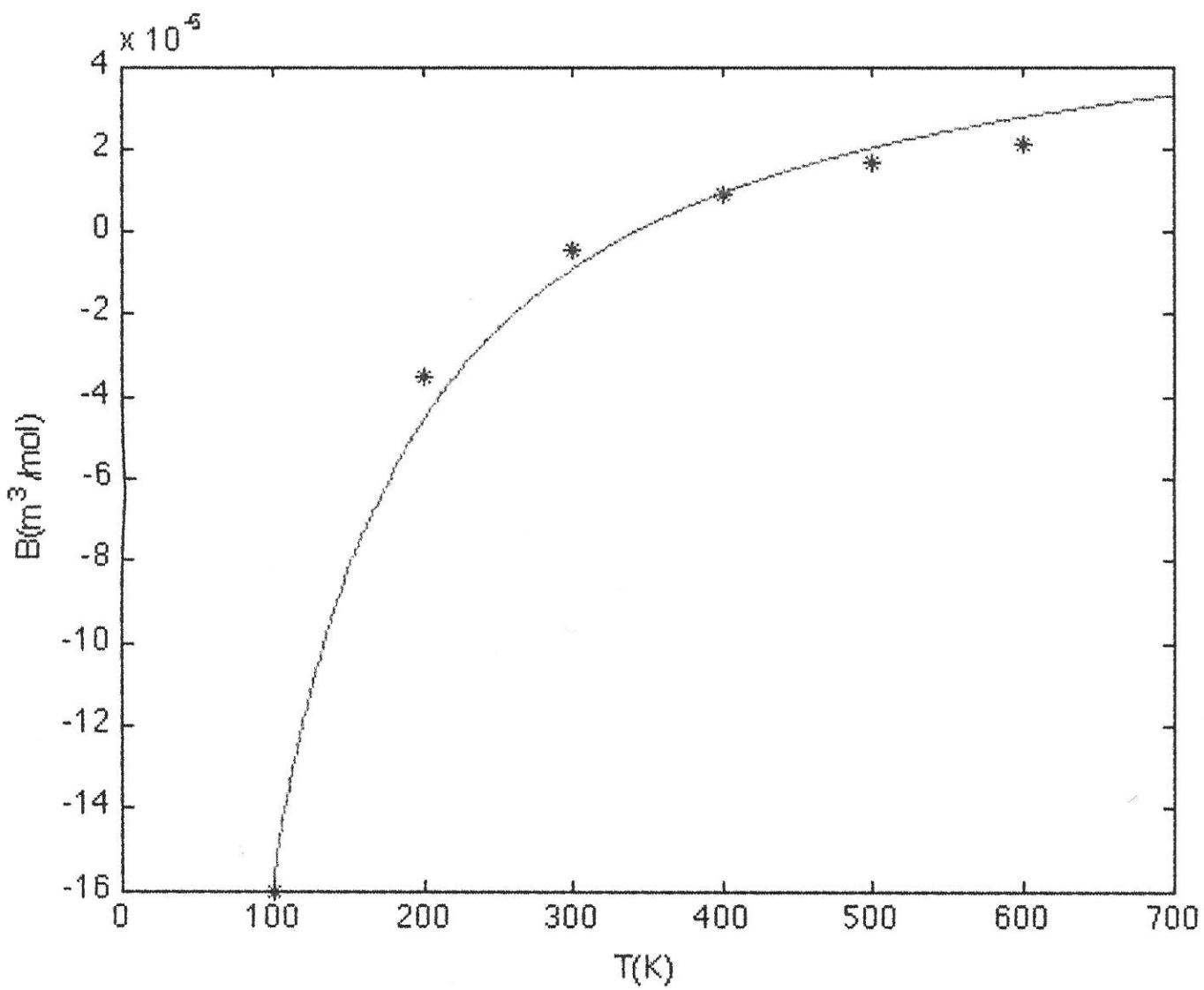
code is:

$$\begin{aligned} X &= \text{ones}(1, 6) ./ (8.315 * [100:100:600]); \quad \% \frac{1}{RT} \\ B &= [-160, -35, -4.2, 9.0, 16.9, 21.3] * 10^{-6}; \quad \% B \end{aligned}$$

$$P = \text{polyfit}(X, B, 1)$$

$$P = \underline{\underline{[-0.1823, 0.0001]}}$$

$$\begin{array}{c} \uparrow \quad \uparrow \\ \boxed{-a} \quad \boxed{b} \end{array}$$



1.22. (12 points)

a) (1.9) $\bar{P} = -\frac{m(\frac{\Delta V_x}{\Delta t})}{A}$ is the average pressure of one molecule colliding on the wall.
(2 pts)

Assume N molecules, elastic collision $\Rightarrow \Delta V_x = -2V_x$

$$\bar{P} = \overline{N \bar{P}} = \frac{m \cdot 2N \bar{V}_x}{A \Delta t} \Rightarrow N = \frac{\bar{P} A \Delta t}{2m \bar{V}_x}$$

b) (1.15) $KT = m \bar{V}_x^2$

(1 pt)
 $\Rightarrow (\bar{V}_x^2)^{1/2} = \sqrt{\frac{KT}{m}}$

c) Now all the molecules colliding the hole will escape
(3 pts)

$$-\Delta N = \frac{\bar{P} A \Delta t}{2m \bar{V}_x} \propto \frac{\bar{P} A \Delta t}{2m (\bar{V}_x^2)^{1/2}} = \frac{\bar{P} A \Delta t}{2m} \sqrt{\frac{m}{KT}}$$

$$\frac{dN}{dt} = \frac{\Delta N}{\Delta t} = - \frac{\bar{P} A \Delta t}{2m} \sqrt{\frac{m}{KT}}$$

Plug $PV = NKT$ into the above equation:

$$\frac{dN}{dt} = - \frac{A}{2V} \sqrt{\frac{KT}{m}} N = -\frac{1}{\tau} N$$

$$N(t) = N(0) e^{-t/\tau} \quad \tau = \frac{2V}{A} \sqrt{\frac{m}{KT}}$$

d)
(2 pts) $\sqrt{\frac{m}{KT}} = \sqrt{\frac{M}{RT}}$: M is the molar mass of air

$$\tau = \frac{2V}{A} \sqrt{\frac{M}{RT}} = \frac{2 \cdot (0.001 \text{ m}^3)}{(10^{-6} \text{ m}^2)} \sqrt{\frac{0.029 \text{ kg/mol}}{8.3 \text{ J/mol} \cdot \text{K} \cdot 300 \text{ K}}} = 6.8254 \text{ sec}$$

e) $\tau = 1 \text{ hour} = 3600 \text{ s}$

(2 pts) Assume the tire can be estimated as a cylinder with radius $r = 3 \text{ cm}$ length = 2 m . $V = \pi (0.03 \text{ m})^2 \cdot (2 \text{ m}) = 0.0057 \text{ m}^3$

$$A = \frac{2V}{\tau} \sqrt{\frac{M}{RT}} = 1.0807 \times 10^{-8} \text{ m}^2$$

f)
(2 pts)

Assume $A = 0.2 \text{ m}^2$ $V = 100 \text{ m}^3$

$$\tau = \frac{2V}{A} \sqrt{\frac{M'}{RT}} = \frac{2 \cdot 100 \text{ m}^3}{0.2 \text{ m}^2} \sqrt{\frac{0.029 \text{ kg/mol}}{8.3 \text{ J/mol} \cdot \text{K} \cdot 300 \text{ K}}} = 3.4127 \text{ s}$$

Assume they open the door for one second

$$N(t) = N(0) e^{-ts/3.4127 \text{ s}} = 0.746 N(0)$$

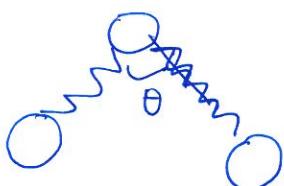
Almost 25% air will be lost.

1.25 (10 pts)

Translation of H_2O molecule: 3

Rotation of H_2O molecule: 3

Vibration: Each "spring" between H atom and O atom
is a vibration mode. "Stretching mode"



The angle between two springs formulate
another mode. Two H atoms move towards
or away from each other.

Three vibration mode, each mode has two freedoms (kinetic
and potential). $3 \times 2 = 6$ freedom.

We have 12 freedom int total.

a) (3 pts)

Isothermal compression: ($PV = NKT$)

$$\frac{dP}{dV} = \frac{d(\frac{NKT}{V})}{dV} = -\frac{NKT}{V^2}$$

$$B_{\text{isothermal}} = -\frac{dP}{dV} \cdot V = \frac{NKT}{V} = P$$

Adiabatic compression: ($PV^\gamma = \text{const}$)

$$\frac{dP}{dV} = \frac{d(\text{constant} \cdot V^{-\gamma})}{dV} = -\frac{\gamma}{V^{\gamma+1}} \cdot \text{constant}$$

$$B_{\text{adiabatic}} = -\frac{dP}{dV} \cdot V = \frac{\gamma}{V^{\gamma+1}} \cdot \text{const.} = \gamma \cdot P$$

b) Sound always travels faster than heat flow. So it is impossible to accomplish any temperature equilibrium during the sound travelling process. We should regard sound travel as an adiabatic process.

c) $\rho = \frac{Nm}{V} = \frac{mP}{kT} = \frac{MP}{RT}$

m is the average molecule mass, M is the molar mass.

$$c = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P RT}{MP}} = \sqrt{\frac{\gamma RT}{M}}$$

At room temperature $\gamma = \frac{f+2}{f} = \frac{5+2}{5} = 1.4$

$$c = \sqrt{\frac{1.4 \times 8.31 \text{ J/K} \times 300 \text{ K}}{0.029 \text{ kg}}} = 347 \text{ m/s} \quad \frac{c}{c_{\text{rms}}} = \sqrt{\frac{1.4}{3}} = 0.68$$

d) For high altitude, P will decrease. But from $c = \sqrt{\frac{\gamma RT}{M}}$ we find speed of sound is only the function of Temperature.

1.45 (12 pts)

a) $w = xy = x\left(\frac{x}{z}\right) = \frac{x^2}{z}$

$$w = xy = (yz) \cdot y = y^2 z$$

b) $\left(\frac{\partial w}{\partial x}\right)_y = y$

$$\left(\frac{\partial w}{\partial x}\right)_z = \frac{2x}{z} = \frac{2(yz)}{z} = 2y$$

} \neq

c) $\left(\frac{\partial w}{\partial y}\right)_x = x$

$$\left(\frac{\partial w}{\partial y}\right)_z = 2yz = 2x$$

} \neq

$$\left(\frac{\partial w}{\partial z}\right)_x = -\frac{x^2}{z^2} = -y^2$$

$$\left(\frac{\partial w}{\partial z}\right)_y = y^2$$

} \neq