Physics Issues for

Baseline + Energy Choice(s)

Stephen Parke
Fermilab
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* Oscillation Probability
* NuMI ν verses $\bar{\nu}$
* JHF ν verses NuMI ν
* Summary + Conclusions:

	LBL	"Competition"
θ_{13}	$\sin\theta_{23}\ \sin\theta_{13}$ + possible degeneracies θ_{13} and δ	Reactors θ_{13} indep of all degeneracies
$\delta m^2_{31} \lessgtr 0$	OK if sufficient matter effect	- possible $0\nu\beta\beta$ decay - supernova - cosmology
$\cancel{CP}(\delta)$	$\cos\theta_{23}\ \sin\delta$ $\cos\theta_{23}\ \cos\delta$	—
$\theta_{23} \lessgtr \pi/4$ $\sin^2 2\theta_{23} \neq 1$	$\sin^2 2\theta_{23} = 1-\epsilon^2$ $\sin^2\theta_{23} = \frac{1\mp\epsilon}{2}$ $\cos^2\theta_{23} = \frac{1\pm\epsilon}{2}$ two sol^n	Reactors + $\sin\theta_{23}\sin\theta_{13}$ — from LBL

Sensitivity to Θ_{13}

Flux $\otimes$ Cross Section $\otimes$ Detector Size

$\otimes$ Osc. Prob

http://www-off-axis.fnal.gov

Best Sensitivity
at E/L above Osc. Max.

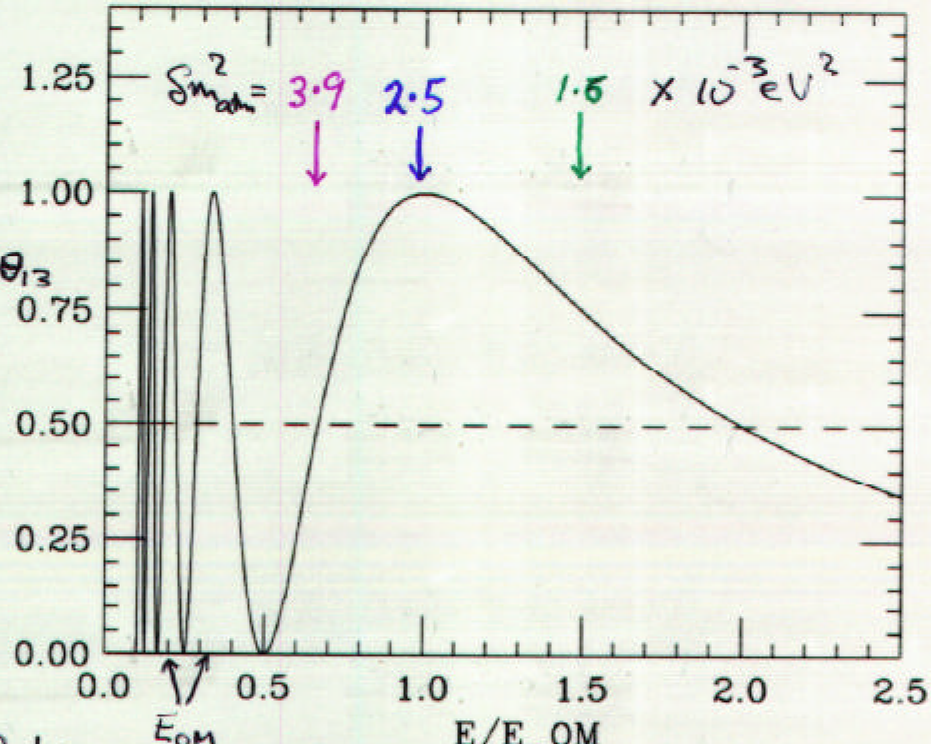
$$\delta m^2_{atm} = 2.5 \times 10^{-3}\,eV^2$$

$$(3.9 - 1.6) \times 10^{-3}\,eV^2$$

$$\text{Prob} = \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\pi}{2}\frac{E_{OM}}{E}\right)$$

$$E_{OM} = \frac{2}{\pi}\,\frac{\delta m^2_{atm} L}{4}$$

in units of $\sin^2\theta_{23} \sin^2 2\theta_{13}$



Peaks $\frac{E_{OM}}{(2n+1)}$

$$E^{NuMI}_{OM} = 1.5\ \text{GeV}\left(\frac{L}{732\ \text{km}}\right)\left(\frac{\delta m^2_{atm}}{2.5 \times 10^{-3}\,eV^2}\right)$$

$$\left(\text{JHF} = 0.6\ \text{GeV}\left(\frac{L}{295\ \text{km}}\right)\left(\frac{\delta m^2_{atm}}{2.5 \times 10^{-3}\,eV^2}\right)\right)$$

$$a_{\mu \to e} = \sum_{i=1,2,3} U_{\mu i} U_{ei}^* e^{-i \frac{m_i^2 L}{4E}}$$

eliminate $U_{\mu 1} U_{e1}^*$

$\Longrightarrow$

3 active flavors

(but can be easily modified to accommodate 3+1)

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

The parameterization used for the unitary MNS matrix, U, is

PDG

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

where $c_{jk} \equiv \cos\theta_{jk}$ and $s_{jk} \equiv \sin\theta_{jk}$.

The primary element of interest here is

$$|U_{e3}|^2 \quad \text{or} \quad \sin^2 2\theta_{13}$$

and δ.

$$\boxed{\nu_\mu \to \nu_e}$$

$a_\otimes$

$$a_{\mu\to e} = \Big[e^{-i\Delta_{32}} \, 2 U_{\mu 3} U_{e3}^* \sin \Delta_{31} + 2 U_{\mu 2} U_{e2}^* \sin \Delta_{21} \Big]$$

$a_\odot$

$$\Delta_{ij} = \frac{\delta m_{ij}^2 L}{4E} = 1.2669\ldots \frac{\delta m_{ij}^2 L}{E}$$

and $\sum U_{\mu i} U_{ei}^* = 0$

$$2 U_{\mu 3} U_{e3}^* = e^{i\delta} \sin\theta_{23} \sin 2\theta_{13}$$

$$2 U_{\mu 2} U_{e2}^* \approx \cos\theta_{13} \cos\theta_{23} \sin 2\theta_{12}$$

green

$$P_{\mu\to e} = \left| e^{-i(\Delta_{32}-\delta)} \sqrt{P_\otimes} + \sqrt{P_\odot} \right|^2$$

interference term

$$= P_\otimes + 2\sqrt{P_\otimes P_\odot} \cos(\Delta_{32} - \delta) + P_\odot$$

$\nu \to \bar{\nu}$

$\delta \to -\delta$

$$P_\otimes = \left(\sin\theta_{23} \sin 2\theta_{13} \sin \Delta_{31} \right)^2$$

$$P_\odot = \left(\cos\theta_{13} \cos\theta_{23} \sin 2\theta_{12} \sin \Delta_{21} \right)^2$$

$$2\sqrt{P_{\otimes} P_{\odot}} \cos(\Delta_{32} - \delta)$$

$$= 8 J_r \sin\Delta_{21} \sin\Delta_{31} \sin\Delta_{32} \sin\delta$$

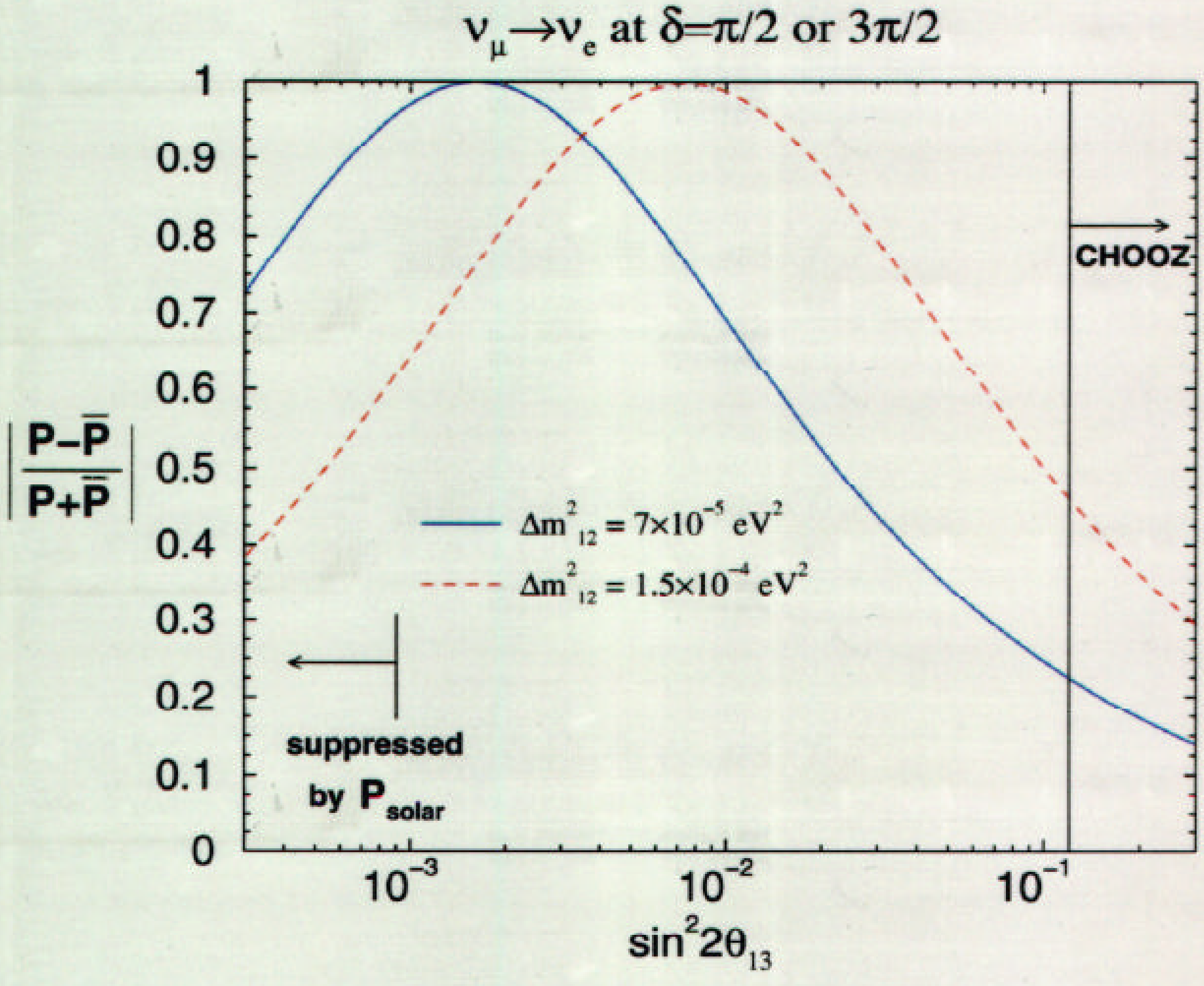
$$+ 8 J_r \sin\Delta_{21} \sin\Delta_{31} \cos\Delta_{32} \cos\delta$$

$$J_r = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2$$

$P_{\mu\to e}$ differs for $\left\{\begin{array}{c}\text{neutrinos}\\ \text{verses}\\ \text{anti-neutrinos}\end{array}\right.$

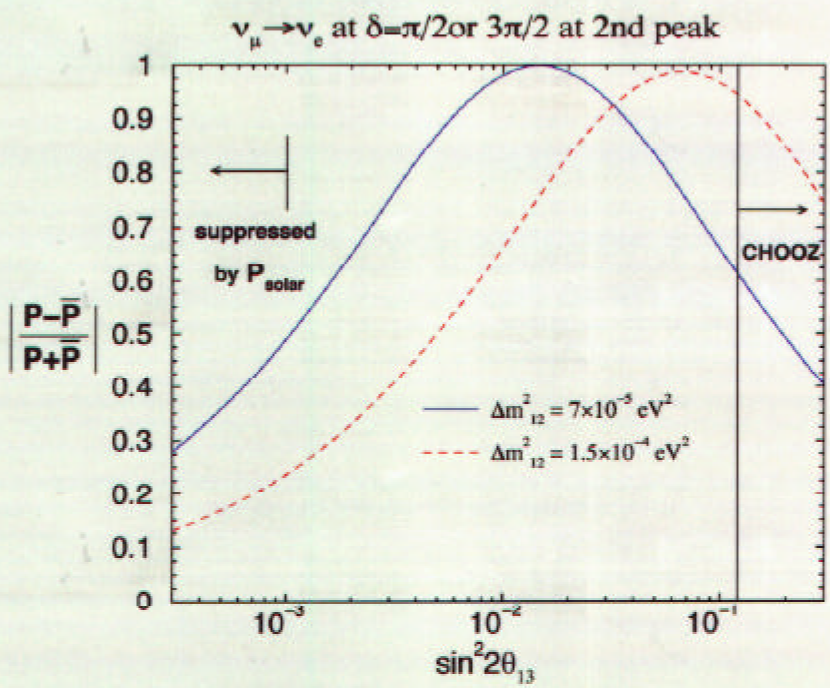
because of a CHANGE in phase of interference term between

$\overset{\otimes}{a}_{\mu\to e}$ and $\overset{\odot}{a}_{\mu\to e}$



$\nu_\mu \rightarrow \nu_e$ at $\delta=\pi/2$ or $3\pi/2$
$\left|\frac{P-\bar{P}}{P+\bar{P}}\right|$
$\sin^2 2\theta_{13}$
$\Delta m^2_{12} = 7\times10^{-5}$ eV^2
$\Delta m^2_{12} = 1.5\times10^{-4}$ eV^2
CHOOZ
suppressed
by P_{solar}

2nd Peak



- Peak occurs at $\sin^2 2\theta_{13} \approx \frac{\sin^2 2\theta_{12}}{\tan^2 \theta_{23}} \left[\frac{3\pi}{2} \; \frac{\delta m^2_{12}}{\delta m^2_{13}}\right]^2$

$P(\nu_\mu \to \nu_e)$

Why Everybody is Excited!

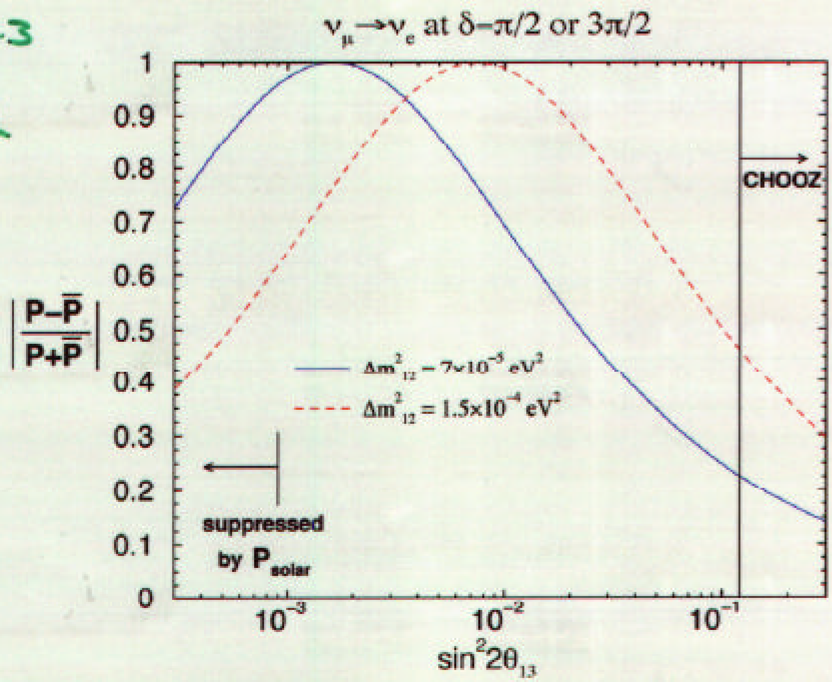
- Maximum Allowed Asymmetry ($\delta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$) for $\nu_\mu \to \nu_e$ at first Oscillation Maximum in vac:

- $P, \; \bar{P} = |a^{atm}_{\mu\to e} + a^{\odot}_{\mu\to e}|^2 \approx (\sin\theta_{23}\sin 2\theta_{13} \pm \sqrt{P_\odot})^2$
  - $|P - \bar{P}| \approx 4\sqrt{P_\odot}\sin\theta_{23}\;\sin 2\theta_{13}$
  - $P + \bar{P} \approx 2\sin^2\theta_{23}\;\sin^2 2\theta_{13} + 2P_\odot$

easily generalized
- $\delta \neq \frac{\pi}{2}$
- off O.M.
- matter

$a^{atm}_{\mu\to e} \propto U_{e3}$

$a^{\odot}_{\mu\to e} \propto U_{e2}$



Asymmetry LARGE: !

- Peak occurs at

$$\sin^2 2\theta_{13} \approx \frac{\sin^2 2\theta_{12}}{\tan^2\theta_{23}}\left[\frac{\pi}{2}\;\frac{\delta m^2_{12}}{\delta m^2_{13}}\right]^2$$

at OM $\sqrt{P_\odot} = \cos\theta_{13}\cos\theta_{23}\sin 2\theta_{12}\sin\left(\frac{\pi}{2}\;\frac{\delta m^2_{12}}{\delta m^2_{13}}\right)$

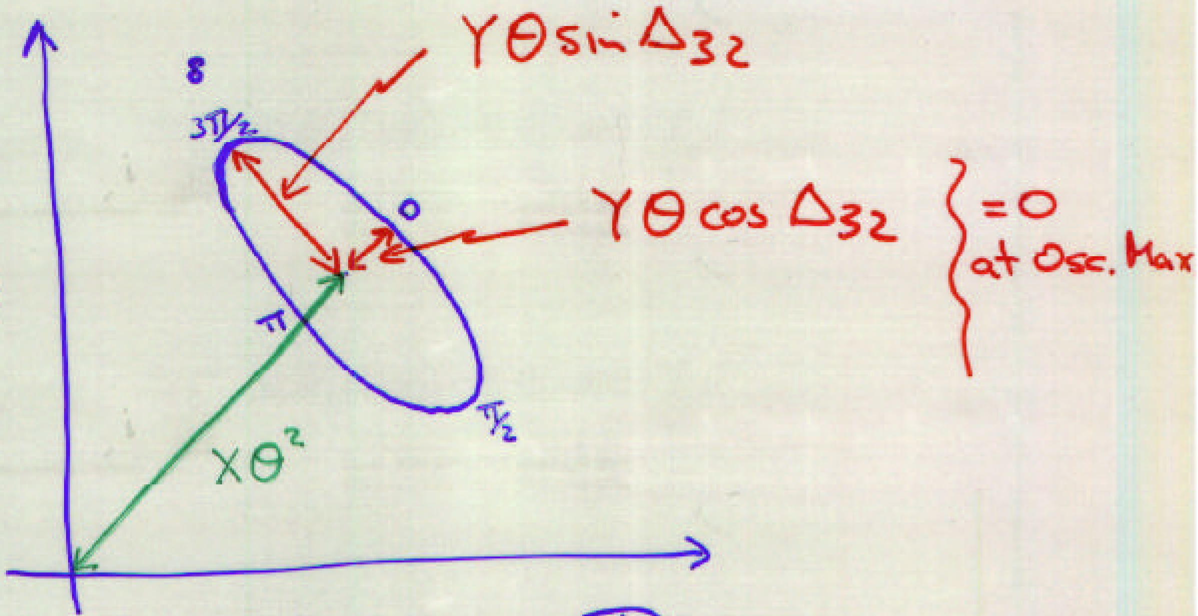
- For BK

for small θ_{13} (dropping 13 indices)

$$P_{\mu\to e} = X\theta^2 + Y\theta\cos(\Delta_{32} \mp \delta) + P_\odot$$

$$X = 4\sin^2\theta_{23}\sin^2\Delta_{31} : \quad P_\odot = \cos^2\theta_{23}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

$$\text{and} \quad Y = 2\sqrt{X P_\odot}$$

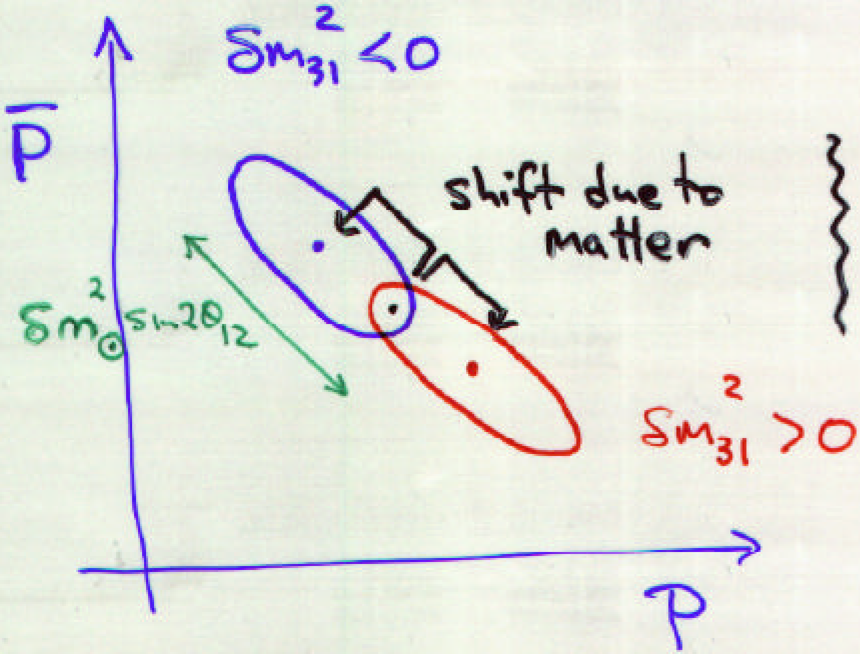


Y depends on solar parameter

$$\delta m^2_{21}\sin 2\theta_{12}$$

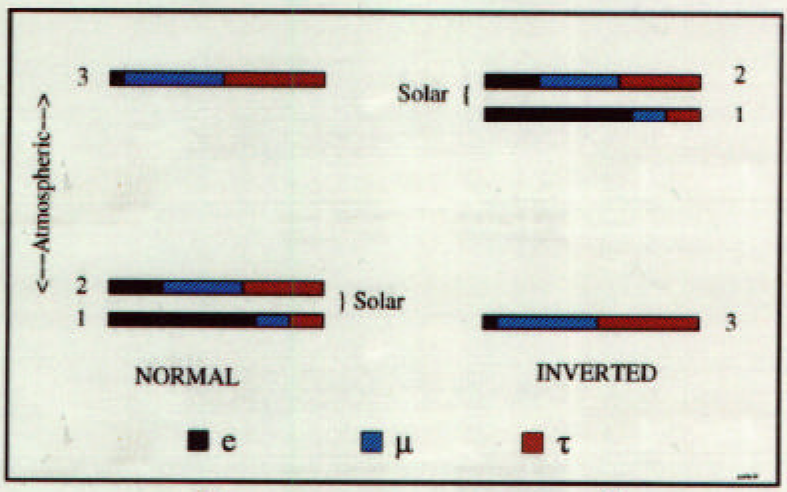
Vacuum:

Matter Effects:



large L bigger effect

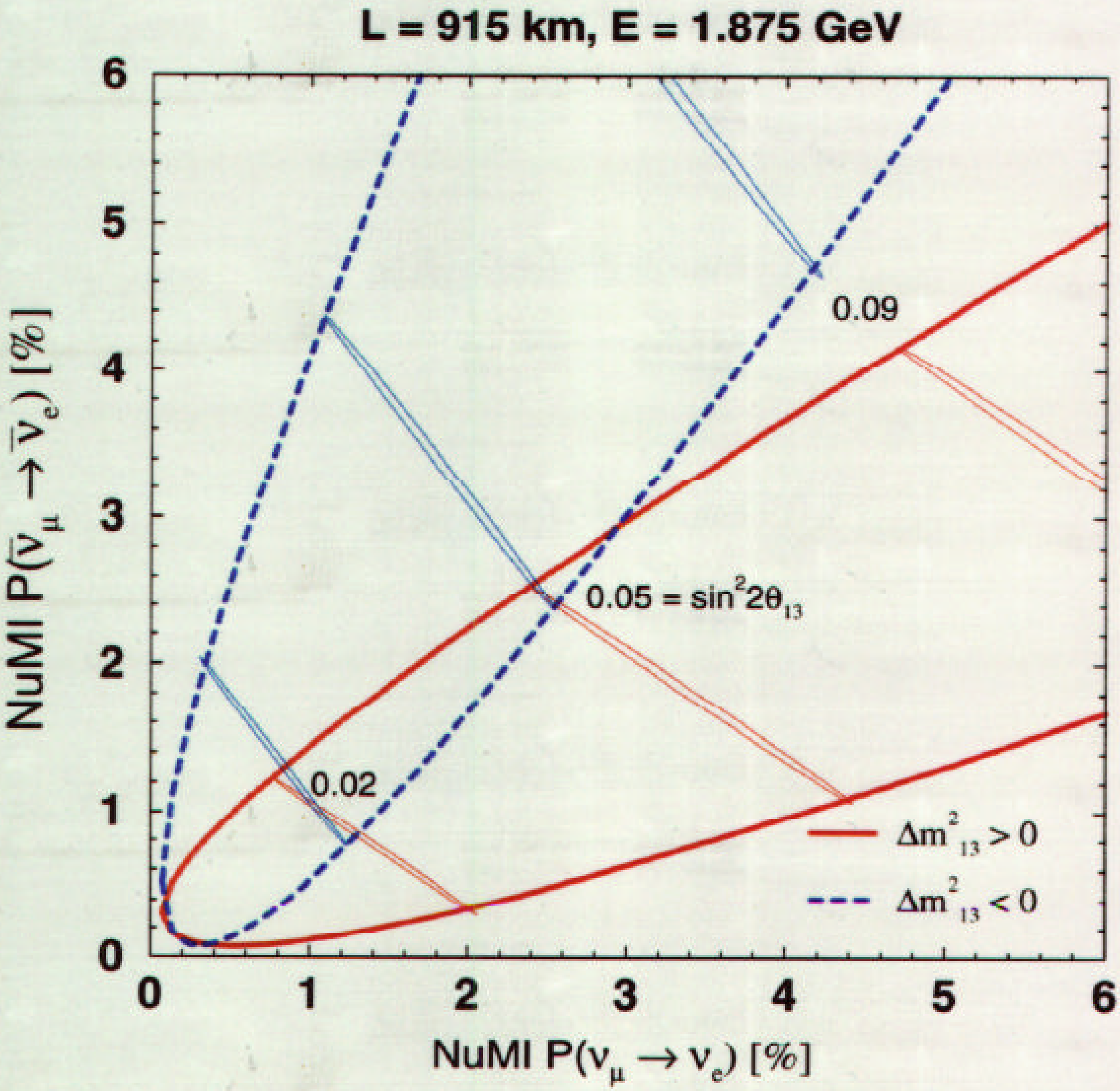
$$P^{center}_{mat} \approx \left(1 \pm \frac{E_\nu}{6\,\mathrm{GeV}}\right) P^{center}_{vac}$$



$\delta m^2_{31} > 0$ $\delta m^2_{31} < 0$

Near Oscillation Maximum:

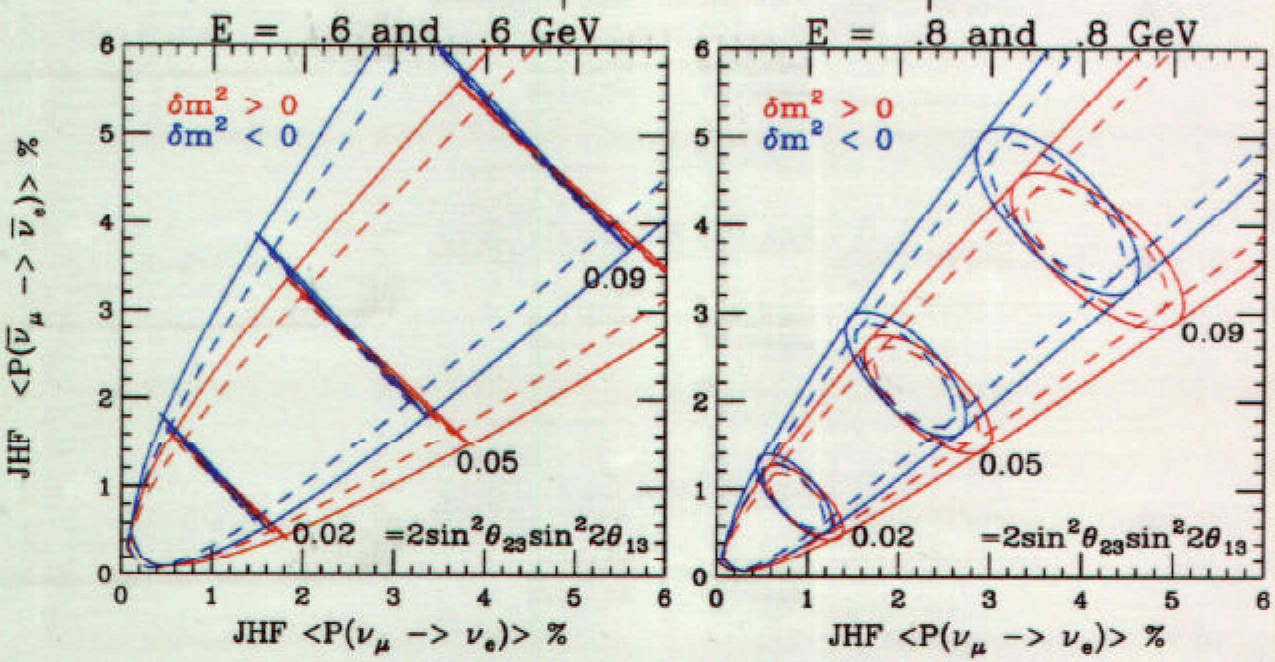
good separation betwen hierachies
but not perfect.



$\sin^2 2\theta_{23}$ "Scaling":

- Using $\sin^2 2\theta_{23} = 0.96 = 4 * (0.4) * (0.6)$:

$2\sin^2\theta_{23}\sin^2 2\theta_{13}$	$\sin^2 2\theta_{13}^{(1)}$ $2\sin^2\theta_{23}^{(1)} = 0.8$	$\sin^2 2\theta_{13}^{(2)}$ $2\sin^2\theta_{23}^{(2)} = 1.2$
0.02	0.0250	0.0167
0.05	0.0625	0.0417
0.09	0.1125	0.075
	solid	dashes



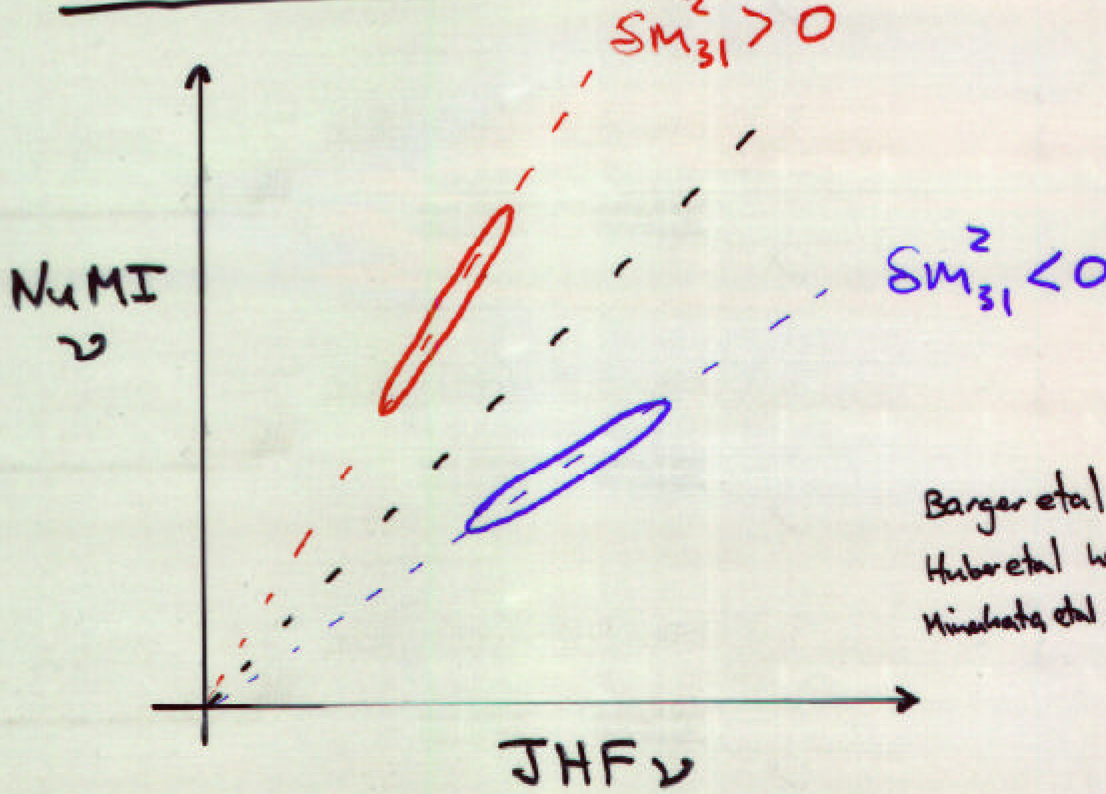
If $\sin^2 2\theta_{23} \neq 1$ then two solution for θ_{13}

are related by $\theta_{13}^{(2)} \approx \dfrac{\sin\theta_{23}^{(1)}}{\sin\theta_{23}^{(2)}}\,\theta_{13}^{(1)}$

- Exact at Oscillation Maximum:

— small corrects at other energies.

JHF ν verses NuMI ν



Barger et al hep-ph/0211
Huber et al hep-ph/0210
Minakata et al hep-ph/0301

Ratio of Slopes:

$$\frac{X_+^N}{X_+^J} \Big/ \frac{X_-^N}{X_-^J} \simeq 1 + 2G(\Delta^N)(aL)^N - 2G(\Delta^J)(a$$

$$G(\Delta) = \left(\Delta^{-1} - \cot\Delta\right)$$ monot. increasing f^n of Δ o

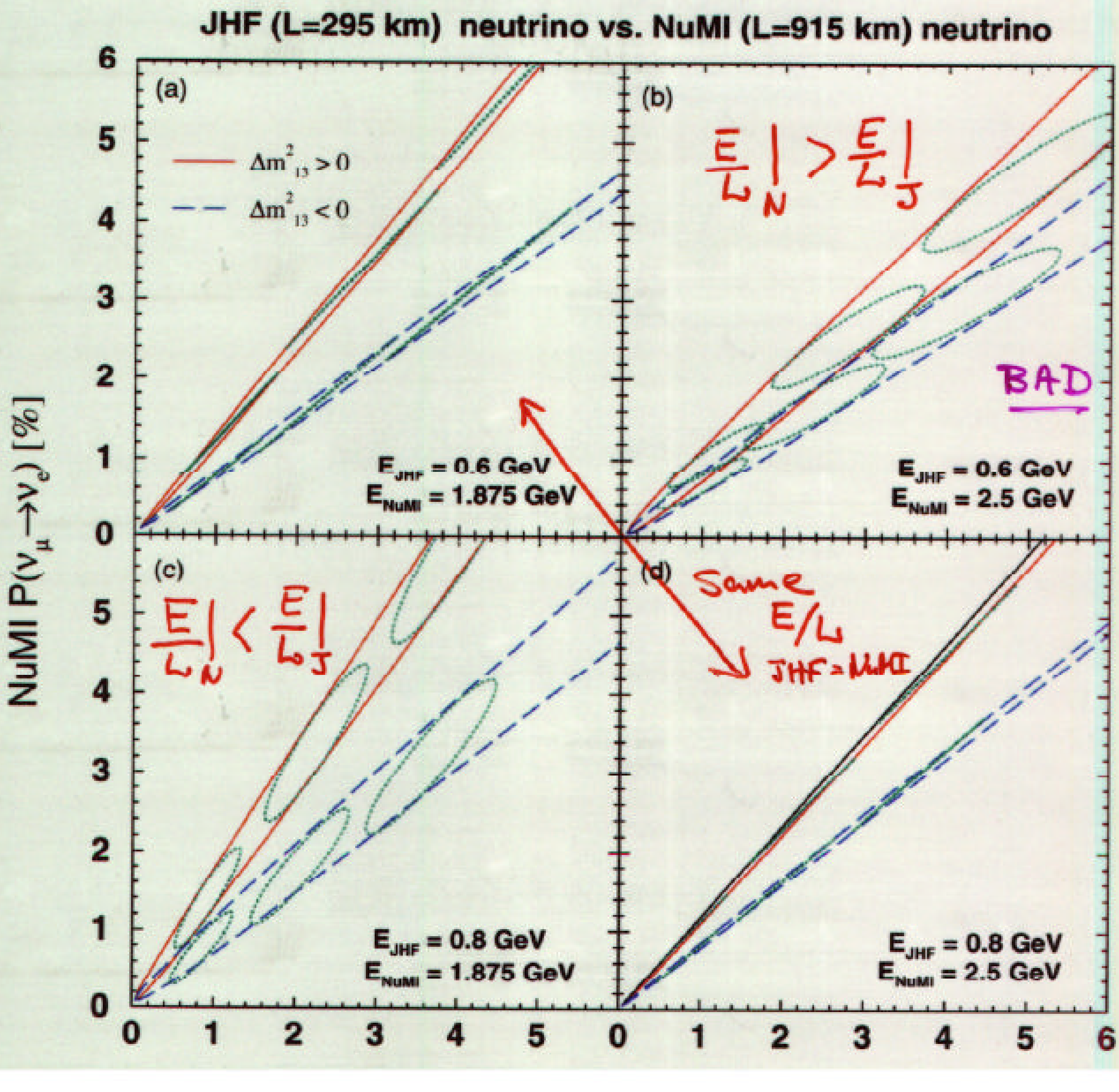
Want larger L^N, smaller E^N
smaller L^J, larger E^J

Width of Ellipses: $\left|\frac{Y^N}{X^N} - \frac{Y^J}{X^J}\right| \Theta$

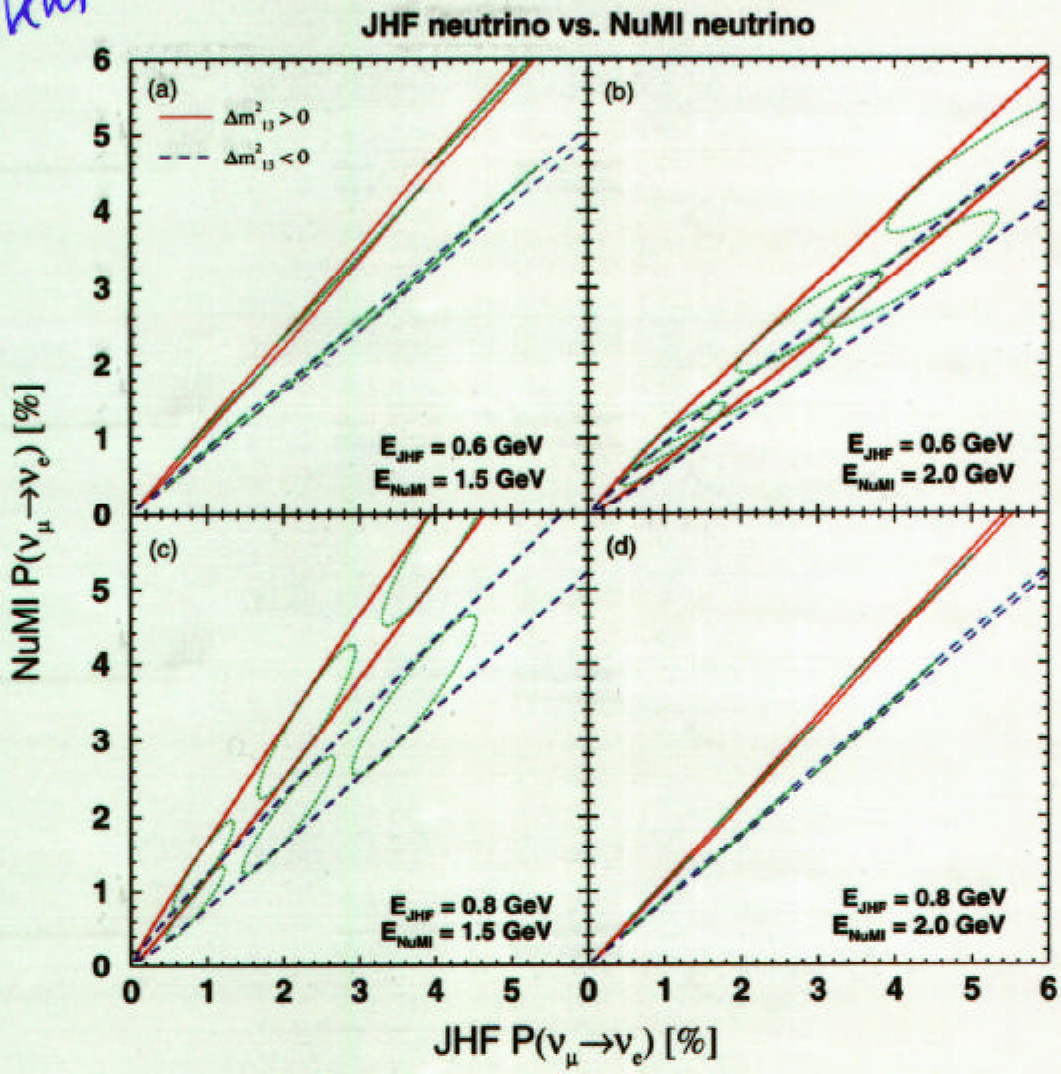
Smallest at SAME $\left(\frac{E}{L}\right)$

cancellation
$\frac{Y}{X}$ the same

Comparison Between
JHF & NuMI neutrinos



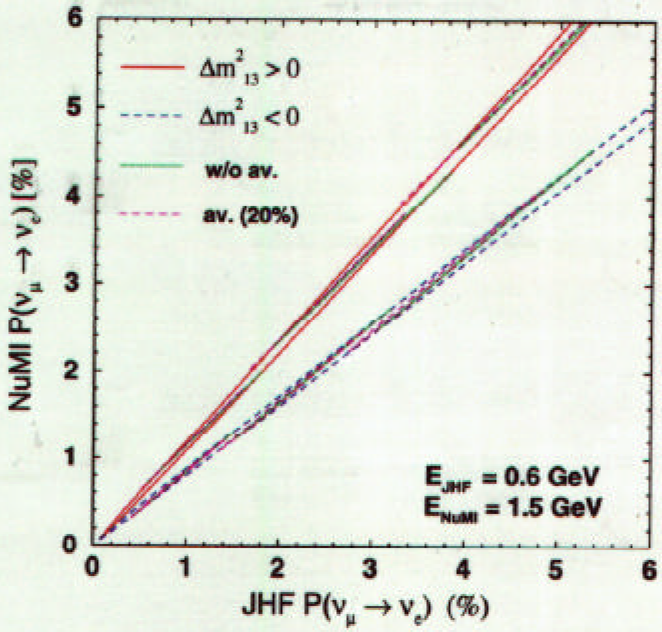
7 32 km

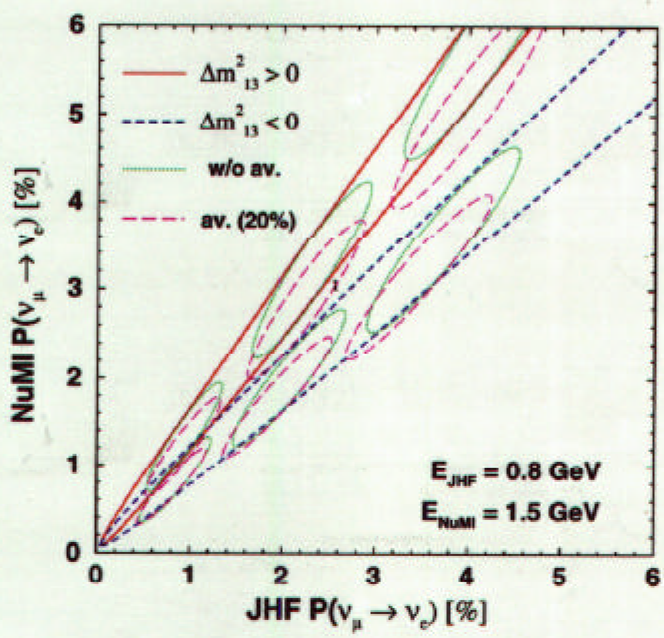


- Separation provided $\left(\frac{E}{L}\right)_{NuMI} \leq \left(\frac{E}{L}\right)_{JHF}$

- Best Separation at Oscillation Maximum

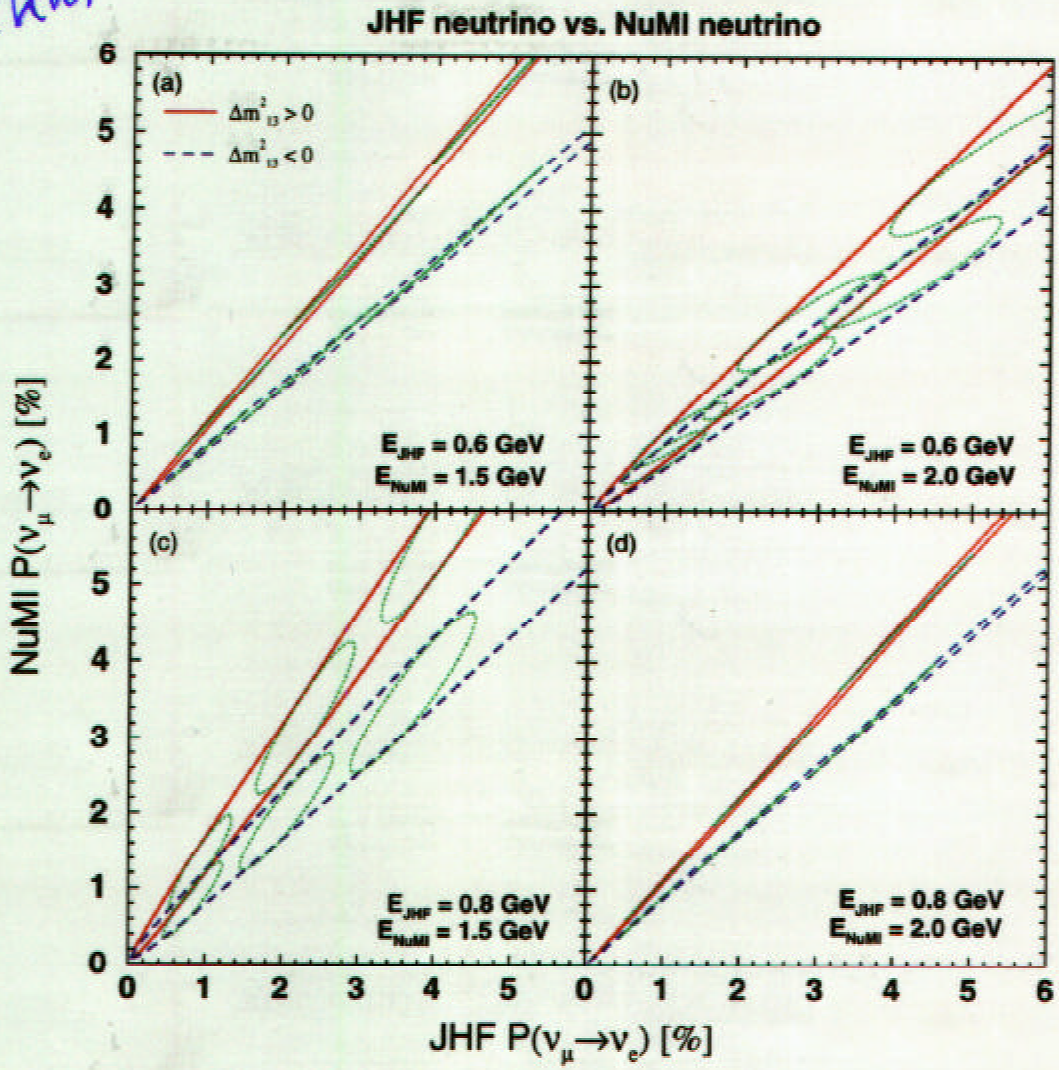
Energy Averaging:





- Energy Averaging does NOT effect separation.

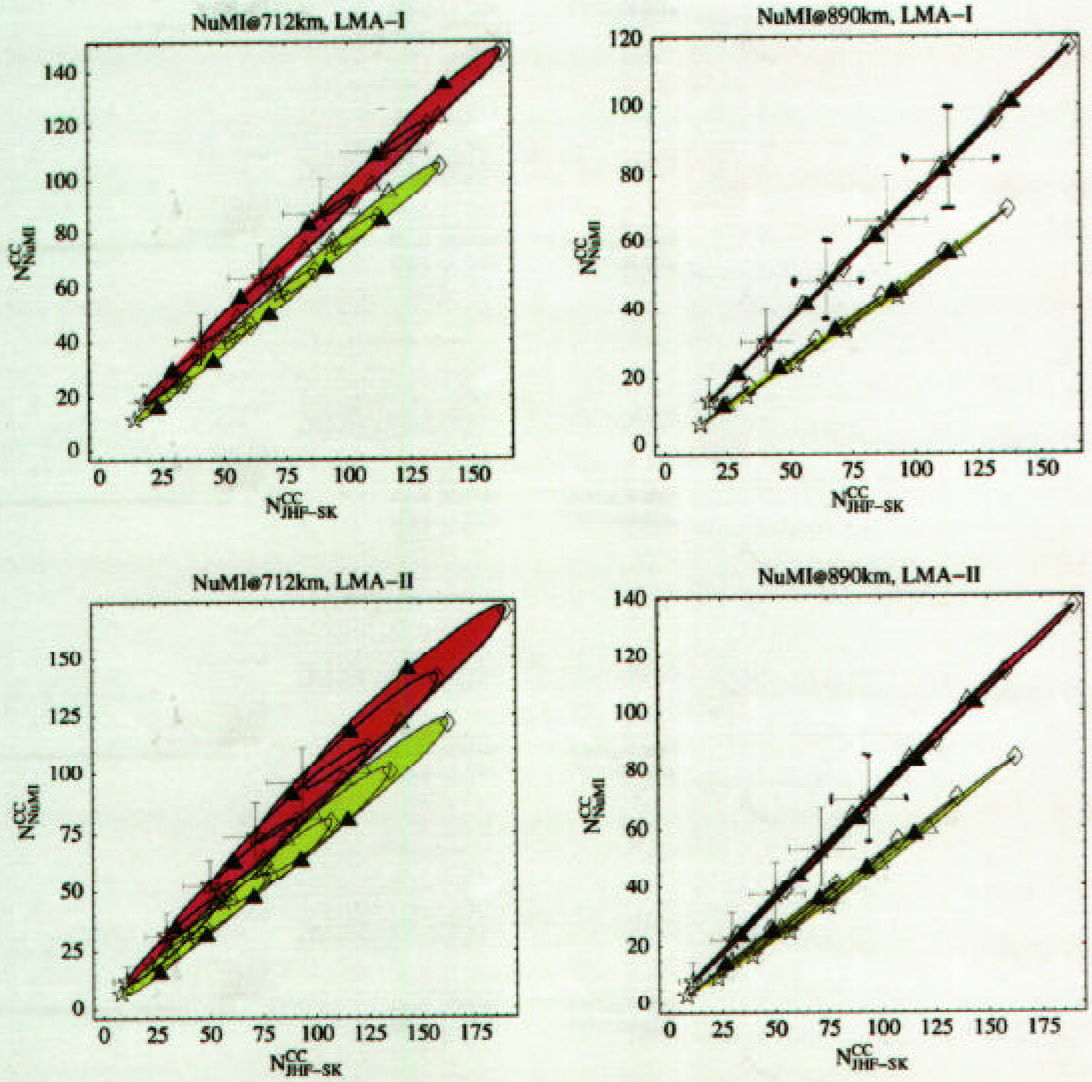
732km



- Separation provided $\left(\frac{E}{L}\right)_{NuMI} \leq \left(\frac{E}{L}\right)_{JHF}$

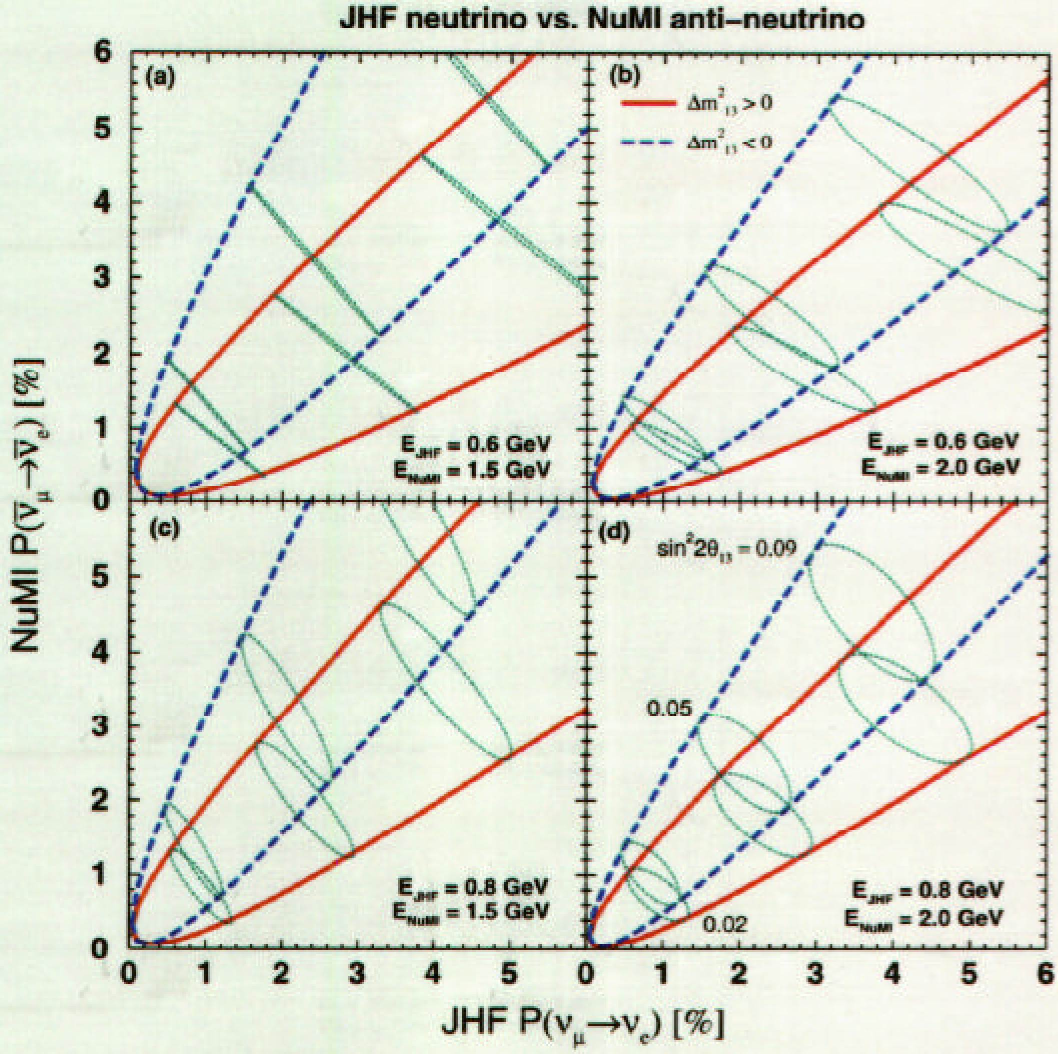
- Best Separation at Oscillation Maximum

Walter Winter from TMU group



$E_{JHF} = 0.76$ GeV

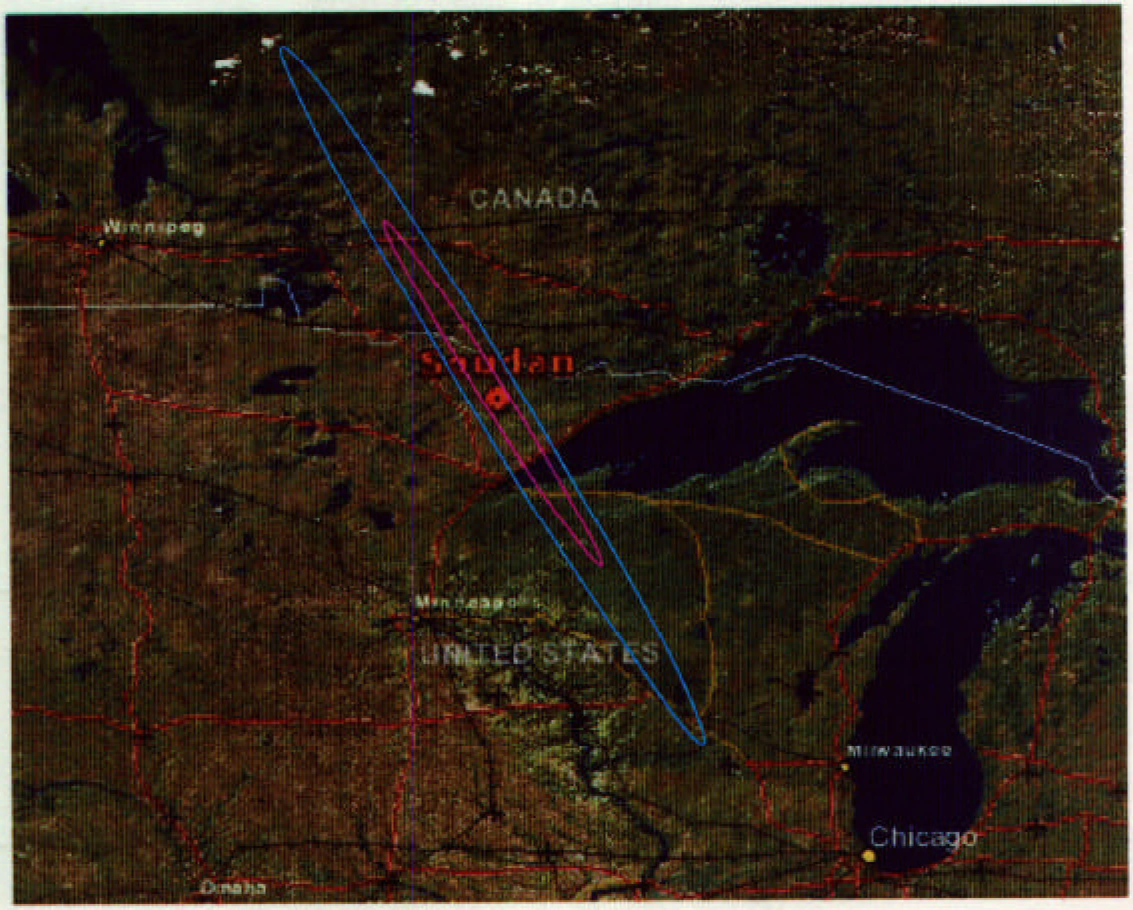
$E_{NuMI} = 2.22$ GeV



- Width of the Cigars: $\left| \dfrac{Y_\pm^N}{X_\pm^N} - \dfrac{Y_\mp^J}{X_\mp^J} \right| \theta$

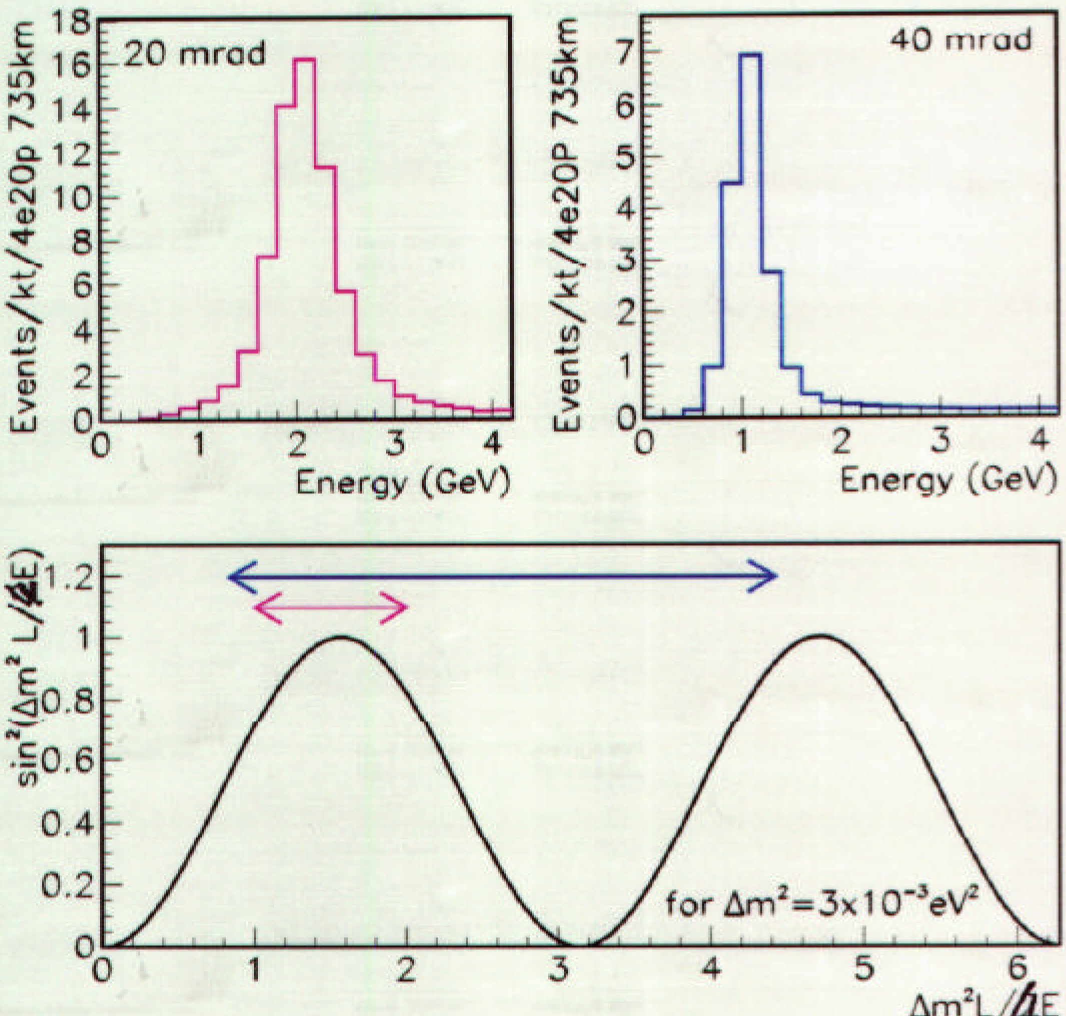
Since $Y_\pm$ have opposite sign NO cancellation.

Flexibility of NuMI Off Axis Beamline



Narrow Beams from 1 to 2 GeV, distances from 400 to 1100 km!

Different Angles ↔ Different Spectra



One could put a detector at second oscillation maximum to look for CP violating effects, with 1 GeV beam...

$$\frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)} \propto L$$

Summary & Conclusions

* To show non-zero θ_{13}:

  $\frac{E}{L} >$ Osc. Max.

  provides best sensitivity

  but allows a wide range of values.

  {reactor exp. & JHF-SK}

* Measurement of $\sin\theta_{23}\sin\theta_{13}$

  independent of δ-phase can be

  made at Osc. Max. with ν & $\bar{\nu}$ running

  (maybe hierarchy)

  {JHF → SK}

* Comparing NuMI ν with JHF ν

with $L_{NuMI} \approx 3 * L_{JHF}$

and $\left.\frac{E}{L}\right|_{NuMI} \leq \left.\frac{E}{L}\right|_{JHF}$

can <u>determine the hierarchy</u>

$\Big\{$ if Majorana $0\nu\beta\beta$ decay ⋮ Supernova ⋮ cosmology $\Big\}$

The Optimization of Energy + Baseline depends on the Physics Question: