WF-NOTE-234
August 2006

Modeling of Flux Concentrator

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1. Motivation

The objective of this study is to develop an equivalent circuit model for modeling a flux concentrator [1], to have a clear understanding of how a flux concentrator works from circuit point of view. In addition, use the equivalent circuit model to help on flux concentrator design for the ILC.

2. Introduction of modeling method

Flux concentrator technique is a viable option to produce high magnetic field for ILC positron matching. It works in a pulsed mode. In general, solenoids are designed and operated in DC state. Due to the pulsed operation nature of the flux concentrator, one needs to have a clear understanding of frequency response for the solenoid. In order to model the transient response of a flux concentrator, we developed an equivalent circuit model based on frequency domain analysis. The procedure of the modeling process, as shown in Fig. 1, is described as below.

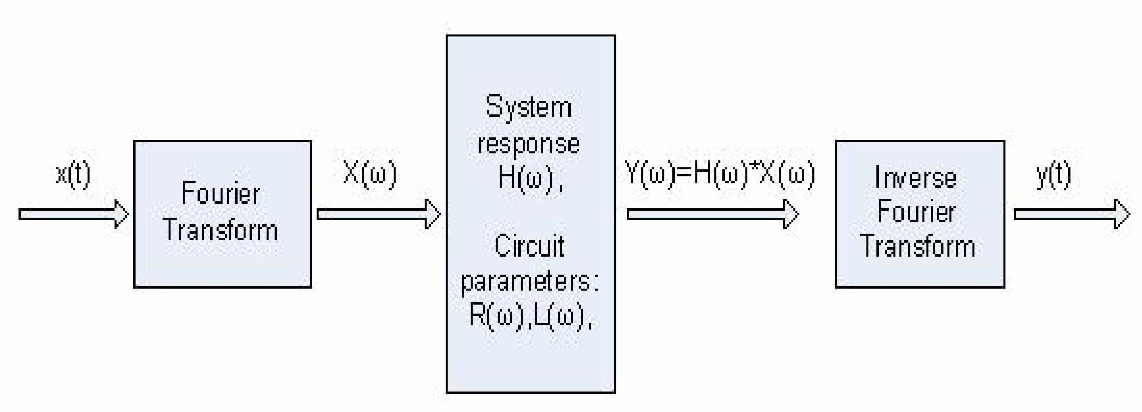

Fig.1 A diagrammatic sketch of modeling the transient response of a pulsed flux concentrator.

- Using Fourier transform to transform the external input pulsed signal, x(t), into a frequency domain signal ,X(ω).
- Calculating the parameters of the frequency-dependent equivalent circuits, such as resistance, R, self inductance, L, and mutual inductance, M.
- Solving the equations of equivalent circuits for the frequency domain response of a flux concentrator, H(ω).

- By multiplying the input signal, X(ω), with the system frequency domain response H(ω), the total system output in frequency domain is then obtained as Y(ω)= X(ω) H(ω).
- Finally, the transient response, y(t), is obtained by applying Inverse Fourier Transform to frequency domain solutions Y(ω).

3. Detailed modeling of an actual flux concentrator

In this section, we present a detailed modeling process by modeling a practical flux concentrator, which is an early-days structure designed by Brechna et al.[1]. Based on this process, we want to confirm the feasibility of the circuit model by comparing to the measurement results.

3.1 Geometric structure of the flux concentrator

Fig. 2 shows the cross section of the flux concentrator. Only upper half part is drawn. The flux concentrator consists of four disks. These disks are 1.12 cm thick at the outer edge and 47.7cm in diameter, with a raised central hub of 2.86cm in thickness and 20.3cm in diameter. Each disk has a central hole. Different from the actual structure, these holes in our model have same diameter of 2.3cm, rather than a bore profile. The dark shadow regions, as shown in the figure, are coils, which are wound of rectangular OFHC copper wire, 0.475×0.381cm in cross section, in a two-layer spiral (26 turns per layer). The coils form four pancakes. The number of turns of each pancake coil is 52, and the total number of turns of the coils is 208. In consideration of the insulation between turns and between layers, the width of each pancake coil is 1cm and thickness is 10.4cm.

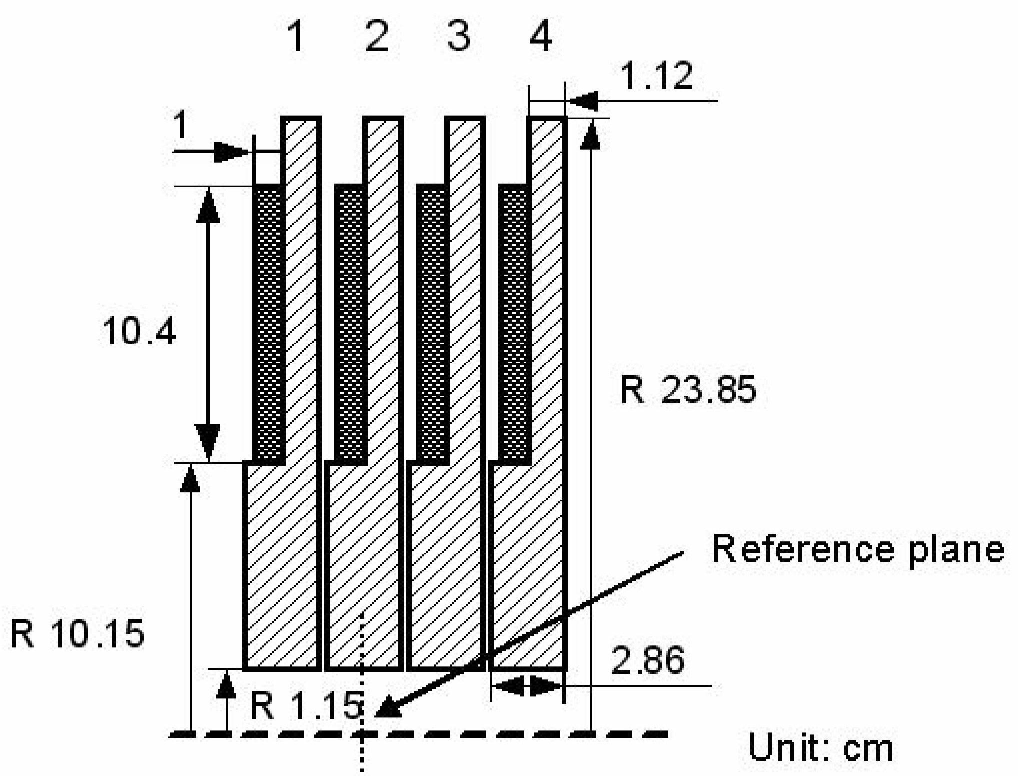

Fig. 2 Cross sectional view of an actual flux concentrator (only upper half is drawn). The coils are dark area and copper disks are the shaded area to be modeled.

3.2 Modeling based on circuit parameters

Due to cylindrical symmetry, we apply a 2-D model to the structure. In r-z plane each disk of flux concentrator is partitioned along longitudinal and radial directions into m rows and n columns, total N ($N=m\times n$) small-size regions, called micro-regions, as shown in Fig. 3. Each micro-region is revolved to form an unclosed ring, represented by a circuit branch. As shown in Fig. 4, each circuit branch consists of resistance R, self-inductance L, and coupling M from primary coil current i_p and from currents of other circuit branches i_k (k=1,4N). The modeling of these circuit parameters will be described in the following section.

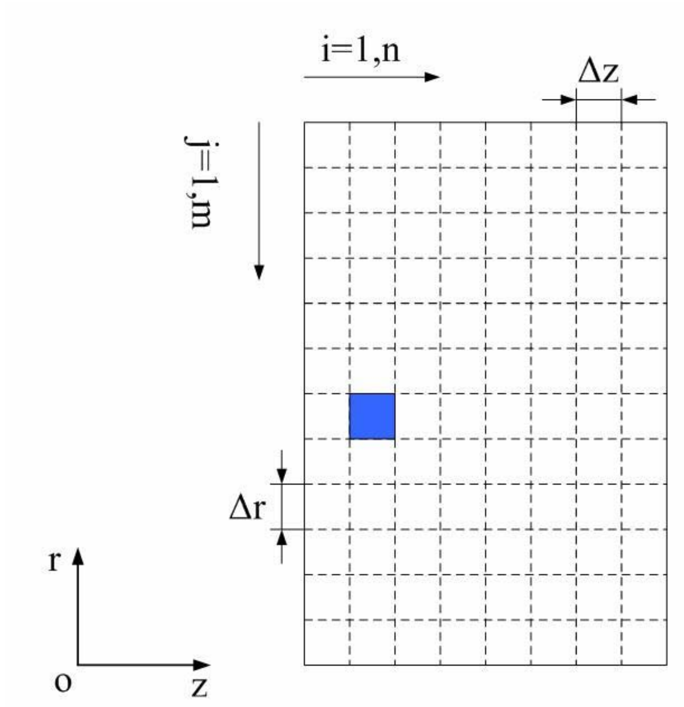

Fig. 3 2-D partitioning of single copper disk of flux concentrator.

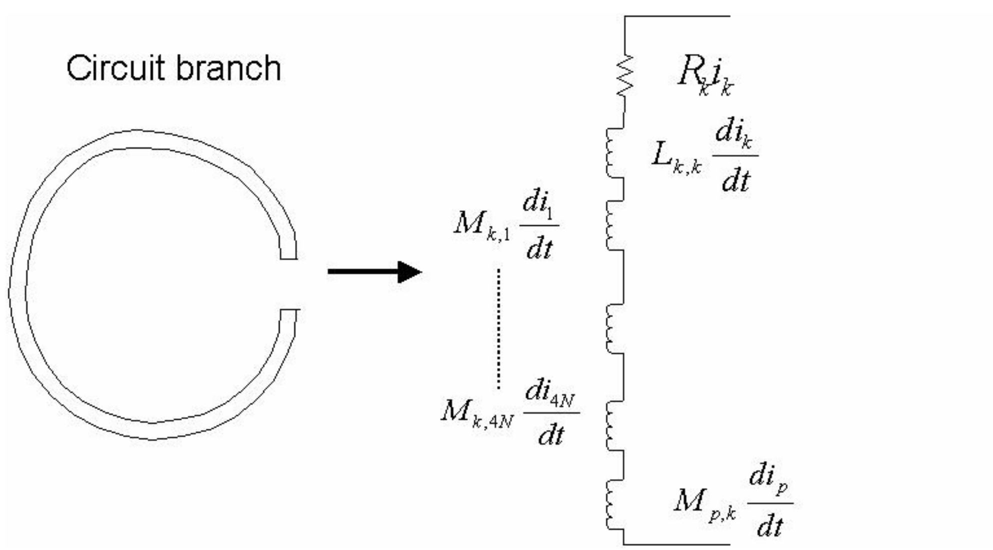

Fig. 4 Modeling of a circuit branch.

After modeling single circuit branch, the modeling of the entire flux concentrator can be obtained, as shown in Fig. 5. In order to abbreviate the drawing, in each circuit branch, the couplings from primary coil and other circuit branches are abstracted into a source in the diagram

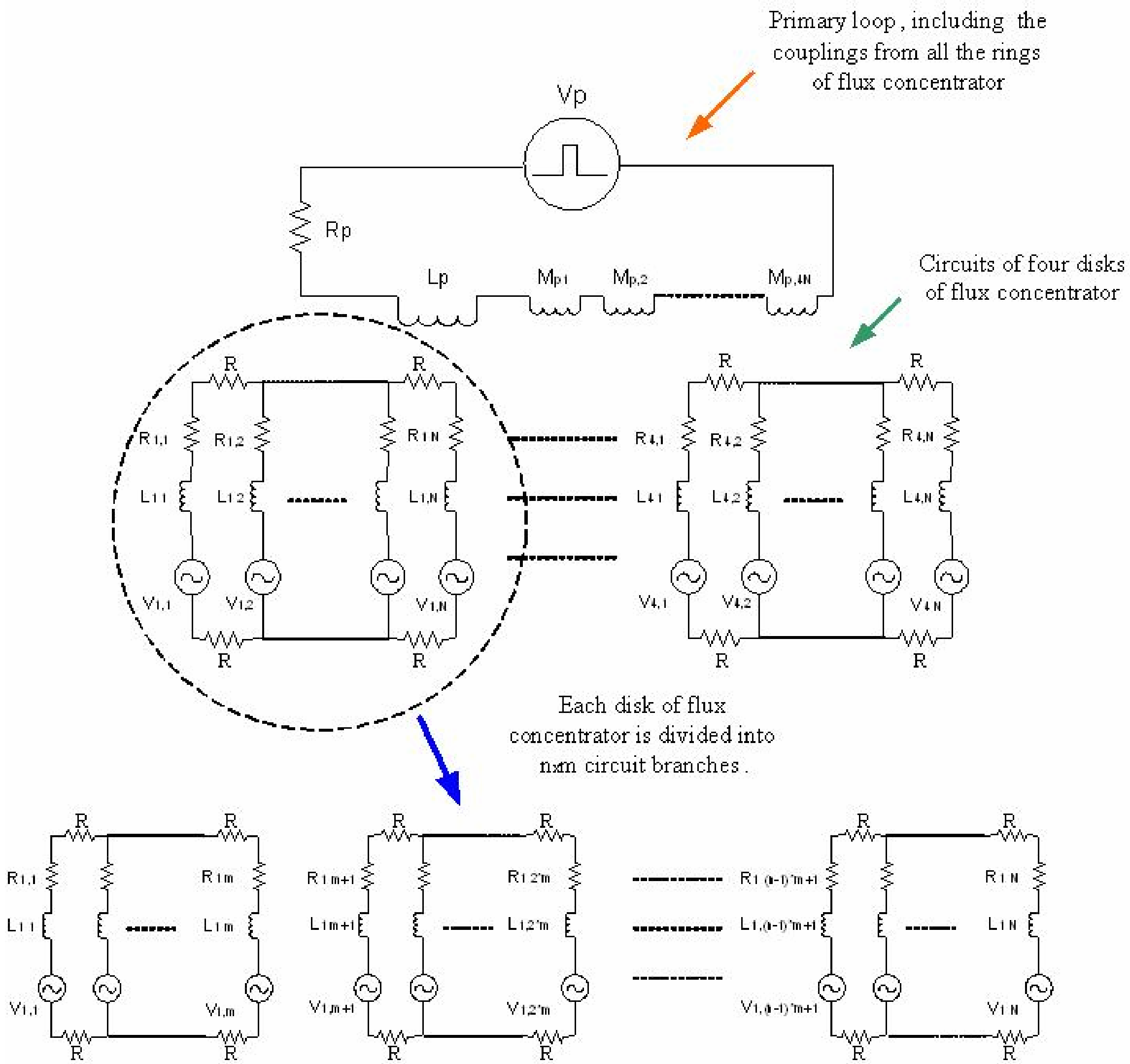

Fig. 5 Modeling of entire flux concentrator.

The upper part in Fig. 5 is the primary loop, which includes voltage source, resistance and inductance of primary coil, and coupling from flux concentrator. According to Kirchholff's Voltage Law, the circuit equation in the primary loop in frequency domain can be expressed as:

$$v_p = Z_p i_p + \sum_{k=1}^{4N} Z_{p,k} i_k \quad (1)$$

where,

Vp, the externally excited voltage signal,

i_p, the current in the primary coil,

i_k, the current of k-th circuit branch,

$Z_p = R_p + j\omega L_p$, the impedance of primary coil,

$Z_{p,k} = j\omega L_{p,k}$, the coupling between primary coil and each circuit branch of flux concentrator.

The flux concentrator consists of four disks, each of which is divided into m rows and n columns. If we assume magnetic field has z-component only, the currents on the side surface of the slit (narrow cut-off of flux concentrator along radial direction), converged from each circuit branch, flow along radial direction. Current coupling along z-direction does not exist. So for each column, sum of currents should be zero. This can be expressed as:

$$\sum_{k=(l-1)*m+1}^{i*m} i_k = 0 \qquad (2)$$

where l=1,n is column number.

Next, for each column, two adjacent circuit branches forms a closed circuit loop through the slit. In order to model the effect that currents flow along the slit, we apply a resistance R, as shown in Fig. 5, into the closed circuit loop. By applying Kirchholff's Voltage Law, the circuit equation for the loop is expressed as:

$$(Z_{p,k+1} - Z_{p,k})i_p + \sum_{m=1}^{4N}(Z_{m,k} - Z_{m,k+1})i_m = 0 \qquad (3)$$

where,

$$Z_{m,k} = \begin{cases} R_k + j\omega L_{k,k} & \text{If m=k} \\ j\omega M_{m,k} & \text{If m}\neq\text{k} \end{cases} \qquad (4)$$

Finally entire circuit equations of the flux concentrator in frequency domain are expressed using a matrix as:

$$\begin{bmatrix} Zp & Z_{mp1} & Z_{mp2} & \cdots & Z_{mpN} & \cdots & \cdots & \cdots & Z_{mp(4N)} \\ Z_{mp1}-Z_{mp2} & Z_{2,1}-Z_{1,1} & Z_{2,2}-Z_{1,2} & \cdots & Z_{2,N}-Z_{1,N} & \cdots & \cdots & \cdots & Z_{2,4N}-Z_{1,4N} \\ Z_{mp2}-Z_{mp3} & Z_{3,1}-Z_{2,1} & Z_{3,2}-Z_{2,2} & \cdots & Z_{3,N}-Z_{2,N} & \cdots & \cdots & \cdots & Z_{3,4N}-Z_{2,4N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{mp(N-1)}-Z_{mpN} & Z_{N,1}-Z_{N-1,1} & Z_{N,2}-Z_{N-1,2} & \cdots & \cdots & \cdots & \cdots & \cdots & Z_{N,4N}-Z_{N-1,4N} \\ 0 & 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{mp(4N-1)}-Z_{mp(4N)} & Z_{4N,1}-Z_{4N-1,1} & Z_{4N,2}-Z_{4N-1,2} & \cdots & \cdots & \cdots & \cdots & \cdots & Z_{4N,4N}-Z_{4N-1,4N} \\ 0 & 0 & 0 & \cdots & 0 & \cdots & \cdots & \cdots & 1 \end{bmatrix} \times \begin{bmatrix} i_p/v_p \\ i_1/v_p \\ i_2/v_p \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ i_{4N}/v_p \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

$$Z_p = R_p + j\omega L_p \qquad Z_{i,k} = \begin{cases} R_i + j\omega M_{ii} & \text{If i=k} \\ j\omega M_{ik} & \text{If i}\neq\text{k} \end{cases}$$

$$Z_{mpi} = j\omega M_{pi}$$

By solving the circuit equation systems, the frequency domain solutions of the currents in primary coil and in all the circuit branches are obtained. With the solution of frequency domain current distribution, the calculation of on-axis magnetic field at reference plane is straight forward in frequency domain. On-axis magnetic field produced by a current ring with radius r at reference plane (0, 0) can be obtained using the following equation:

$$B_z = \frac{\mu_0}{2} \frac{r^2 I}{(r^2 + z^2)^{3/2}} \qquad (5)$$

By summing the effects from all the current rings of flux concentrator and primary coil, the frequency domain solution on axis magnetic field can be obtained and the transient response of on-axis magnetic field at reference plane can be found by doing Inverse Fourier Transform.

3.3 Calculation methods of circuit parameters

In the following sub-sections, the calculation methods of different circuit parameters, like resistance, self and mutual inductance will be presented.

3.3.1 Resistance

Due to the skin effect, the current intensity inside the flux concentrator can be expressed as:

$$J = J_0 \exp(-d/\delta) \qquad (6)$$

where

d is the distance off the surface.

Skin depth, δ, is expressed as:

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}} \qquad (7)$$

ρ is the conductor resistivity, for OFHC copper at 78°K, $\rho = 0.218 \times 10^{-8} \Omega \cdot m$, μ is the material permeability, and f is the frequency (Hz).

The resistance in k-th circuit branch as shown in Fig. 4 is represented as:

$$R_k = \rho \frac{l_k}{A_k} \qquad (8)$$

Here l_k is the length of the ring, and A_k is the effective area of k-th micro-region, which depends on current intensity distribution at that location. As an example, the effective area of the shadow region, as shown in Fig. 3, can be calculated as:

$$A_k = \Delta r \int_{(i-1)\cdot\Delta z}^{i\cdot\Delta z} \exp(-\frac{z}{\delta}) dz \qquad (9)$$

The resistance, R, on the side surface of slit between two adjacent circuit rings, can be calculated as:

$$R = \rho \frac{\Delta r}{\Delta z \cdot \int_0^{\delta} \exp(-\frac{h}{\delta}) dh} \quad (10)$$

Where h is the depth off the slit surface. Here just one skin depth is considered in the calculation.

3.3.2 Mutual inductance

The couplings between primary coil and each circuit branch, and between any two circuit branches may be obtained by calculating mutual inductance. In our model, the method of computing mutual inductance comes from Neumann's formula. The mutual inductance between two coaxial circular filaments, one with radius a and another with radius A, with distance between centers b, as shown in Fig. 6, can be calculated as:

$$M = \frac{\mu_0}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{Aa \cos(\varphi - \varphi') d\varphi d\varphi'}{\sqrt{A^2 + a^2 + b^2 - 2Aa \cos(\varphi - \varphi')}} \quad (11)$$

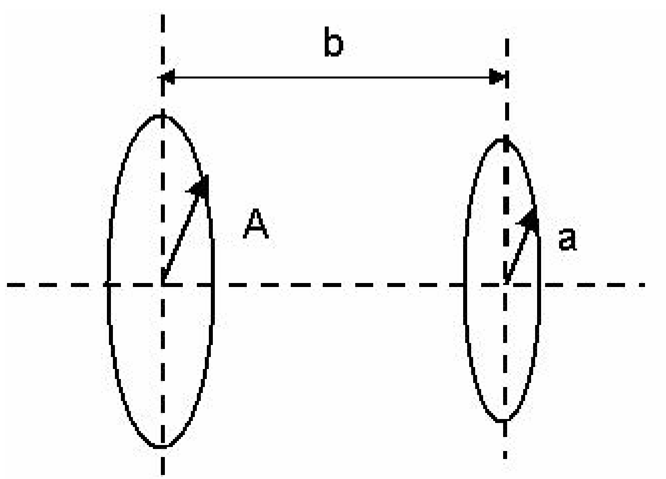

Fig. 6 A diagrammatic sketch of two coaxial circular filaments.

The integral from (11) can be exactly solved analytically as:

$$M = -\mu_0 \sqrt{Aa} \left[\left(k - \frac{2}{k} \right) K + \frac{2}{k} E \right] \quad (12)$$

where the modulus k is:

$$k = \frac{2\sqrt{Aa}}{\sqrt{(A+a)^2 + b^2}}$$

K and E are the complete elliptic integrals of first and second kinds:

$$K = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}}$$

$$E = \int_0^{\pi/2} \sqrt{1-k^2\sin^2\varphi}\,d\varphi$$

The mutual inductance between two concentrical coils with integer number of turns n1 and n2 is:

$$M_{tot} = \sum_{i=1}^{n1}\sum_{j=1}^{n2} M_{ij} \qquad (13)$$

The self-inductance can be calculated by computing the mutual inductance between two identical coils placed at a distance equal to the geometrical mean distance. The geometrical mean distance, D, for a round conductor with radius r is:

$$D = re^{-\frac{1}{4}} = 0.7788 \cdot r \qquad (14)$$

Note that in the actual application of this method to calculate the self and mutual inductance in the flux concentrator, in order to obtain accurate results, the ring size must be re-partitioned small enough to guarantee that the radius of curvature is much larger compared with the dimensions of the transverse section of each ring.

4. Results and comparison

Based on the method described above, the flux concentrator in [1] is calculated. Due to the limitation of computation resource, the flux concentrator in 2-D r-z plane is partitioned into micro-regions of 4mm×4mm in size, and frequency is calculated from 0 up to 500Hz. Fig. 7 shows the calculation results from our circuit model. The excited voltage, Vp, is 600V with pulse width 50ms. i_p is current response in primary coil. Bz is the transient response of on-axis magnetic field at reference plane generated by primary coil and flux concentrator. Compared to the experimental measurement results [1], as shown in Fig. 8, the circuit model can give a reasonable agreement.

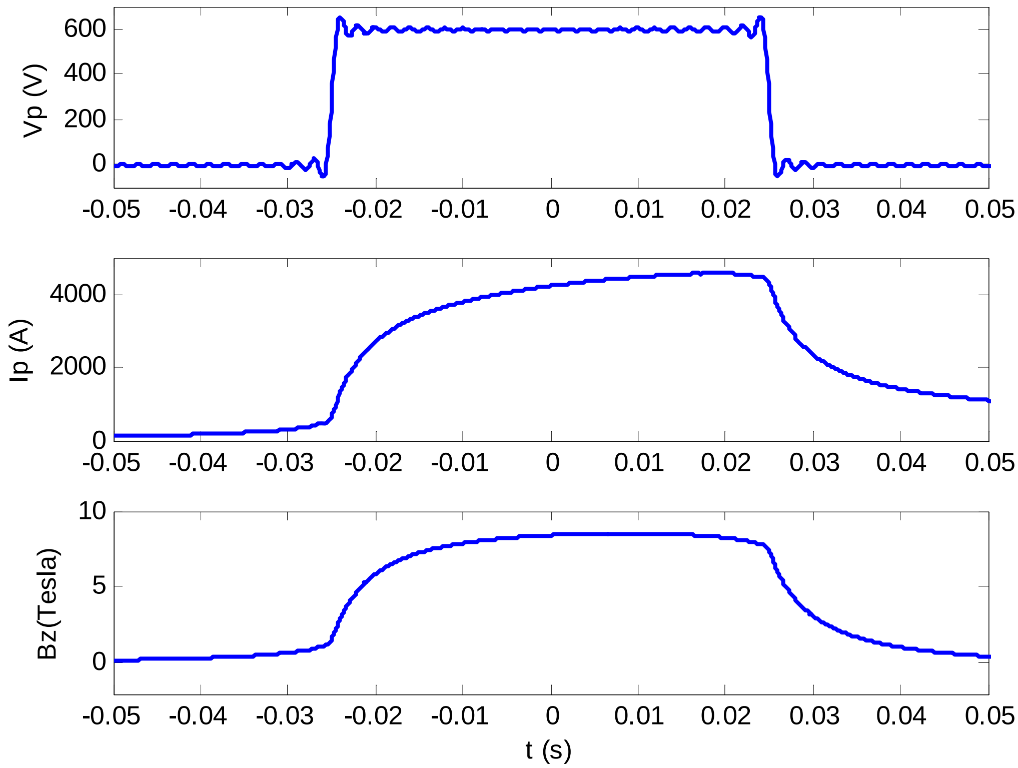

Fig. 7 The calculation results based on the circuit model.

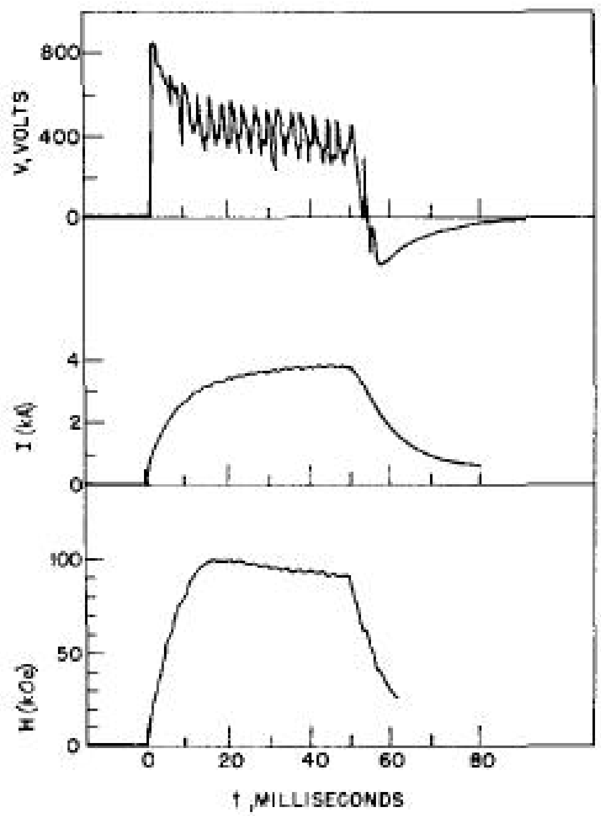

FIG. 8. Voltage and current waveforms at magnet terminals, and corresponding axial field at reference plane.

Fig. 8 The experimental measurement results of an early-days flux concentrator [1].

5. Summary

A circuit model has been developed to model the transient response of a pulsed flux concentrator. After modeling an actual flux concentrator, the result from the circuit model has a reasonable agreement to the measurement result. We will try to improve the algorithm in calculating the equivalent circuit parameters to improve the accuracy of the model. Based on this model, we will attempt a preliminary design for the AMD with pulse length of ~ 5 ms and amplitude of 5 – 10 Tesla.

References

[1] H. Brechna, D. A. Hill and B. M. Bally, “150 kOe Liquid Nitrogen Cooled Flux-Concentrator Magnet”, Rev. Sci. Instr., 36 1529, 1965.