Wakefields in Dielectric-Loaded Rectangular Waveguide Accelerating Structures

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Abstract. Employing modal analysis, an accurate formulation to represent the electromagnetic wakefields generated by an electron charge bunch traveling along an off-center axis in a dielectric loaded rectangular waveguide accelerating structure is presented. The resulting x-dipole and y-dipole modes are described in detail. Specific analytical results were obtained for the dispersion characteristics, the transverse force, and the longitudinal wakefields.

I. INTRODUCTION

Following the successful wakefield analysis of center-beam excited dielectric-loaded rectangular waveguides [1], an effort has been made to investigate the case of off-center excitation. Based on Fourier theorem, the wakefield resulted from traveling electron bunch can be decomposed into an infinite series of normal modes, each satisfying the boundary conditions required by the waveguiding structure [2]. Such a representation is more accurate than the single monopole mode approach, and is a necessity to account for the asymmetry in the total field caused by an off-center charge beam. At the same time, the analysis and computation become inevitably more involved. From a computation standpoint, the summation over the normal modes must be truncated at some point. Under the assumption of a very small off-center displacement for the charge beam, it can be expected that the monopole and the dipole modes should be adequate to provide an accurate account of the wakefields for practical considerations.

It is known [3] that the modes in a dielectric-loaded rectangular waveguide can be classified as LSM (longitudinal section magnetic) and LSE (longitudinal section electric) modes that have no H or E components normal to the interface respectively. This corresponds to assuming the transverse direction to the interface normal vector to be the direction of propagation. Applied to our case, when a short bunch moves through dielectric-loaded rectangular waveguide at transverse position (x_0, y_0) instead of on-axis where the coordinate is (0,0), the modes contributing to the longitudinal

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wake field should consist of monopole modes LSM_{1n} plus LSE_{1n} [1] and the dipole mode in x-direction, (which we denote as x-dipole) and the dipole mode in y-direction (y-dipole). Here, we neglect the other higher order modes because we assume that the bunch trajectory does not deviate far from the axis, and keeping terms to this order is sufficient to ensure the accuracy of the final results. We have verified this by comparing our analytical results with a numerical simulation using a commercial EM code.

The organization of this paper is as follows. In section II, we obtain the analytical expressions for the EM field components of multipole modes through potential functions, and then we calculate R over Q [R/Q], a very important structure parameter, for each related mode. Because R/Q has a very simple relation to the loss factor k_l of beam loading [4], we can obtain the longitudinal component of the wakefield amplitude excited by a charged particle traveling beam easily, rather than computation of the wakefield using the Panofsky-Wenzel theorem as [5].

II. EIGENMODES

The field components of some special accelerating modes in a dielectric-loaded rectangular waveguide have been analyzed using a mode matching method in our previous work [1]. The cross-section of a dielectric-loaded rectangular waveguide is shown in Figure 1. The general structure considered here is limited to the case of two *H*-plane slabs placed symmetrically. In [1], we only considered those modes with nonvanishing longitudinal electric components at the central point since any other modes will not couple to an on-axis beam. This implies that the central plane in the y-z view is an open plane. Using symmetries, the remaining modes with vanishing longitudinal electric field components on axis can be obtained by considering the central plane in the y-z view as a short plane.

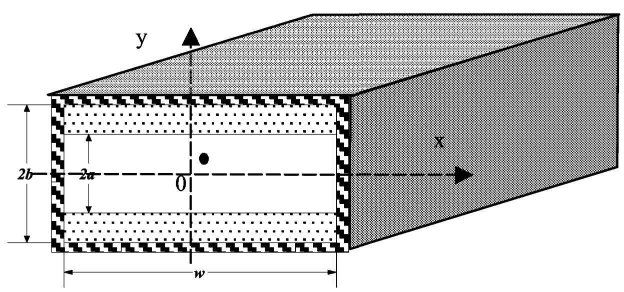


Fig.1 Dielectric-loaded rectangular guide

The transverse equivalent circuit corresponding to open and short planes for this symmetrical H-plane waveguide can be established as shown in Fig.2 (a) and (b) respectively. In this paper, we will use the superscript (open) and (short) to represent these modes with and without the longitudinal electric components nonvanishing at the central point. To clarify our procedure, we summarize all fundamental electromagnetic field physics in this kind of H-plane dielectric slab loaded rectangular waveguide. First of all, we obtain dispersion relations of the open and short central plane cases respectively by using transverse resonance method [6].

From [6], we can deduce the dispersion relations as follows:

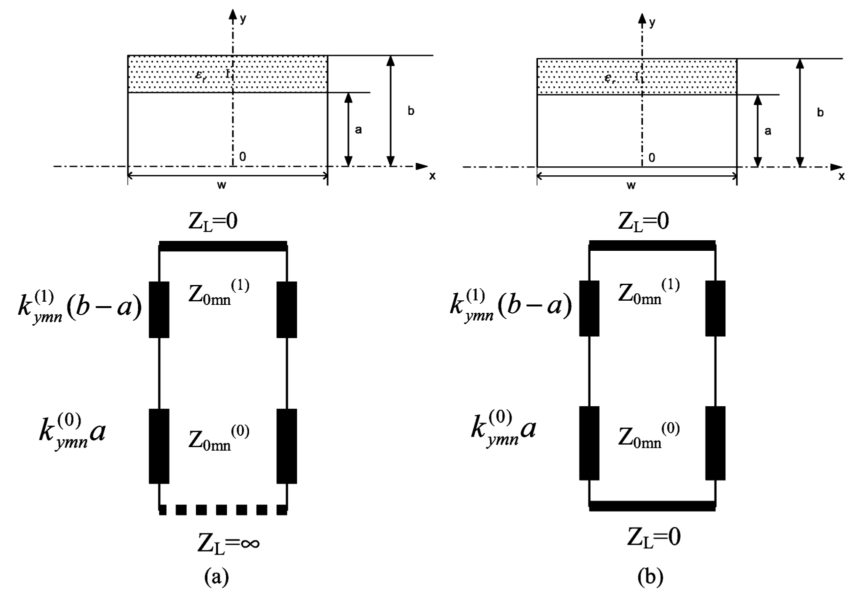


Fig.2 a) open plane and its equivalent circuit b) short plane and its equivalent circuit

$$\text{(open)} \quad -Z_{0mn}^{(0)} \cot(k_{ymn}^{(0)} a) + Z_{0mn}^{(1)} \tan[k_{ymn}^{(1)} (b-a)] = 0 \qquad \text{(1a)}$$

$$\text{(short)} \quad Z_{0mn}^{(0)} \tan(k_{ymn}^{(0)} a) + Z_{0mn}^{(1)} \tan[k_{ymn}^{(1)} (b-a)] = 0 \qquad \text{(1b)}$$

where $Z_{0mn}^{(0)} = \dfrac{k_{ymn}^{(0)}}{\omega\varepsilon_0}$, $Z_{0mn}^{(1)} = \dfrac{k_{ymn}^{(1)}}{\omega\varepsilon_0\varepsilon_r}$ (LSM_{mn})

$Z_{0mn}^{(0)} = \dfrac{\omega\mu_0}{k_{ymn}^{(0)}}$, $Z_{0mn}^{(1)} = \dfrac{\omega\mu_0}{k_{ymn}^{(1)}}$ (LSE_{mn})

are the values of the characteristic impedances of each mode, and the transverse propagation constants are expressed in terms of the longitudinal propagation constant β_{mn} using the following conditions:

$$k_{ymn}^{(0)\,2} = k^2 - \left(\frac{m\pi}{w}\right)^2 - \beta_{mn}^2$$

and $$k_{ymn}^{(1)\,2} = \varepsilon_r k^2 - \left(\frac{m\pi}{w}\right)^2 - \beta_{mn}^2 . \qquad (2)$$

Here k= 2πf/c is the propagation constant in free space and the notation of superscript (0) or (1) indicates the vacuum or dielectric region of the waveguide respectively.

Transcendental equation (1) is a complex function of β_{mn} and f, which gives the field components and dispersion relations of all the eigenmodes for a dielectric-loaded rectangular waveguide. For the inhomogeneous guide considered here, the dispersion relation must be solved at each frequency. An infinite number of discrete solutions exist, but we are interested only in those consisting of monopole and dipole modes.

The field components of $LSM^{(open)}$, $LSE^{(open)}$, $LSM^{(short)}$ and $LSE^{(short)}$ modes in a dielectric-loaded rectangular waveguide can be derived from the solution of vector potential wave equations as the following formulations[6].

LSM mode: $\vec{\psi}_e = \vec{\alpha}_y \psi_e(x,y,z)$

$$\begin{cases} E_x = -j\dfrac{1}{\omega\mu\varepsilon}\dfrac{\partial^2\psi_e}{\partial x\partial y} & H_x = -\dfrac{1}{\mu}\dfrac{\partial\psi_e}{\partial z} \\ E_y = -j\dfrac{1}{\omega\mu\varepsilon}(\dfrac{\partial^2}{\partial y^2}+\beta^2)\psi_e & H_y = 0 \\ E_z = -j\dfrac{1}{\omega\mu\varepsilon}\dfrac{\partial^2\psi_e}{\partial y\partial z} & H_z = -\dfrac{1}{\mu}\dfrac{\partial\psi_e}{\partial x} \end{cases} \quad (3a)$$

LSE mode: $\vec{\psi}_h = \vec{\alpha}_y \psi_h(x,y,z)$

$$\begin{cases} E_x = \dfrac{1}{\varepsilon}\dfrac{\partial\psi_h}{\partial z} & H_x = -j\dfrac{1}{\omega\mu\varepsilon}\dfrac{\partial\psi_h}{\partial x\partial y} \\ E_y = 0 & H_y = -j\dfrac{1}{\omega\mu\varepsilon}(\dfrac{\partial^2}{\partial y^2}+\beta^2)\psi_h \\ E_z = -\dfrac{1}{\varepsilon}\dfrac{\partial\psi_h}{\partial x} & H_z = -j\dfrac{1}{\omega\mu\varepsilon}\dfrac{\partial\psi_h}{\partial y\partial z} \end{cases} \quad (3b)$$

where ψ_e and ψ_h must satisfy the scalar wave equations of

$$\nabla^2\psi_q(x,y,z)+\beta^2\psi_q(x,y,z)=0 \qquad q=e,h. \quad (4)$$

Applying the boundary conditions at the perfectly conducting guide walls (x=±w/2, y=b) and the boundary condition at the magnetic or electric wall(y=0), the potential function ψ_e for open or short modes are

$$\psi_{emn}^{(open)} = \begin{cases} A_{mn}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(0)}y\cdot e^{-j\beta_{mn}z}, & 0<y<a \\ B_{mn}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(1)}(b-y)\cdot e^{-j\beta_{mn}z}, & a<y<b \end{cases} \quad (5a)$$

$$\psi_{emn}^{(short)} = \begin{cases} A_{mn}\sin\frac{m\pi}{w}(x+\frac{w}{2})\cos k_{ymn}^{(0)}y \cdot e^{-j\beta_{mn}z}, & 0<y<a \\ B_{mn}\sin\frac{m\pi}{w}(x+\frac{w}{2})\cos k_{ymn}^{(1)}(b-y) \cdot e^{-j\beta_{mn}z}, & a<y<b \end{cases} \qquad (5b)$$

In each region, the fields for the $LSE^{(open)}_{mn}$ or $LSE^{(short)}_{mn}$ derive from an magnetic-type potential function. Using the boundary conditions at the perfectly conducting guide walls (x=±w/2, y=b) and the boundary condition at the magnetic or electric wall (y=0), the potential function $\psi_{\mathbf{h}}$ is

$$\psi_{hmn}^{(open)} = \begin{cases} C_{mn}\frac{1}{j\omega\mu_0}\cos\frac{m\pi}{w}(x+\frac{w}{2})\cos k_{ymn}^{(0)}y \cdot e^{-j\beta_{mn}z}, & 0<y<a \\ D_{mn}\frac{1}{j\omega\mu_0}\cos\frac{m\pi}{w}(x+\frac{w}{2})\sin k_{ymn}^{(1)}(b-y) \cdot e^{-j\beta_{mn}z}, & a<y<b \end{cases} \qquad (6a)$$

$$\psi_{hmn}^{(short)} = \begin{cases} C_{mn}\frac{1}{j\omega\mu_0}\cos\frac{m\pi}{w}(x+\frac{w}{2})\sin k_{ymn}^{(0)}y \cdot e^{-j\beta_{mn}z}, & 0<y<a \\ D_{mn}\frac{1}{j\omega\mu_0}\cos\frac{m\pi}{w}(x+\frac{w}{2})\sin k_{ymn}^{(1)}(b-y) \cdot e^{-j\beta_{mn}z}, & a<y<b \end{cases} \qquad (6b)$$

The corresponding field components derived from equation (3) are reviewed briefly and listed separately as the Appendix of this paper.

III. TRANSVERSE WAKEFIELDS

The monopole and dipole modes are of greatest interest in determining beam stability. We have shown that the $LSM^{(open)}_{11}$ mode is the lowest luminal mode in our the structures under consideration, and its longitudinal electric field is distributed symmetrically in both x and y. In [1], the general acceleration properties of the $LSM^{(open)}_{11}$ mode had been calculated, such as the ratio of the peak surface electric field to the axial acceleration field E_s/E_0, the group velocity v_g, the attenuation constant α, and R/Q which measures the efficiency of acceleration in term of the given stored energy, etc. We also have analyzed the longitudinal wake fields of monopole modes $LSM^{(open)}_{1n}+LSE^{(open)}_{1n}$. Some results are presented and compared with numerical results from the MAFIA code and are found to be in good agreement.

In this paper, we will consider the transverse wake fields of dipole modes first by using the same method as used in [1] for the longitudinal wake fields of the monopole modes, then together with result from monopole analysis, we can obtain accurate wake field numerical simulations.

The longitudinal electric field components of $LSM^{(open)}_{mn}$, $LSE^{(open)}_{mn}$, $LSM^{(short)}_{mn}$ and $LSE^{(short)}_{mn}$ modes have the following form in the vacuum region:

$$E^{(open)}_{z,LSM} = A^{(open)}_{mn}(-j\beta_{mn})k^{(0)}_{y,mn} \sin\frac{m\pi}{w}(x+\frac{w}{2})\cos k^{(0)}_{y,mn}y \qquad (7a)$$

$$E^{(open)}_{z,LSE} = C^{(open)}_{mn}\frac{m\pi}{w}\sin\frac{m\pi}{w}(x+\frac{w}{2})\cos k^{(0)}_{y,mn}y \qquad (7b)$$

$$E^{(short)}_{z,LSM} = A^{(short)}_{mn}(j\beta_{mn})k^{(0)}_{y,mn} \sin\frac{m\pi}{w}(x+\frac{w}{2})\sin k^{(0)}_{y,mn}y \qquad (7c)$$

$$E^{(short)}_{z,LSE} = C^{(short)}_{mn}\frac{m\pi}{w}\sin\frac{m\pi}{w}(x+\frac{w}{2})\sin k^{(0)}_{y,mn}y \qquad (7d)$$

Obviously, $LSM^{(open)}_{2n}/LSE^{(open)}_{2n}$ modes are the x-dipole modes and $LSM^{(short)}_{1n}/LSE^{(short)}_{1n}$ are the y-dipole modes.

We assume a Gaussian longitudinal beam shape (with bunch length σ_z and charge e). The transverse forces can be directly calculated from E_z by using the Panofsky-Wenzel theorem [2]

$$\frac{\partial \vec{F}_{\perp}}{\partial z} = e\nabla_{\perp}E_z, \qquad (8)$$

For synchronous x-dipole modes ($k^{(0)}_y$=-j(2π/w)), the longitudinal electric field component and transverse forces can be expressed as

$$\begin{aligned} E_z^{x-dipole}(x,y,z) &= \sum_i E_{0i}(x_0,0)\sin\frac{2\pi}{w}(x+\frac{w}{2})\cos k_y^{(0)}y\cos\beta_i z \\ &= \sum_i E_{0i}(x_0,0)\sin\frac{2\pi}{w}x\cos k_y^{(0)}y\cos\beta_i z \end{aligned} \qquad (9)$$

where $E_{0i}(x_0, 0)$ is the off-axis $(x_0, 0)$ accelerating/ decelerating gradient of the i^{th} mode. Thus,

$$\begin{aligned} F_x(x_0,0,z) &= e\frac{2\pi}{w}\cdot\frac{1}{\sqrt{2\pi}\cdot\sigma_z}\int_{-\infty}^{z}\sum_i E_{0i}\cos(\frac{2\pi}{w}x_0)\cos\beta_i(z-z')\exp(-\frac{(z')^2}{2\sigma_z^2})dz' \\ \vec{F}_y(x_0,0,z) &= 0 \end{aligned} \qquad (10)$$

For synchronous y-dipole modes ($k^{(0)}_y$=-j(π/w)), the longitudinal electric field component and transverse forces can be expressed as

$$\begin{aligned} E_z^{y-dipole}(x,y,z) &= \sum_i E_{0i}(0,y_0)\sin\frac{\pi}{w}(x+\frac{w}{2})\sin k_y^{(0)}y\cos\beta_i z \\ &= \sum_i E_{0i}(0,y_0)\cos\frac{\pi}{w}x\sin k_y^{(0)}y\cos\beta_i z \end{aligned} \qquad (11)$$

where $E_{0i}(0, y_0)$ is the off-axis $(0, y_0)$ accelerating/ decelerating gradient of the i^{th} mode. Then,

$$F_y(0,y_0,z) = e\frac{1}{\sqrt{2\pi}\cdot\sigma_z}\int_{-\infty}^{z}\sum_i E_{0i}k_y^{(0)}\cos(k_y^{(0)}y_0)\cos\beta_i(z-z')\exp(-\frac{(z')^2}{2\sigma_z^2})dz' \quad (12)$$

$$\vec{F}_y(0,y_0,z) = 0$$

However, since we have already obtained the field distribution, and the normalized shunt impedance R/Q of each mode can be calculated using mode analysis [1], the longitudinal component of the wakefield amplitude excited by a charged particle beam traveling on axis can be easily obtained as follows [2].

$$E_{0i}(x_0, y_0) = \frac{q\omega_i}{4}(\frac{R}{Q})_i \quad (13)$$

where $$(\frac{R}{Q}) = \frac{E_z^2(x_0, y_0)v_g}{\omega P}.$$

$E_z(x_0, y_0)$ is the value of the longitudinal electrical field component at point (x_0, y_0), P is the power flow in cross-section of waveguide $P=\frac{1}{2}\int\int \vec{E}_t\times\vec{H}_t dxdy$, and v_g is the group velocity.

We assume a Gaussian longitudinal beam shape (with bunch length σ_z and charge q). The longitudinal wake $W_z(z)$ at distance z behind the drive electron beam is then given by $$W_z(z) = \frac{1}{\sqrt{2\pi}\sigma_z}\int_{-\infty}^{z}\sum_{i=1}^{\infty}E_{0i}\cos\beta_i(z-z')\exp(-\frac{z'^2}{2\sigma_z^2})dz' \quad (14).$$

Together with the monopole mode [1], the whole wakefield generated by the electron bunch traveling at the off-center position (x_0, y_0) can be obtained accurately.

IV. CALCULATED RESULTS

This section we will be concerned with presenting the data by applying the theory we introduced above to a specific case. Here, the dimensions of the X-Band waveguide are a =3mm, b =5mm and w =23mm, and the dielectric constant = 10. Fig.3 shows the dispersion characteristics of x-dipole and y-dipole modes, and we found the corresponding synchronous accelerating parameter for each mode, i.e. the intersection points between the dispersion curve of each mode and the light speed line.

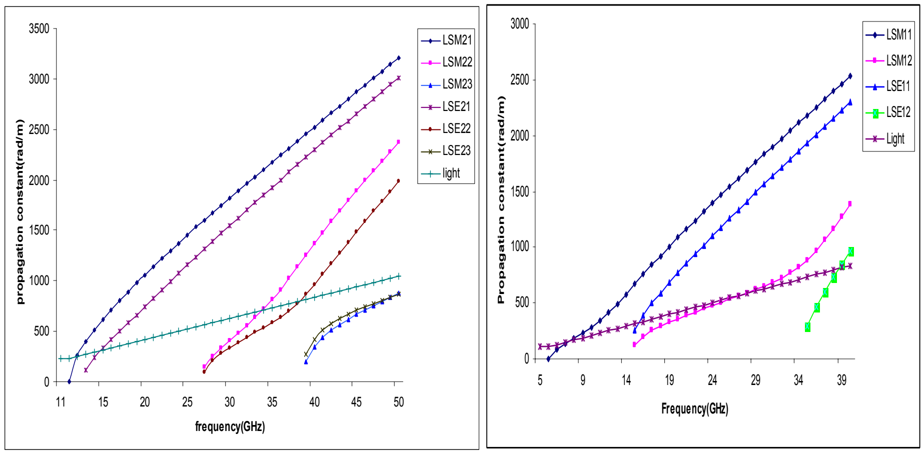


Fig.3 a) dispersion characteristics of x-dipole mode b) dispersion characteristics of y-dipole mode

Table (1) shows the calculated results of x-dipole modes synchronous with and acting upon an ultra-relativistic electron ($\beta=2\pi f/c$), in which $LSM^{(open)}_{21}$, $LSE^{(open)}_{21}$, $LSM^{(open)}_{22}$, $LSM^{(open)}_{23}$, $LSM^{(open)}_{24}$, and $LSE^{(open)}_{22}$ are considered.

TABLE 1. Parameters of x-dipole synchronous accelerating modes

Mode	Freq. (GHz)	β_i(rad/m)	$(R/Q)_i$
$LSM^{(open)}_{21}$	11.95	250.413	313.224
$LSE^{(open)}_{21}$	14.77	309.617	268.956
$LSM^{(open)}_{22}$	33.92	710.998	171.053
$LSE^{(open)}_{22}$	38.33	803.381	27.304
$LSM^{(open)}_{23}$	56.88	1192	70.91
$LSM^{(open)}_{24}$	80.44	1686	32.785

In Figure 4, the transverse wake fields obtained using equations (10) is shown. For this example, a q =1nC bunch with length σ_z =2mm located at x_0 =1mm and y_0 =0 is moving along the axis with a speed of light in dielectric-loaded waveguide. The first six x-dipole modes are used in the sum over modes; higher order modes do not contribute significantly.

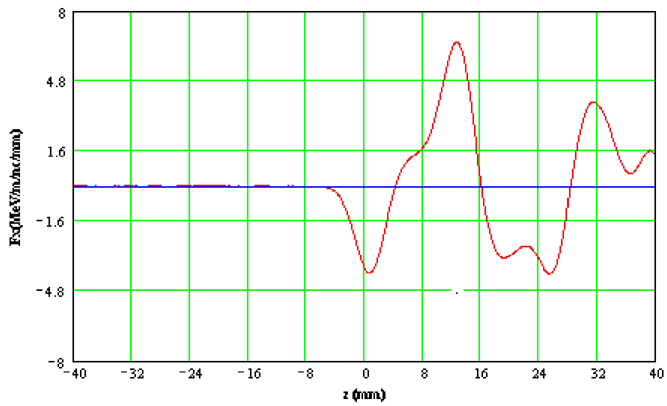


Fig. 4. Calculated transverse wakefield in an X-Band structure of x-dipole mode (a=3mm, b=5mm, w=23mm, ε_r=10, and σ_z=2 mm, q=1nc)

The corresponding results for y-dipole modes are shown in table (2), and here only four modes are considered.

TABLE 2. Parameters of y-dipole synchronous accelerating modes

Mode	Freq. (GHz)	β_i(rad/m)	$(R/Q)_i$
$LSM^{(short)}_{11}$	7.319	153.392	461.556
$LSE^{(short)}_{11}$	15.44	323.512	227.808
$LSM^{(short)}_{12}$	27.45	575.244	24.98
$LSE^{(short)}_{12}$	38.68	810.657	20.542

The transverse wake fields obtained using equations (12) are shown in Figure 5. The bunch is located at x_0=0 and y_0=1mm. The first four y-dipole modes are used in the sum over modes, and again higher order modes do not contribute significantly.

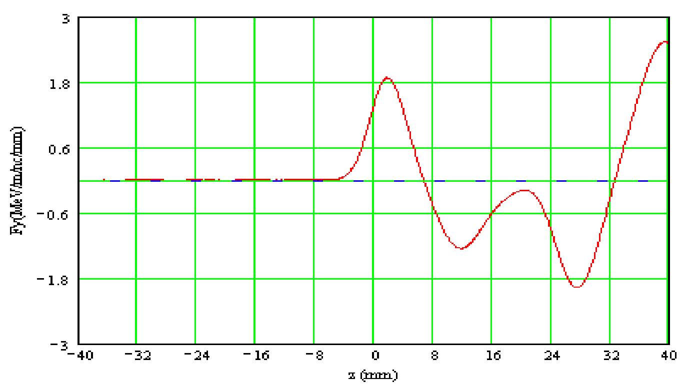
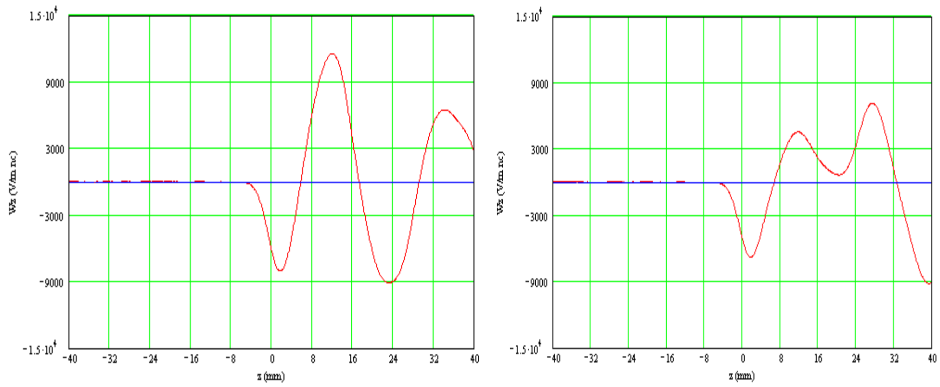


Fig. 5. Calculated transverse wakefield in an X-Band structure of y-dipole mode

In Fig.6, analytical simulation results for the longitudinal wakefields for x and y-dipole modes obtained by using Eq.14 are shown respectively. It is evidently that the x-dipole has a larger contribution to the bunch wakefield than the y-dipole.



(a) (b)

Fig.6 Longitudinal wakefield analysis (a) x-dipole case (b) y-dipole case

The modes that we choose to compute the whole wakefield are with respect to the contribution of $(R/Q)_i$ for each corresponding mode which includes both monopole and dipole. The table (3) is the list of mode and their parameters we choose. Fig.7 is final behavior of the wakefield including all contributions from the dipole modes.

TABLE 3. Modes used for calculating wakefield

Mode	Freq. (GHz)	β_i(rad/m)	$(R/Q)_i$
$LSM_{11}^{(mono)}$	11.17	234	12610
$LSM_{12}^{(mono)}$	33.28	697	3743
$LSE_{11}^{(mono)}$	13.19	276	3009
$LSM_{13}^{(mono)}$	56.24	1179	1334
$LSM_{14}^{(mono)}$	79.84	1673	574
$LSM_{11}^{(y\text{-}dipole)}$	7.319	153	462
$LSM_{21}^{(x\text{-}dipole)}$	11.95	250	313
$LSM_{15}^{(mono)}$	103.9	2177	288
$LSE_{21}^{(x\text{-}dipole)}$	14.77	310	269
$LSE_{11}^{(y\text{-}dipole)}$	15.44	324	228
$LSM_{22}^{(x\text{-}dipole)}$	33.92	711	171

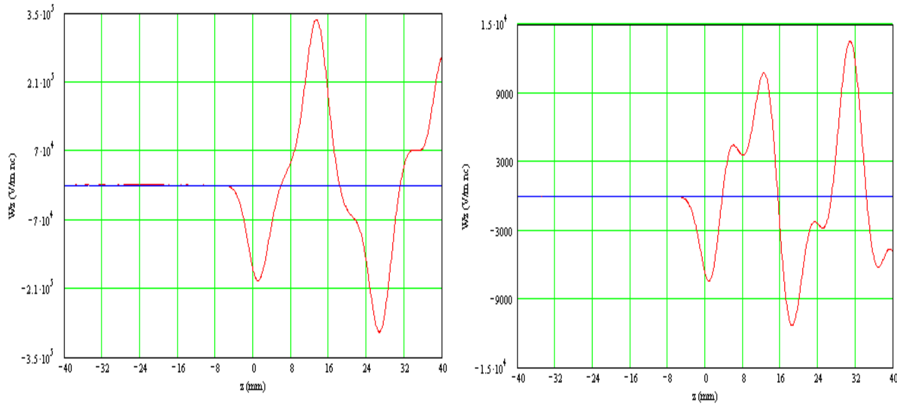


Fig.7 (a)analytical result of full wakefield (b) contribution of dipole modes to wakefield

V. CONCLUSIONS

A exact solution for wakefields generated by a bunch traversing a dielectric loaded rectangular waveguide accelerating structure off axis is developed. It is an extension of the previous approach for the on axis beam case. The idea behind this effective and accurate analytical method to solve the complicated problem of wakefields is based on the fact that there exists an orthogonal EM field modal decomposition and the dependence of wakefield on the linear combination of accelerating structure dimension factors R/Q.

ACKNOWLEDGEMENTS

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APPENDIX

The formulas of EM components for inhomogeneous H-plane dielectric loaded rectangular waveguide are listed here. The z dependence $\exp(-j\beta_{mn}z)$ has been omitted in these expressions for simplicity.

$LSM^{(open)}{}_{mn}$ modes:

$$
\left.
\begin{aligned}
E_{xmn} &= \begin{cases} A_{mn}\dfrac{m\pi}{w}k_{ymn}^{(0)}\cos\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(0)}y, & 0<y<a \\ B_{mn}\dfrac{m\pi}{w}k_{ymn}^{(1)}\cos\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(1)}(b-y), & a<y<b \end{cases} \\
E_{ymn} &= \begin{cases} A_{mn}k_{cmn}^{2}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(0)}y, & 0<y<a \\ B_{mn}k_{cmn}^{2}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(1)}(b-y), & a<y<b \end{cases} \\
E_{zmn} &= \begin{cases} A_{mn}(-j\beta_{mn})k_{ymn}^{(0)}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(0)}y, & 0<y<a \\ B_{mn}(-j\beta_{mn})k_{ymn}^{(1)}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(1)}(b-y), & a<y<b \end{cases} \\
H_{xmn} &= \begin{cases} -A_{mn}\omega\varepsilon_0\beta_{mn}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(0)}y, & 0<y<a \\ -B_{mn}\omega\varepsilon_0\varepsilon_r\beta_{mn}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(1)}(b-y), & a<y<b \end{cases} \\
H_{ymn} &= 0 \\
H_{zmn} &= \begin{cases} A_{mn}(j\omega\varepsilon_0)\dfrac{m\pi}{w}\cos\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(0)}y, & 0<y<a \\ B_{mn}(j\omega\varepsilon_0\varepsilon_r)\dfrac{m\pi}{w}\cos\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(1)}(b-y), & a<y<b \end{cases}
\end{aligned}
\right\} \quad (1)
$$

where $\dfrac{A_{mn}}{B_{mn}}=\dfrac{k_{ymn}^{(1)}\sin k_{ymn}^{(1)}(b-a)}{k_{ymn}^{(0)}\cos k_{ymn}^{(0)}a}, or\ \dfrac{A_{mn}}{B_{mn}}=\dfrac{\varepsilon_r\cos k_{ymn}^{(1)}(b-a)}{\sin k_{ymn}^{(0)}a},$ (2)

$LSE^{(open)}{}_{mn}$ modes:

$$
\left.
\begin{aligned}
E_{xmn} &= \begin{cases} C_{mn}(-j\beta_{mn})\cos\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(0)}y, & 0<y<a \\ D_{mn}(-j\beta_{mn})\cos\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(1)}(b-y), & a<y<b \end{cases} \\
E_{ymn} &= 0 \\
E_{zmn} &= \begin{cases} C_{mn}\dfrac{m\pi}{w}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(0)}y, & 0<y<a \\ D_{mn}\dfrac{m\pi}{w}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(1)}(b-y), & a<y<b \end{cases} \\
H_{xmn} &= \begin{cases} C_{mn}\dfrac{k_{ymn}^{(0)}}{j\omega\mu_0}\dfrac{m\pi}{w}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(0)}y, & 0<y<a \\ D_{mn}\dfrac{k_{ymn}^{(1)}}{j\omega\mu_0}\dfrac{m\pi}{w}\sin\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(1)}(b-y), & a<y<b \end{cases} \\
H_{ymn} &= \begin{cases} C_{mn}\dfrac{k_{cmn}^{2}}{j\omega\mu_0}\cos\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(0)}y, & 0<y<a \\ D_{mn}\dfrac{k_{cmn}^{2}}{j\omega\mu_0}\cos\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(1)}(b-y), & a<y<b \end{cases} \\
H_{zmn} &= \begin{cases} C_{mn}\dfrac{\beta_{mn}k_{ymn}^{(0)}}{\omega\mu_0}\cos\dfrac{m\pi}{w}(x+\dfrac{w}{2})\sin k_{ymn}^{(0)}y, & 0<y<a \\ D_{mn}\dfrac{\beta_{mn}k_{ymn}^{(1)}}{\omega\mu_0}\cos\dfrac{m\pi}{w}(x+\dfrac{w}{2})\cos k_{ymn}^{(1)}(b-y), & a<y<b \end{cases}
\end{aligned}
\right\} \quad (3)
$$

where $\dfrac{C_{mn}}{D_{mn}}=\dfrac{k_{ymn}^{(1)}\cos k_{ymn}^{(1)}(b-a)}{k_{ymn}^{(0)}\sin k_{ymn}^{(0)}a}, or\ \dfrac{C_{mn}}{D_{mn}}=\dfrac{\sin k_{ymn}^{(1)}(b-a)}{\cos k_{ymn}^{(0)}a},$ (4)

LSM$^{(short)}{}_{mn}$ modes:

$$
\left.
\begin{array}{l}
E_{xmn} = \begin{cases} -A_{mn} \dfrac{m\pi}{w} k^{(0)}_{ymn} \cos \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \sin k^{(0)}_{ymn} y, & 0<y<a \\ B_{mn} \dfrac{m\pi}{w} k^{(1)}_{ymn} \cos \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \sin k^{(1)}_{ymn} (b-y), & a<y<b \end{cases} \\
E_{ymn} = \begin{cases} A_{mn} k^2_{cmn} \sin \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \cos k^{(0)}_{ymn} y, & 0<y<a \\ B_{mn} k^2_{cmn} \sin \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \cos k^{(1)}_{ymn} (b-y), & a<y<b \end{cases} \\
E_{zmn} = \begin{cases} A_{mn} j\beta_{mn} k^{(0)}_{ymn} \sin \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \sin k^{(0)}_{ymn} y, & 0<y<a \\ B_{mn} (-j\beta_{mn}) k^{(1)}_{ymn} \sin \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \sin k^{(1)}_{ymn} (b-y), & a<y<b \end{cases} \\
H_{xmn} = \begin{cases} -A_{mn} \omega\varepsilon_0 \beta_{mn} \sin \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \cos k^{(0)}_{ymn} y, & 0<y<a \\ -B_{mn} \omega\varepsilon_0 \varepsilon_r \beta_{mn} \sin \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \cos k^{(1)}_{ymn} (b-y), & a<y<b \end{cases} \\
H_{ymn} = 0 \\
H_{zmn} = \begin{cases} A_{mn} (j\omega\varepsilon_0) \dfrac{m\pi}{w} \cos \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \cos k^{(0)}_{ymn} y, & 0<y<a \\ B_{mn} (j\omega\varepsilon_0 \varepsilon_r) \dfrac{m\pi}{w} \cos \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \cos k^{(1)}_{ymn} (b-y), & a<y<b \end{cases}
\end{array}
\right\} \quad (5)
$$

where $\dfrac{A_{mn}}{B_{mn}} = \dfrac{-k^{(1)}_{ymn} \sin k^{(1)}_{ymn}(b-a)}{k^{(0)}_{ymn} \sin k^{(0)}_{ymn} a}, or \dfrac{A_{mn}}{B_{mn}} = \dfrac{\varepsilon_r \cos k^{(1)}_{ymn}(b-a)}{\cos k^{(0)}_{ymn} a},$ (6)

LSE$^{(short)}{}_{mn}$ modes

$$
\left.
\begin{array}{l}
E_{xmn} = \begin{cases} C_{mn} (-j\beta_{mn}) \cos \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \sin k^{(0)}_{ymn} y, & 0<y<a \\ D_{mn} (-j\beta_{mn}) \cos \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \sin k^{(1)}_{ymn} (b-y), & a<y<b \end{cases} \\
E_{ymn} = 0 \\
E_{zmn} = \begin{cases} C_{mn} \dfrac{m\pi}{w} \sin \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \sin k^{(0)}_{ymn} y, & 0<y<a \\ D_{mn} \dfrac{m\pi}{w} \sin \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \sin k^{(1)}_{ymn} (b-y), & a<y<b \end{cases} \\
H_{xmn} = \begin{cases} C_{mn} \dfrac{k^{(0)}_{ymn}}{j\omega\mu_0} (-\dfrac{m\pi}{w}) \sin \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \cos k^{(0)}_{ymn} y, & 0<y<a \\ D_{mn} \dfrac{k^{(1)}_{ymn}}{j\omega\mu_0} \dfrac{m\pi}{w} \sin \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \cos k^{(1)}_{ymn} (b-y), & a<y<b \end{cases} \\
H_{ymn} = \begin{cases} C_{mn} \dfrac{k^2_{cmn}}{j\omega\mu_0} \cos \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \sin k^{(0)}_{ymn} y, & 0<y<a \\ D_{mn} \dfrac{k^2_{cmn}}{j\omega\mu_0} \cos \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \sin k^{(1)}_{ymn} (b-y), & a<y<b \end{cases} \\
H_{zmn} = \begin{cases} C_{mn} \dfrac{-\beta_{mn} k^{(0)}_{ymn}}{\omega\mu_0} \cos \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \cos k^{(0)}_{ymn} y, & 0<y<a \\ D_{mn} \dfrac{\beta_{mn} k^{(1)}_{ymn}}{\omega\mu_0} \cos \dfrac{m\pi}{w}(x+\dfrac{w}{2}) \cos k^{(1)}_{ymn} (b-y), & a<y<b \end{cases}
\end{array}
\right\} \quad (7)
$$

where $\dfrac{C_{mn}}{D_{mn}} = \dfrac{-k^{(1)}_{ym} \cos k^{(1)}_{ym}(b-a)}{k^{(0)}_{ym} \cos k^{(0)}_{ym} a}, or \dfrac{C_{mn}}{D_{mn}} = \dfrac{\sin k^{(1)}_{ym}(b-a)}{\sin k^{(0)}_{ym} a},$ (8)

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