

# Coherent Multimoded Dielectric Wakefield Accelerators

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**Abstract.** There has recently been a study of the potential uses of multimode dielectric structures for wakefield acceleration [1]. This technique is based on adjusting the wakefield modes of the structure to constructively interfere at certain delays with respect to the drive bunch, thus providing an accelerating gradient enhancement over single mode devices. In this report we examine and attempt to clarify the issues raised by this work in the light of the present state of the art in wakefield acceleration.

## INTRODUCTION

The utility of dielectric loaded structures as wakefield accelerators has been investigated for the past decade [2–4]. Some progress has been made towards demonstrating the feasibility of this technique [5,6].

Previous work on dielectric devices has in general assumed that only a single  $TM_{01}$  accelerating mode is present in the structure. Recently, a new scheme of wakefield acceleration using a dielectric loaded waveguide which makes use of many  $TM_{0n}$  modes excited by a drive bunch train to enhance the accelerating gradient [1]. The structure is cleverly designed so that the coherent sum of the modes Fourier synthesizes a longitudinal wake which on the axis of the structure approximates a train of alternating sign delta functions.

In this report, we examine this interesting scheme, considering only the longitudinal wakefields. We attempt to clarify some of the confusion in terminology which has arisen from this work.

## WAKEFIELDS IN DIELECTRIC LOADED WAVEGUIDES

Consider a dielectric channel with an axial vacuum channel and conductor on the exterior. The dielectric channel has inner radius  $a$  and outer radius  $b$  with dielectric constant  $\epsilon$ . When a single charged particle beam (rms pulse length  $\sigma_z$

and total charge  $Q$ ) passes through the channel, it will produce longitudinal wake which can be expressed as

$$W_z(r, z, t) = Q \sum_n W_{zn}(a, b, \epsilon) \exp[-(\frac{2\pi\sigma_z}{\lambda_n})^2] \cos(\frac{2\pi z}{\lambda_n}) \quad (1)$$

where  $\lambda_n$  is the wavelength of the  $n$ th mode, and the coefficients  $W_{zn}$  depend only on the geometry of the structure.

If multiple bunches with longitudinal separation  $d$  are used then the beam distribution can be expressed as

$$f(z) = \frac{1}{\sqrt{2\pi}\sigma_z} \sum_l \exp[-\frac{(z - ld)^2}{2\sigma_z^2}]. \quad (2)$$

Then the wakefield can be expressed anywhere (including the region inside the bunch) as

$$W_z(z) = \sum_l \sum_n W_{zn}(a, b, \epsilon) \int_{-\infty}^z f(z') \cos[\frac{2\pi(z - z')}{\lambda_n}] dz'. \quad (3)$$

If  $\sigma_z$  is comparable to  $\lambda_n$ , then modes up to the  $n$ th mode will be excited.

## TRANSFORMER RATIO

The conventional transformer ratio in a collinear wakefield accelerator is defined as [7]

$$R = -E_z^+ / E_z^- \quad (4)$$

Where  $E_z^-$  is the *peak* deceleration field within the drive bunch and  $E_z^+$  is the maximum accelerating field a test particle can see while travelling behind the drive bunch. In general, for a Gaussian drive beam profile, the transformer ratio cannot exceed 2.

(There are a number of circumstances where  $R > 2$  can be attained through the use of noncollinear structures, nonlinearity, and multimoding. Reference [7] gives an example where a modest enhancement in transformer ratio beyond 2 is obtained in a multimode structure, but it is difficult to obtain significant enhancements to  $R$  in this manner.)

However, if one uses an alternative definition of transformer ratio

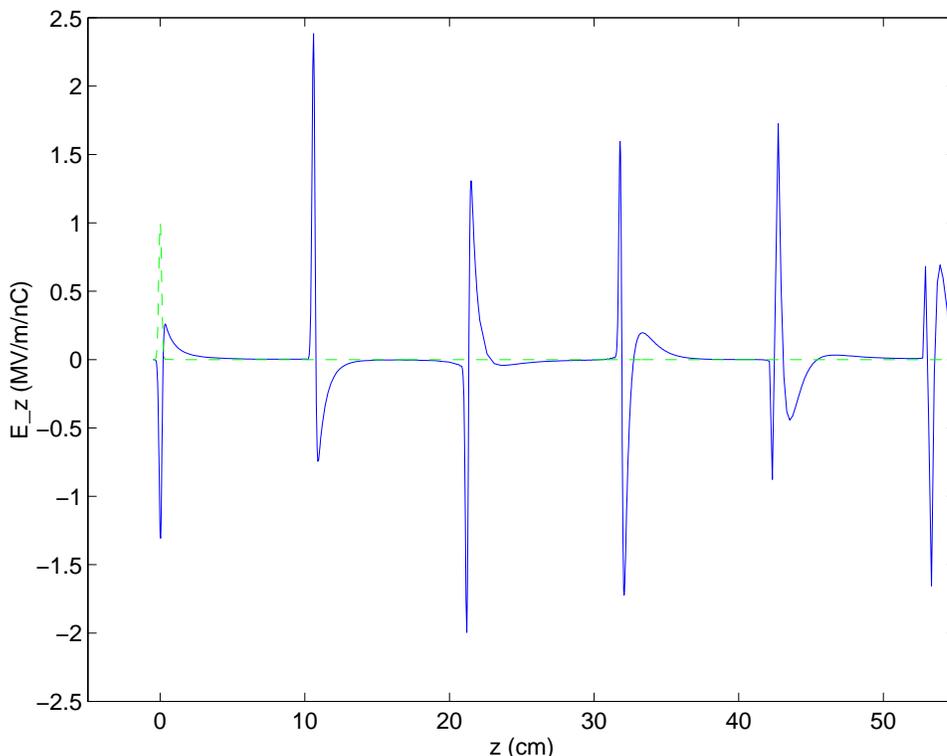
$$R' = -E_z^+ / \overline{E_z^-} \quad (5)$$

where  $\overline{E_z^-}$  is the average energy loss (or average deceleration field) in the drive beam (defined as the “drag field” in [1]), then by this alternative definition of the transformer ratio indeed can exceed 2. It is this definition which is used by the authors of [1].

## NUMERICAL EXAMPLES

In this section we will show some examples of wakefields in multimoded structures. The wakes are calculated using the analytic approach of reference [3]. The transcendental equation obtained for the dispersion relation is solved numerically for each mode and the mode amplitude  $W_{zn}(a, b, \epsilon)$  is evaluated. The modes are then summed to obtain the wake potential according to Equation 3.

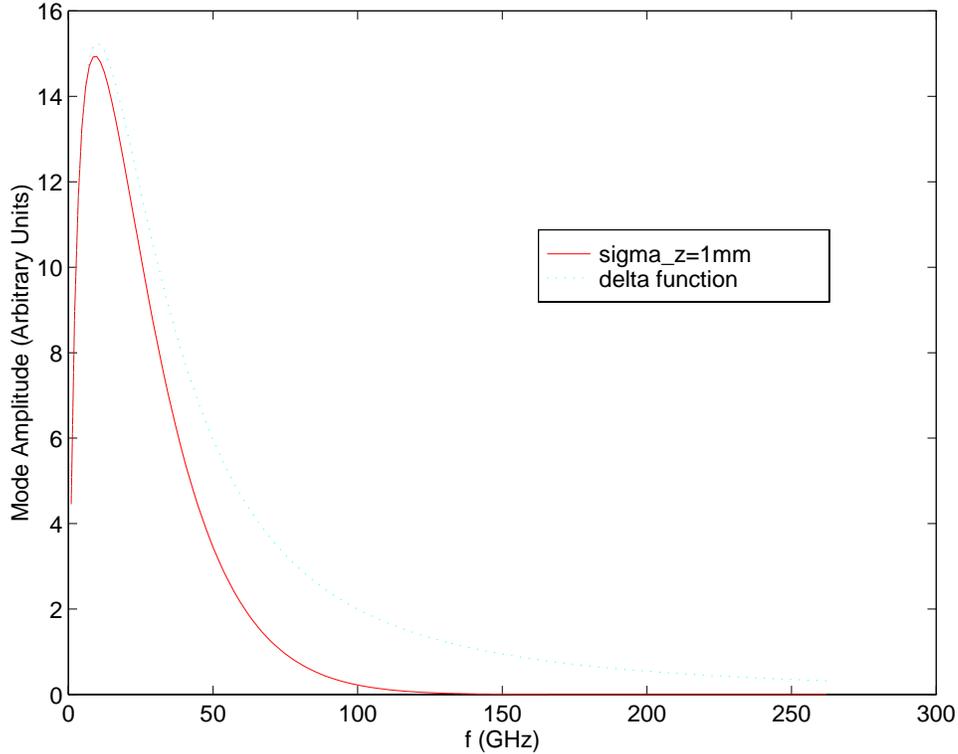
We first calculate the wakefield in a  $\lambda = 21$  cm device designed by Hirshfield as a multimode device. This is a circular dielectric waveguide with  $a = 0.15$  cm,  $b = 1.97$  cm,  $L = 60$  cm, and  $\epsilon = 9.43$ . The wake potential for a  $\sigma_z = 0.1$  cm bunch (including the region inside the beam) is shown in figure 1. The transformer ratio  $R = 2.384/1.307 = 1.824 < 2$ .



**FIGURE 1.** Wake potential for multimode 21 cm wavelength structure. The dashed curve shows the beam charge distribution.

Figure 2 shows the mode spectrum for this structure weighted by the current form factor (Fourier transform of  $f(z)$ ) for 200 modes. It is interesting to note that in contrast to most structures considered for wakefield accelerators, the lowest lying modes do not dominate the spectrum.

Another aspect of the work in reference [1] is the suggestion that driving a multimode structure with a train of periodically spaced bunches would allow stimulated



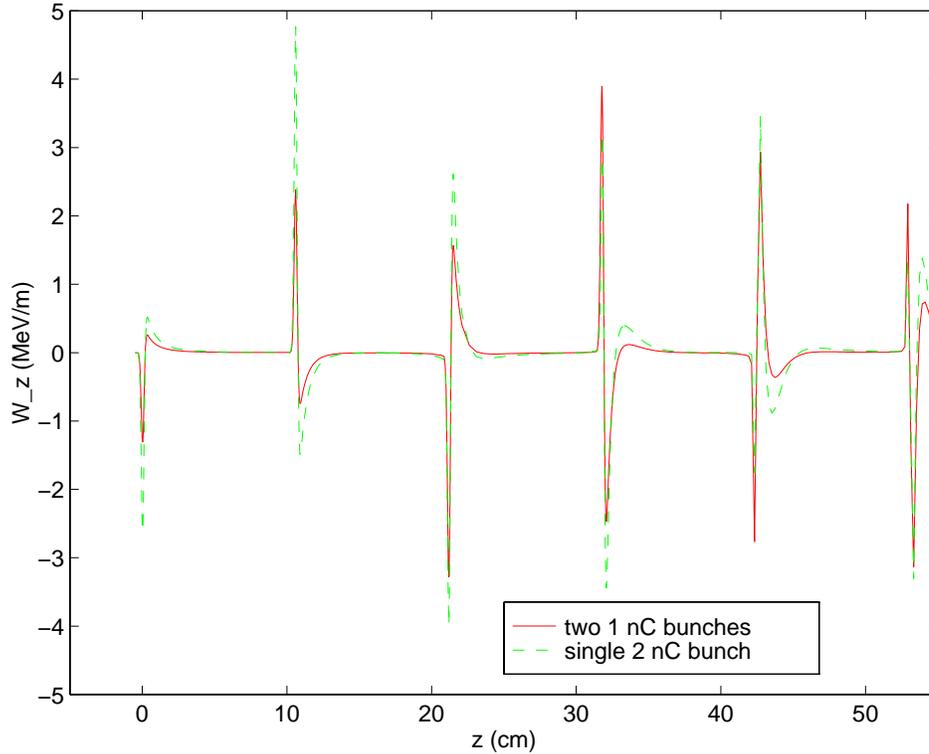
**FIGURE 2.** Current form factor weighted amplitude distribution for the first 200 modes of the 21 cm wavelength structure.

emission of wakefield energy to occur at a rate larger than the coherent radiation from a single bunch with the same total charge. Our calculations suggest that this is not the case. Figure 3 shows a comparison of the wake potentials of two 1 nC bunches spaced at 21 cm (the distance between the leading bunch and its first decelerating maximum) and that of a single 2 nC bunch. There is no apparent enhancement in the two bunch over the one bunch wake, and in fact this is to be expected by the principle of superposition.

We are planning to measure the wakes in a multimoded device optimized for the parameters of the AWA. For a structure with  $a = 0.5$  cm,  $b = 1.467$  cm,  $\epsilon = 36$ , the wavelength is 23.05 cm. The wake potential for this structure (taking  $\sigma_z = 1$  mm) is shown in figure 4. Materials for this device have been ordered and the experiment is planned for later this year. The gradient expected for this device, using a train of  $4 \times 10$  nC electron bunches is 16 MV/m.

## SUMMARY

We have studied the newly proposed multimode wakefield acceleration scheme. We find that using the usual definition of the transformer ratio, the wakefield

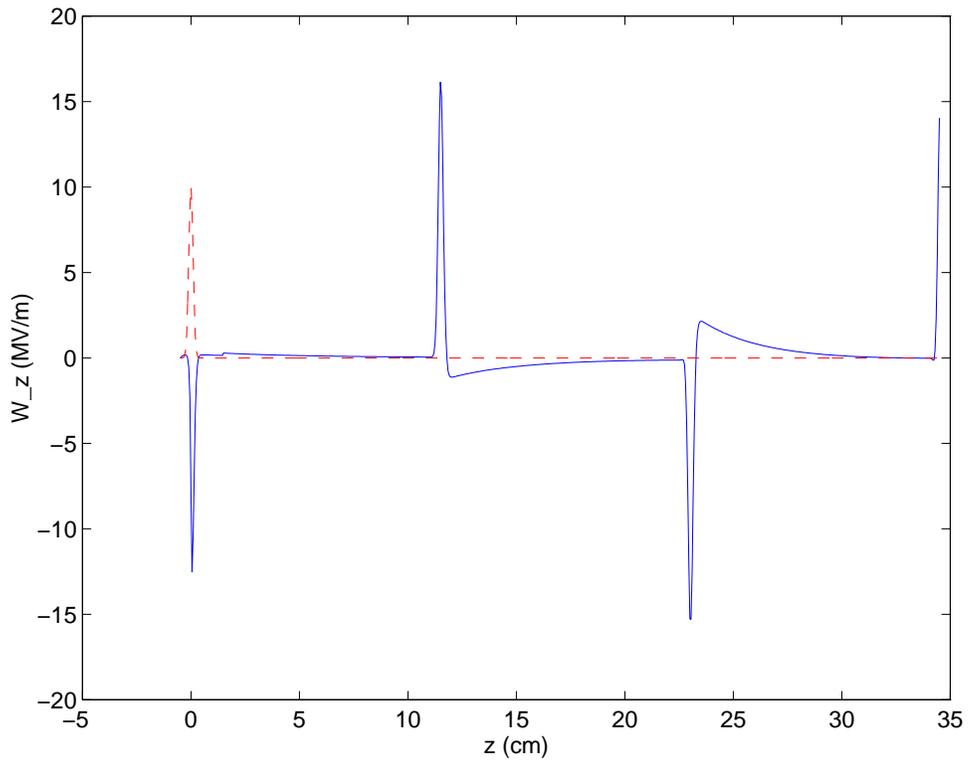


**FIGURE 3.** Comparison of wakes in multimode structure driven by two 1 nC bunches separated by 21 cm vs wake driven by a single 2 nC bunch.

theorem still holds, even though our numerical calculations agree with reference [1]. We also found the wakefield enhancement due to the use of multiple drive bunches simply scales with the total charge if one assumes all the beam parameters are the same for the individual bunches in the train. However, we believe that the multiple drive bunch scheme has an advantage considering both the capabilities of rf photocathode electron sources to produce bunch trains and the reduced sensitivity of multiple beams to parasitic wakefields in dielectric structures. We have designed an experiment at AWA to demonstrate this technique.

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**FIGURE 4.** Wake potential for the planned AWA multimode structure experiment.

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