Q Calculation for dielectric loaded SW cavity in TM-01p mode

WF-188
John Power
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O) Must define bessel functions this way - it is a quirk of mathcad.

$J_0(x) \equiv \text{if}(\text{Im}(x)=0, J0(x), I0(\text{Im}(x)))$ $\quad Y_0(x) \equiv \text{if}(\text{Im}(x)=0, Y0(x), K0(\text{Im}(x)))$

$J_1(x) \equiv \text{if}(\text{Im}(x)=0, J1(x), I1(\text{Im}(x)))$ $\quad Y_1(x) \equiv \text{if}(\text{Im}(x)=0, Y1(x), K1(\text{Im}(x)))$

$TOL := 10^{-10}$

I) Cylindrical Standing Wave Cavity

Inner radius

$a := 0.003$ m

Longitudunal mode

$p := 3$

$v_p := 1.0 \cdot c$

Material Constants

$\kappa \equiv 20.0$ $\quad \mu := 1$

$\tan\delta := 10^{-4}$

Constants

$\varepsilon_0 \equiv 8.854187817 \cdot 10^{-12}$ $\quad \sigma \equiv 5.7 \cdot 10^{7}$

$c \equiv 2.99792485 \cdot 10^{8}$ $\quad \mu_0 \equiv 4 \cdot \pi \cdot 10^{-7}$

frequency

$f_0 := 11.424 \cdot 10^{9}$

$\omega_0 := 2 \cdot \pi \cdot f_0$ $\quad \lambda_g := \frac{v_p}{f_0}$ $\quad \lambda_g = 0.0262423394$

$L := p \cdot \frac{\lambda_g}{2}$ $\quad L = 0.0393635091$ $\quad k_z := p \cdot \frac{\pi}{L}$

$R_s(\omega) := \sqrt{\frac{\mu_0 \cdot \omega}{\sigma \cdot 2}}$ $\quad E_0 := 10^{6}$

$R_s(\omega_0) = 0.0281288215$

$\gamma_1 := \omega_0 \cdot \sqrt{\frac{1}{c^2} - \frac{1}{v_p^2}}$ $\quad \gamma_2 := \omega_0 \cdot \sqrt{\frac{\kappa}{c^2} - \frac{1}{v_p^2}}$

$\gamma_2 = 1043.6481827323$ $\quad \gamma_1 = 0$

II) Starting with following functions

$f1(x,y) := Y_0(x \cdot \gamma_2) \cdot J_0(y \cdot \gamma_2) - J_0(x \cdot \gamma_2) \cdot Y_0(y \cdot \gamma_2)$

$f2(x,y) := Y_0(x \cdot \gamma_2) \cdot J_1(y \cdot \gamma_2) - J_0(x \cdot \gamma_2) \cdot Y_1(y \cdot \gamma_2)$

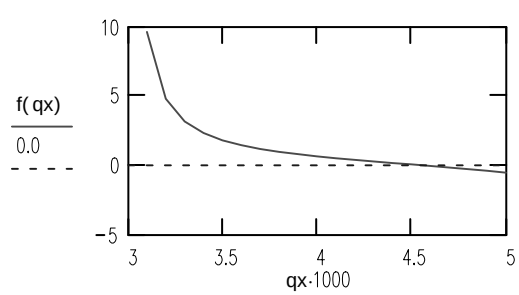
$qx := a, 0.0031 .. 0.005$

$f(x) := \frac{f2(x,a)}{f1(x,a)} \cdot 1 - \frac{\gamma_2}{\kappa} \cdot \text{if}\left(|\gamma_1| = 0, \frac{a}{2}, \frac{J_1(a \cdot \gamma_1)}{|\gamma_1| \cdot J_0(a \cdot \gamma_1)}\right)$

$z := 0.004$

$b := \text{root}(f(z), z)$

$b \cdot 1000 = 4.5670098782$ $\quad b = 0.0045670099$



III) **r < a** Peak values (i.e. time dependence = 1) of the field.

$$E_{z1}(r,z) := E_0 \cdot J_0(r \cdot \gamma_1) \cdot \cos(k_z \cdot z)$$

$$B_{\theta 1}(r,z) := -\left(\frac{\omega_0}{c^2}\right) \cdot E_0 \cdot \cos(k_z \cdot z) \cdot \mathrm{if}\left(|\gamma_1| = 0, \frac{r}{2}, \frac{J_1(r \cdot \gamma_1)}{|\gamma_1|}\right)$$

$$E_{r1}(r,z) := k_z \cdot E_0 \cdot \sin(k_z \cdot z) \cdot \mathrm{if}\left(|\gamma_1| = 0, \frac{r}{2}, \frac{J_1(r \cdot \gamma_1)}{|\gamma_1|}\right)$$

a < r < b Peak values (i.e. time dependence = 1) of the field.

$$E_{z2}(r,z) := E_0 \cdot J_0(a \cdot \gamma_1) \cdot \frac{f1(b,r)}{f1(b,a)} \cdot \cos(k_z \cdot z)$$

$$B_{\theta 2}(r,z) := -\left(\frac{\omega_0 \cdot \kappa}{\gamma_2 \cdot c^2}\right) \cdot E_0 \cdot J_0(a \cdot \gamma_1) \cdot \frac{f2(b,r)}{f1(b,a)} \cdot \cos(k_z \cdot z)$$

$$E_{r2}(r,z) := \frac{k_z}{|\gamma_2|} \cdot E_0 \cdot J_0(a \cdot \gamma_1) \cdot \frac{f2(b,r)}{f1(b,a)} \cdot \sin(k_z \cdot z)$$

IV) Stored Energy --------> Recall: U_tot = Ue(t) + Um(t) = Ue(t=0) = constant

$$U_{vac} := \frac{1}{2} \cdot \varepsilon_0 \cdot 2 \cdot \pi \cdot \int_0^L \int_0^a r \cdot \left(E_{z1}(r,z)^2 + E_{r1}(r,z)^2\right) dr\,dz$$

$$U_{vac} = 0.0000026225$$

$$U_{diel} := \frac{1}{2} \cdot \kappa \cdot \varepsilon_0 \cdot 2 \cdot \pi \cdot \int_0^L \int_a^b r \cdot \left(E_{z2}(r,z)^2 + E_{r2}(r,z)^2\right) dr\,dz$$

$$U_{diel} = 0.0000286911$$

$$100 \cdot \frac{U_{vac}}{U_{diel}} = 9.1404966465$$

$$U_{tot} := U_{vac} + U_{diel}$$

$$U_{tot} = 0.0000313136$$

V) Power Lost per rf cycle in cavity walls

-----> I^2*R = 0.5* R*Int[H^2*dS] = 0.5 * R * mu^-2 * Int[B^2*dS]

$$P_{outer} := \frac{1}{2} \cdot R_s(\omega_0) \cdot \frac{1}{\mu_0^2} \cdot 2 \cdot \pi \cdot \int_0^L b \cdot B_{\theta 2}(b,z)^2 \, dz$$

$P_{outer} = 791.5085393964$

$$P_{ends} := 2 \cdot \frac{1}{2} \cdot R_s(\omega_0) \cdot \frac{1}{\mu_0^2} \cdot 2 \cdot \pi \cdot \left(\int_0^a r \cdot B_{\theta 1}(r,0)^2 \, dr + \int_a^b r \cdot B_{\theta 2}(r,0)^2 \, dr \right)$$

$P_{ends} = 71.2264536779$

$$P_{tot} := P_{outer} + P_{ends}$$

$P_{tot} = 862.7349930742$

VI) Q Calculation

$$Q_{diel} := \frac{U_{tot}}{U_{diel}} \cdot \frac{1}{\tan\delta}$$

$Q_{diel} = 10914.0496646535$

$$Q_{Cu} := \frac{\omega_0 \cdot U_{tot}}{P_{tot}}$$

$Q_{Cu} = 2605.2787105191$

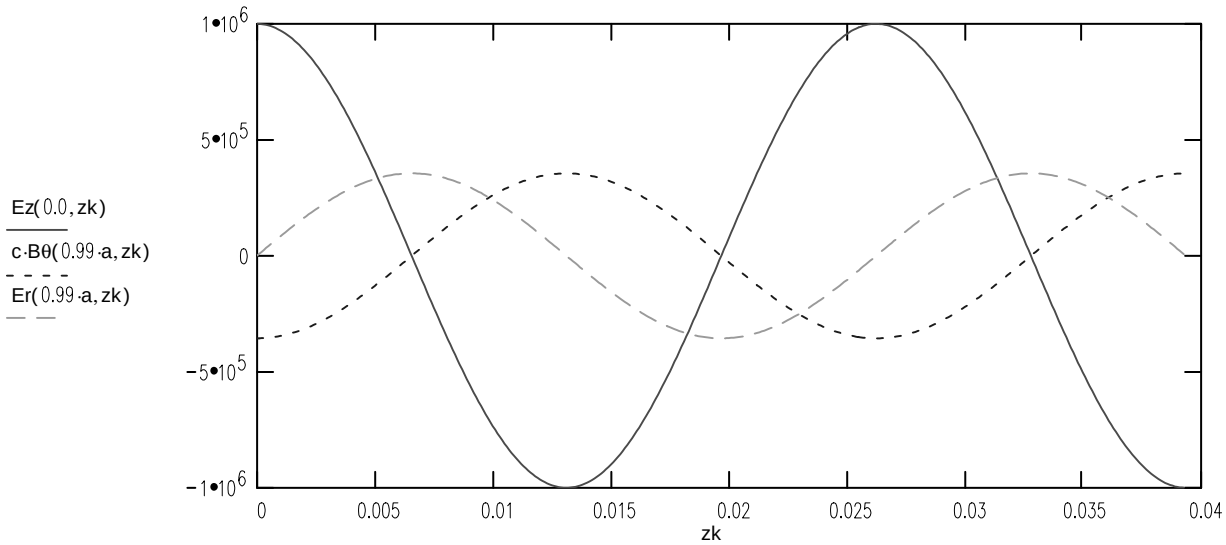
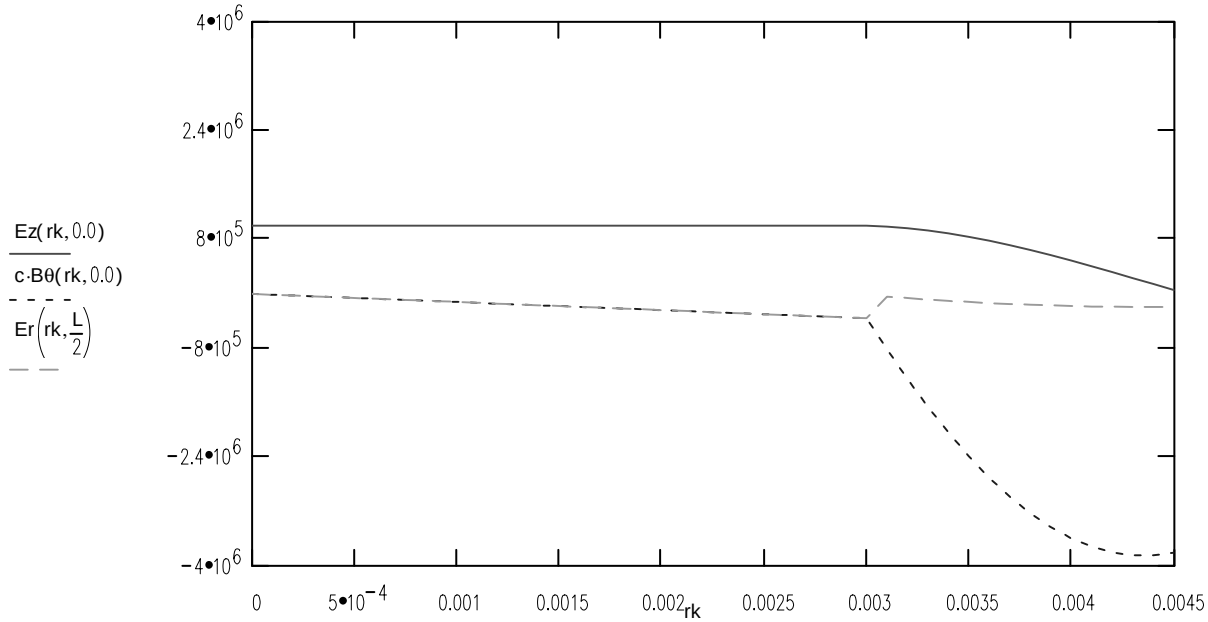
$$Q_{tot} := \frac{1}{\frac{1}{Q_{Cu}} + \frac{1}{Q_{diel}}}$$

$Q_{tot} = 2103.2214358434$

VII) Plot the 3 field components of the TM_01p SW cavity

$a = 0.003$ $b = 0.0045670099$ $L = 0.0393635091$

$Ez(r,z) := if\left(r<a, E_{z1}(r,z), E_{z2}(r,z)\right)$ $B\theta(r,z) := if\left(r<a, B_{\theta 1}(r,z), B_{\theta 2}(r,z)\right)$

$Er(r,z) := if\left(r<a, E_{r1}(r,z), E_{r2}(r,z)\right)$

$zk := 0, 0.0001 .. L$ $rk := 0, 0.0001 .. b$