K Corrections for Dummies

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January 26, 2005

Contents

1 Motivation

- How do we combine SDSS results with other Hubble diagrams using different magnitude systems ?
- Develop software and verify K-corrections quoted in earlier papers.
- Global check of simulation + K-cor + fitter:
  $\Omega_{\Lambda,M}$ → simulation → observer magnitude →
  K-correction → rest frame magnitude [B or g] →
  fitter → $\Omega_{\Lambda,M}$

2 Magnitudes for Dummies

From Bessell et al., AAS, **333** 231 (1998),

$$m_\lambda(i) = -2.5\log\left[\frac{\int f_\lambda(\lambda)S_i(\lambda)d\lambda}{\int S_i(\lambda)d\lambda}\right] - 21.100 - \mathrm{zp}_\lambda \quad (1)$$

$$m_\nu(i) = -2.5\log\left[\frac{\int f_\nu(\nu)S_i(\nu)d\nu}{\int S_i(\nu)d\nu}\right] - 48.598 - \mathrm{zp}_\nu \quad (2)$$

A few comments:

- $''21.100'' = -2.5\log(3.63 \times 10^{-9})$
  (defines `STMAG` at $\bar{\lambda}_V = 5450$Å for Vega, $erg/s/cm^2/$Å)
- $10^{(48.598-21.100)/2.5} = 10^{11} = d\nu/d\lambda = c/\bar{\lambda}_V^2$
  (defines `ABmag`)
- rare to find magnitude definition
  (only Bessell-98 and SDSS calib paper).
- "zp" obtained from Vega SED (which one ??)
- Note that λ and ν flux integrals are the same, but filter integrals are not!

$$\int f_\lambda(\lambda)S(\lambda)d\lambda = \int f_\nu(\nu)S(\nu)d\nu$$
$$\int S(\lambda)d\lambda \neq \int S(\nu)d\nu$$

For SDSS magnitude :

- $d\nu \rightarrow d\nu/\nu$ and $48.598 \rightarrow 48.600$ (and no zero-points).
- note that

$$f_\nu \frac{1}{\nu} d\nu = \frac{dE}{d\nu}\frac{1}{\nu} d\nu = h\frac{dN_\gamma}{d\nu} d\nu = hN_\gamma \qquad (3)$$

SDSS magnitude is a "count-flux" magnitude.

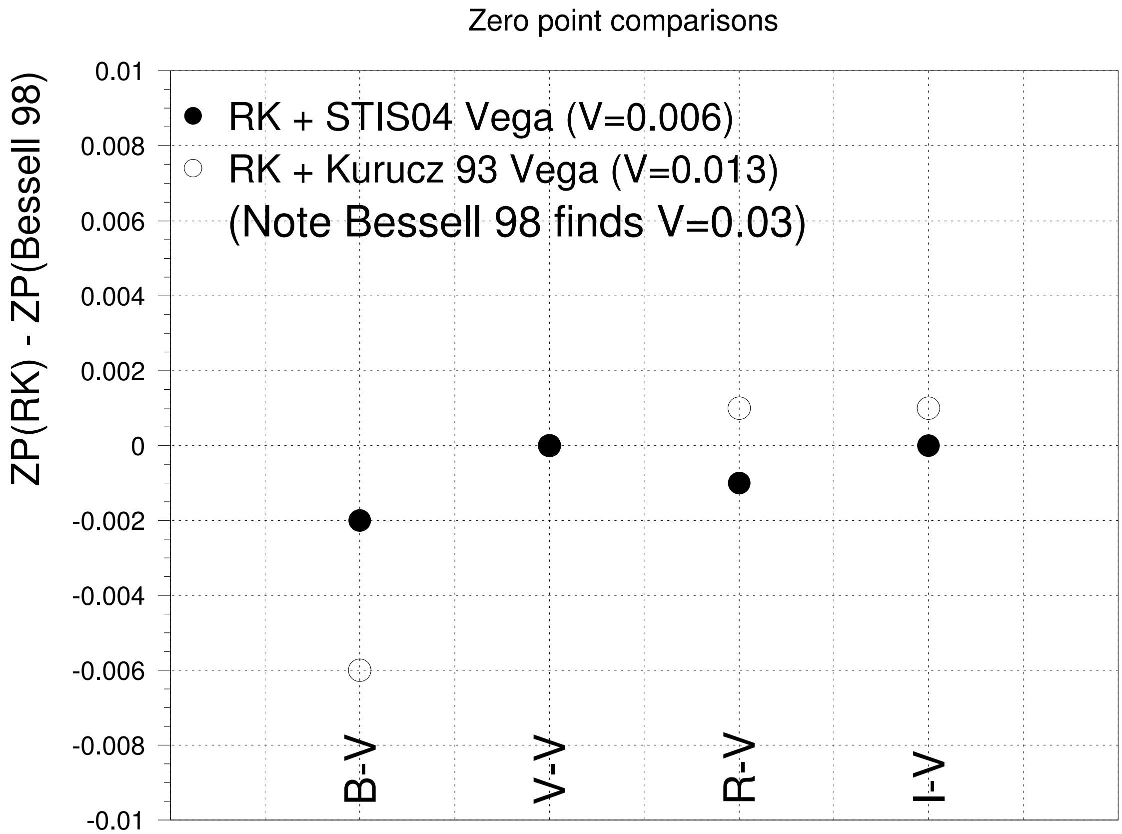

Figure 1: Zero point comparisons between RK and Bessell98. RK uses filter transmissions from Table 2 in Bessell90 [Bessell, PASP, **102** 1181 (1990)] and uses Vega models from STIS94 (**`alpha_lyr_stis_002.fits`**) and also from downloading model from Kurucz93. Bessell98 quotes Kurucz94 Vega model. $m_\lambda(V)$ differs by ~ 0.02 mags. Colors agree to < 0.002 mags, except for B–V when RK uses Kurucz93.

3 Definition of K Correction

K correction "K_{ro}" defined by

$$m_o = M_r + K_{ro} + \mu + A_r + A_o$$

where

- m_o = observed magnitude (in redshifted frame).
- M_r = absolute magnitude in rest frame.
- $\mu = -5\log[10pc/d_L(z)]$ depends on cosmological parameters $\Omega_\Lambda,\ \Omega_m,\ w$, etc ...
- A_r and A_o are extinctions for the SN-host (rest) and our galaxy (observer frame).

Below I ignore μ and $A_{r,o}$.

Discussion follows closely from
Nugent2002: astro-ph/0205351

3.1 Luminosity Distance

$$d_{L,\epsilon} = (1+z)\int dz'/H(z') \quad (4)$$
$$d_{L,\gamma} = \sqrt{1+z}\int dz'/H(z') \quad (5)$$

where ϵ, γ refer to "energy" and "photon".

- Note that redshifting of energy causes difference: $[d_{L,\epsilon}/d_{L,\gamma}]^2 = 1+z$
- Standard literature: $d_{L,\epsilon} \to d_L$.
- in Hubble diagram,

$$m_B^{\rm eff} = M_B + \mu$$

  where $\mu = -5\log[10pc/d_L(z)]$, and $M_B \sim -18.5$

3.2 K Correction Formula

$$K^{\epsilon}_{ro} = 2.5\log\left[(1+z) \times \frac{\int f_\lambda(\lambda) S_r(\lambda) d\lambda}{\int f_\lambda(\lambda/(1+z))) S_o(\lambda) d\lambda}\right] - 2.5\log\left[\frac{\int \mathcal{Z}^{\epsilon}_r(\lambda) S_r(\lambda) d\lambda}{\int \mathcal{Z}^{\epsilon}_o(\lambda) S_o(\lambda) d\lambda}\right] \tag{6}$$

$$K^{\gamma}_{ro} = 2.5\log\left[(1+z) \times \frac{\int \lambda f_\lambda(\lambda) S_r(\lambda) d\lambda}{\int \lambda f_\lambda(\lambda/(1+z)) S_o(\lambda) d\lambda}\right] - 2.5\log\left[\frac{\int \lambda \mathcal{Z}^{\gamma}_r(\lambda) S_r(\lambda) d\lambda}{\int \lambda \mathcal{Z}^{\gamma}_o(\lambda) S_o(\lambda) d\lambda}\right] \tag{7}$$

where

- $f_\lambda(\lambda) = dE/d\lambda$ (SED) for SN
- $\mathcal{Z}(\lambda)$ = SED of reference star with $m_\lambda = 0$ in all filters.
- $S_o(\lambda)$ = transmission of observer filter "o."
- $S_r(\lambda)$ = transmission of rest-frame filter "r."

Comments:

- Use Johnson filters as indicated in Nugent 2002.
- Could not find any discussion of $\mathcal{Z}(\lambda)$; I use Kurucz93 Vega SED with zero points applied (Eq. 1)
- $K_{ro}^{\epsilon,\gamma}$ are both defined to be used with $d_L = d_{L,\epsilon}$.
- $|K_{ro}^{\epsilon} - K_{ro}^{\gamma}|$ is typically < 0.01 mag.
- K_{ro}^{ϵ} valid for both m_λ and m_ν systems.
  (note K_{ro}^{ϵ} definition has no $\int S(\lambda)d\lambda$ terms)
- In limit when $S_r = S_o = 1.000$ at all λ:

$$K_{ro}^{\epsilon} = 0.0$$
$$K_{ro}^{\gamma} = -2.5\log(1+z)$$

- Beware: "$\int \mathcal{Z}_r^{\gamma}(\lambda)S_r(\lambda)d\lambda$" is sometimes called a zero-point, but it is NOT a zero-point [because it's not divided by $\int S(\lambda)d\lambda$]. Must do brute-force integration.

3.3 K Correction Formula for SDSS

- Since SDSS uses photon-count magnitude system, use K^{γ}_{ro} $(r, o = u, g, r, i, z)$.
- But we do not have SEDs for zero-mag stars.
- Therefore, re-write K correction as:

$$K^{\gamma}_{ro} = 2.5\log\left[(1+z) \times \frac{\int f_{\nu}(\nu) S_r(\nu)\frac{1}{\nu}d\nu}{\int f_{\nu}(\nu(1+z)) S_o(\nu)\frac{1}{\nu}d\nu}\right] - 2.5\log\left[\frac{\int S_r(\nu)\frac{1}{\nu}d\nu}{\int S_o(\nu)\frac{1}{\nu}d\nu}\right] \quad (8)$$

- Raises analysis issue. To include SDSS data on other [high-z] Hubble diagrams, do we
  - convert observed SDSS mag to Johnson, then K-correct within Johnson system
    ** or ***
  - K-correct SDSS magnitudes, then convert rest frame SDSS magnitudes to Johnson system.

4 Comparisons with Nugent 2002

Results from RK code are compared with results from astro-ph/0205351.

- Values from figures extracted using ruler and eyeballs.
- SN templates from `supernova.lbl.gov/~nugent/spectra.html`
- SN templates include smooth fudge to match photometry, to correct for host galaxy redenning, and to correct for color vs. stretch.
- RK code written in plain simple C.
  (input file specifies filters, SN templates, Vega SED, etc ...)

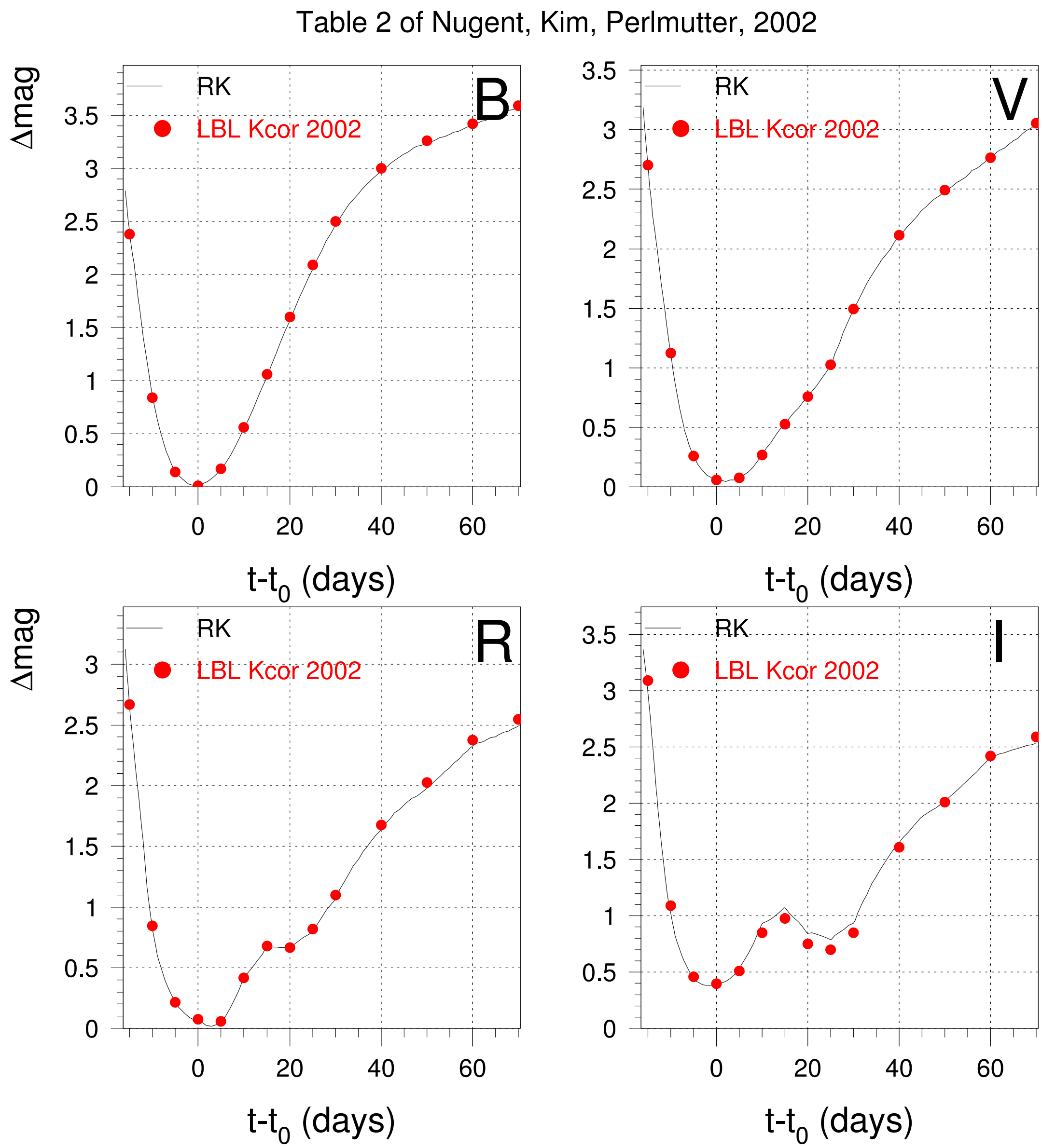

Figure 2: SN template magnitude vs. epoch for RK and from Nugent 2002 magnitude table. RK uses Johnson filters [as indicated in Nugent2002] and Bessell definition. Although shapes look good, offsets were needed in B,V,R,I-bands of $0.01, -0.005, 0.007, -0.02$ mag, respectively. We likely have different magnitude definitions ???

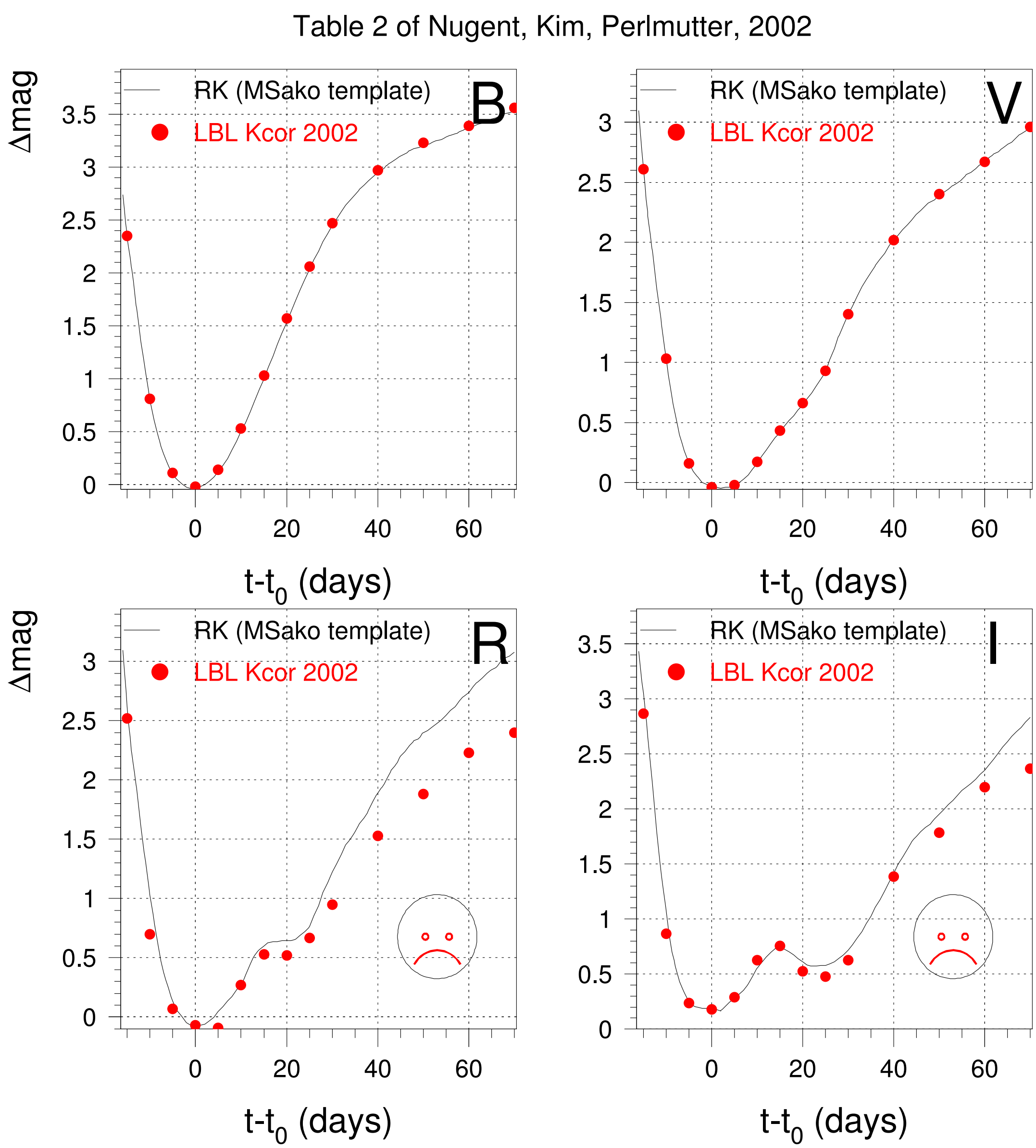

Figure 3: Same as previous plot, but RK curve uses SN templates from Sako (i.e., snap) that were used for SDSS/SN id in Fall 2004. There is some discrepancy at late epochs in R and I-bands. B,V,R,I offsets differ by -0.04, -0.1, -0.16, -0.25 mag, respectively.

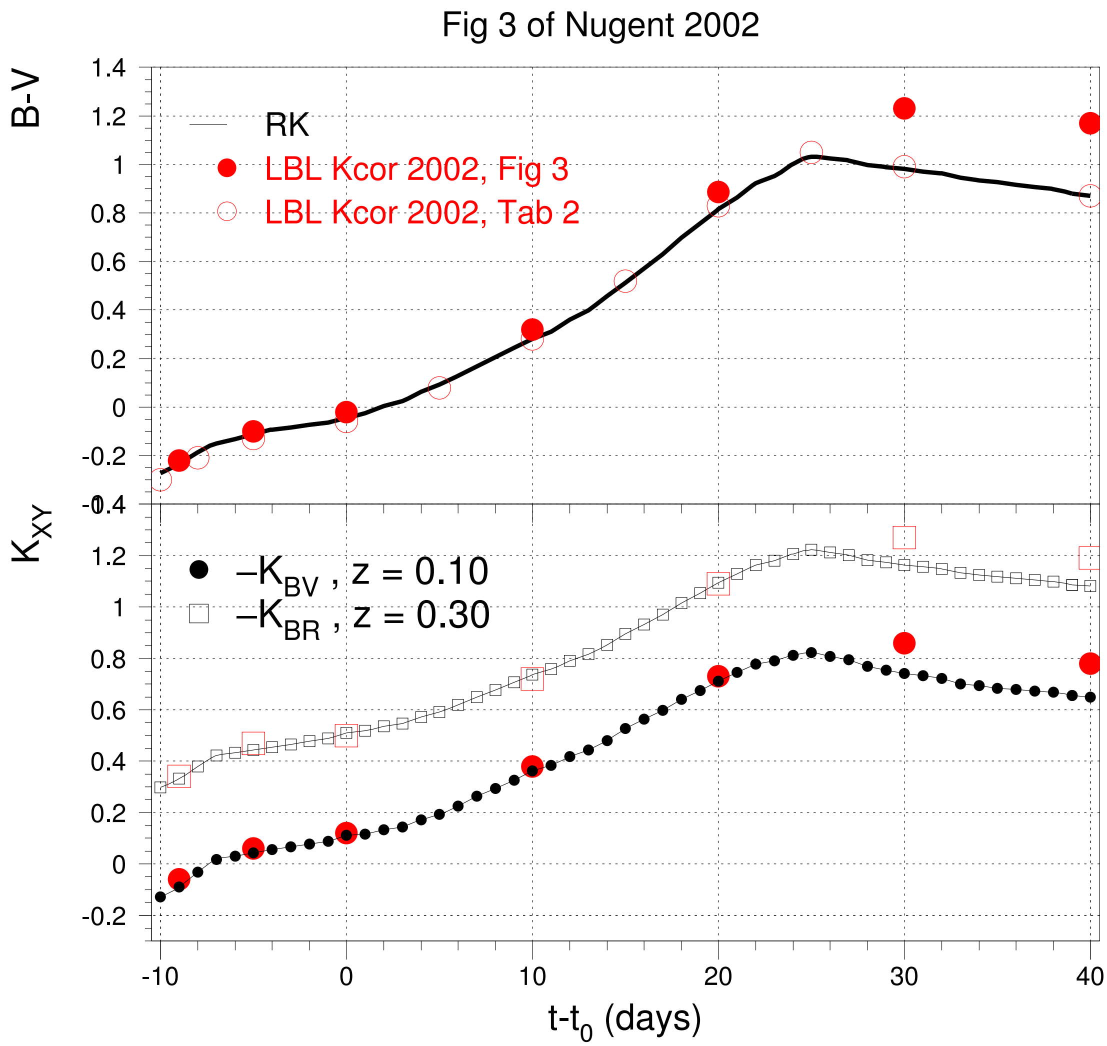

Figure 4: Top: B–V vs. epoch in days, for SN Ia template. RK B–V compares well with Table 2 of Nugent2002, but shows discrepancy in his Fig.3 beyond 25 days; this indicates internal discrepancy in Nugent 2002. Bottom: $-K_{BV}$ and $-K_{BR}$ vs. day for redshifts indicated on plot. Red squares and solid dots are read from Fig.3 in Nugent2002.

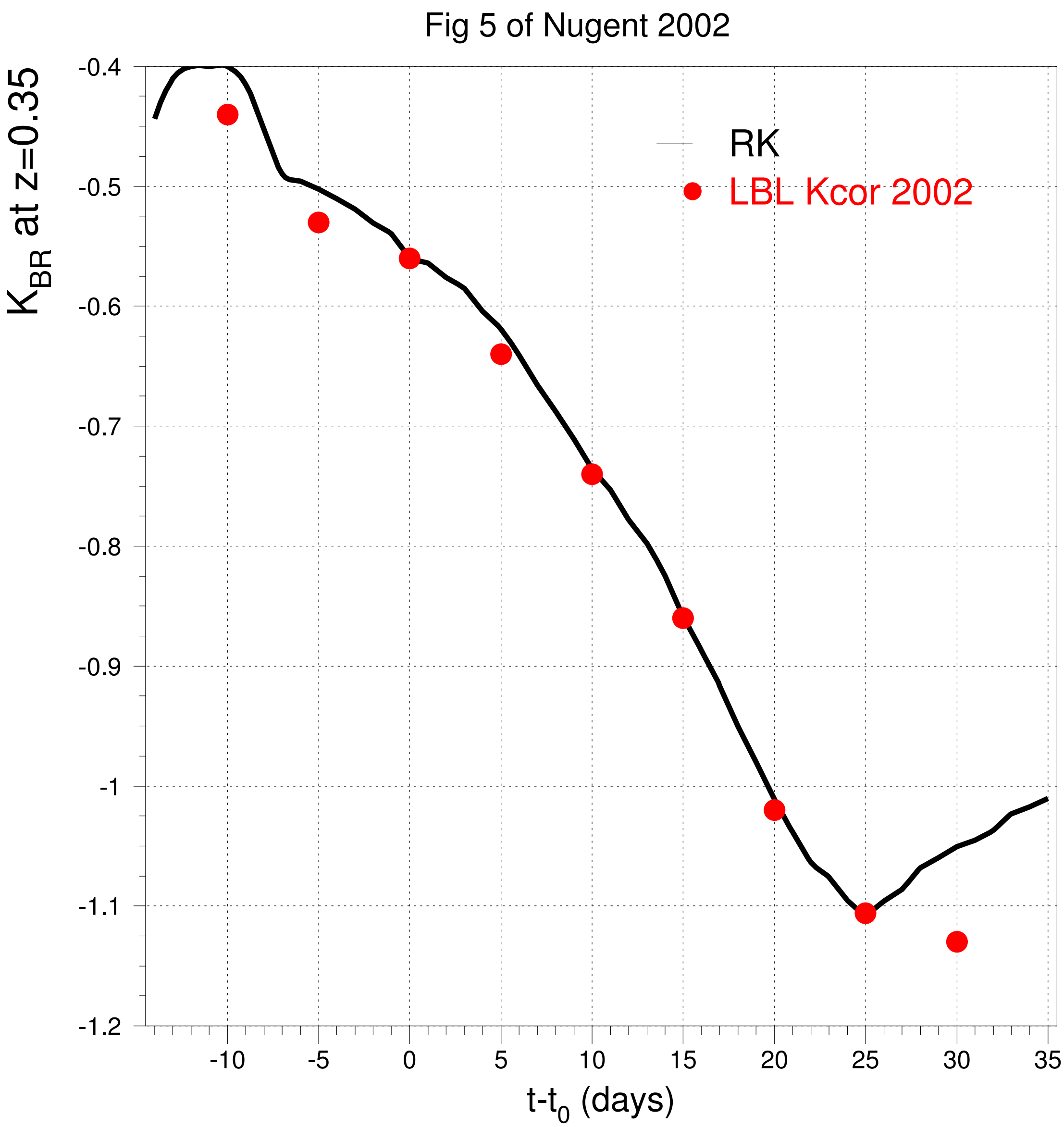

Figure 5: K_{BR} vs. day at $z = 0.35$.

5 Nugent-Riess SN Template Comparisons

- Nugent templates are from Nugent's public web-site, and they are described in a published technical article.
- Riess templates obtained through middle-man (John M.). Description based on John's memory.
- Comparison is probably unfair, but let's compare them anyway.

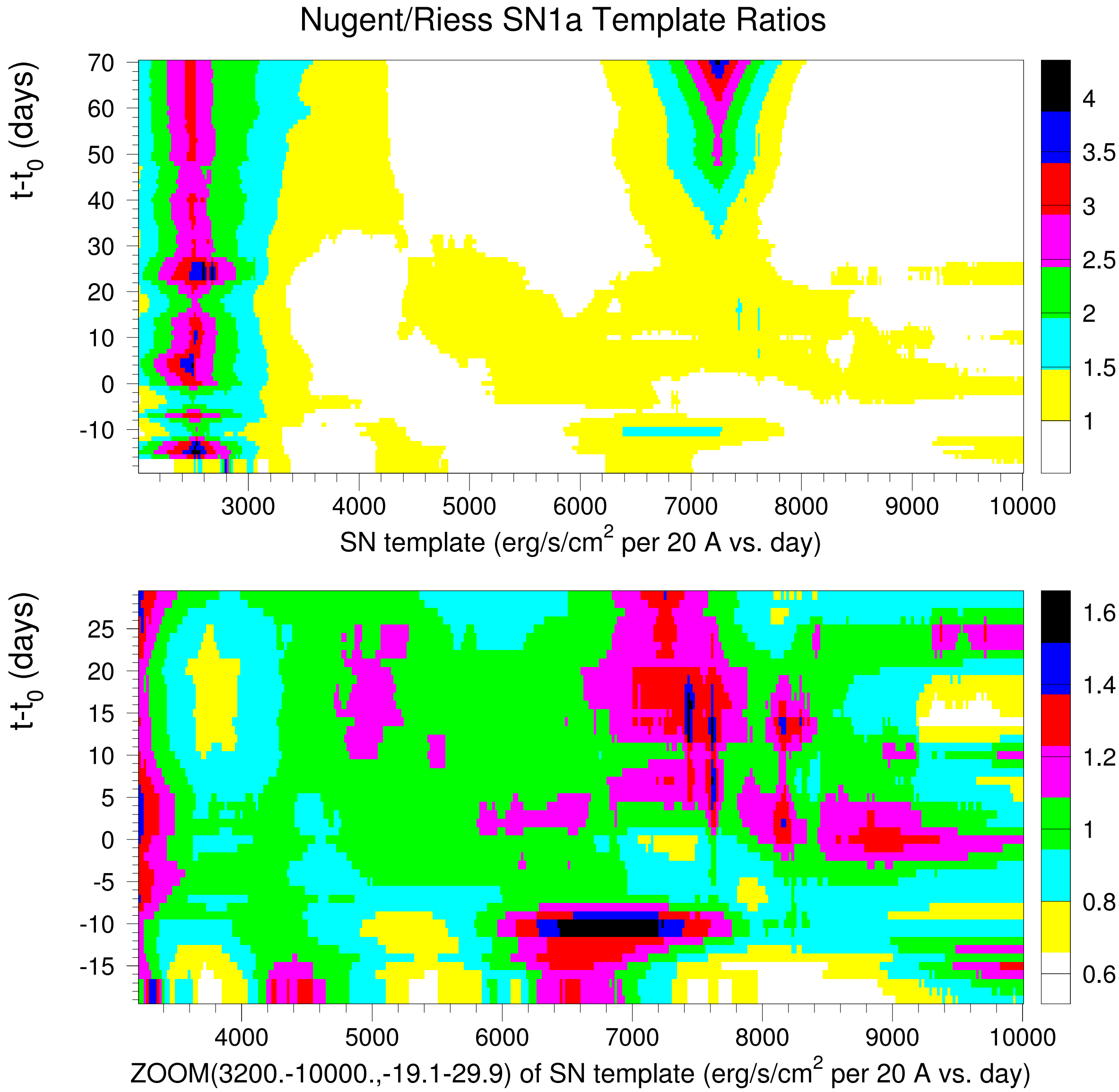

Figure 6: SN Ia flux ratio for Nugent/Riess templates. Flux ratio is shown in bins of wavelength and epoch.

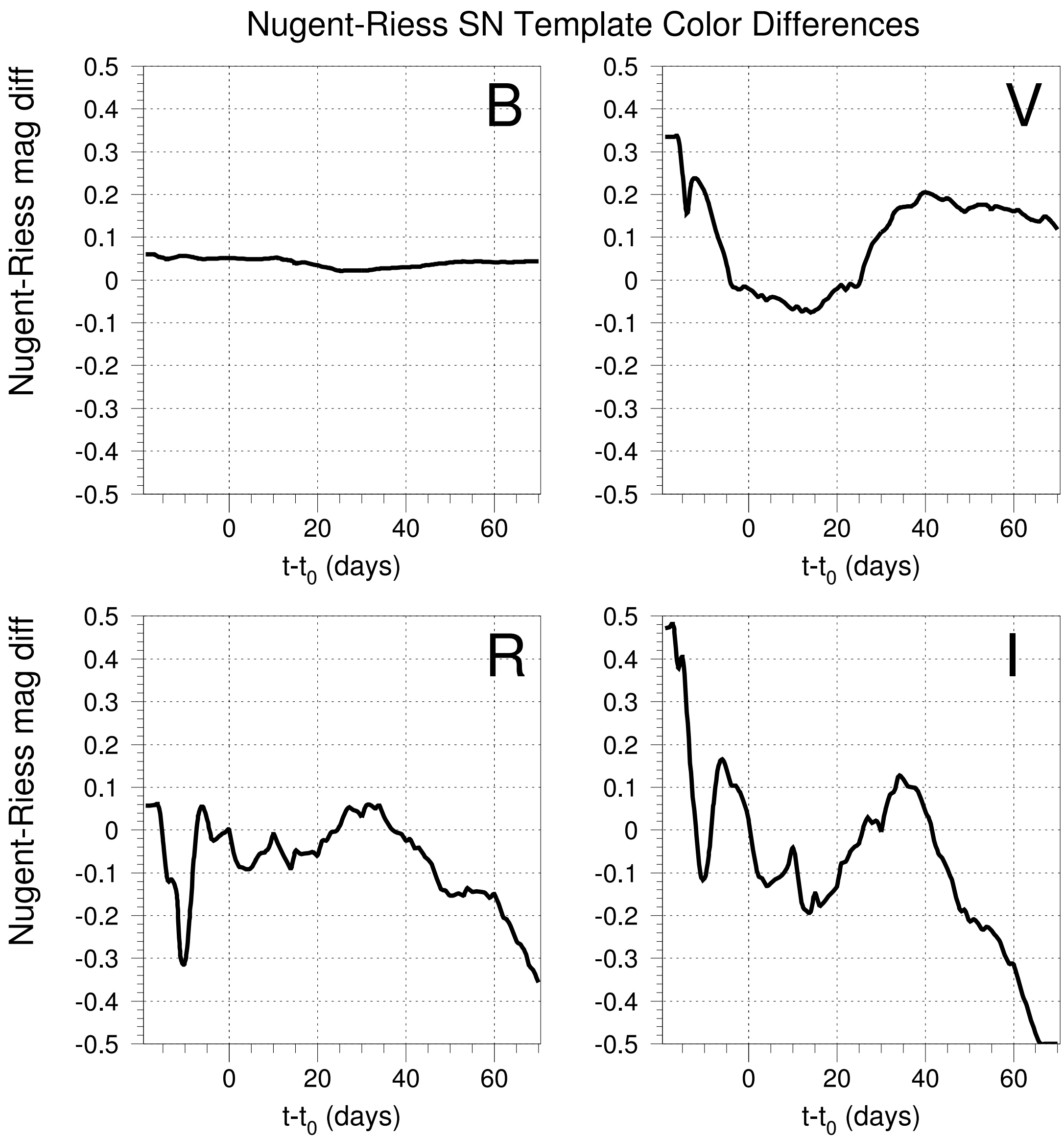

Figure 7: Synthetic color difference between Nugent and Riess SN Ia templates. m_B discrepancy is flat with epoch, while V,R,I discrepancies vary by few tenths of a magnitude.

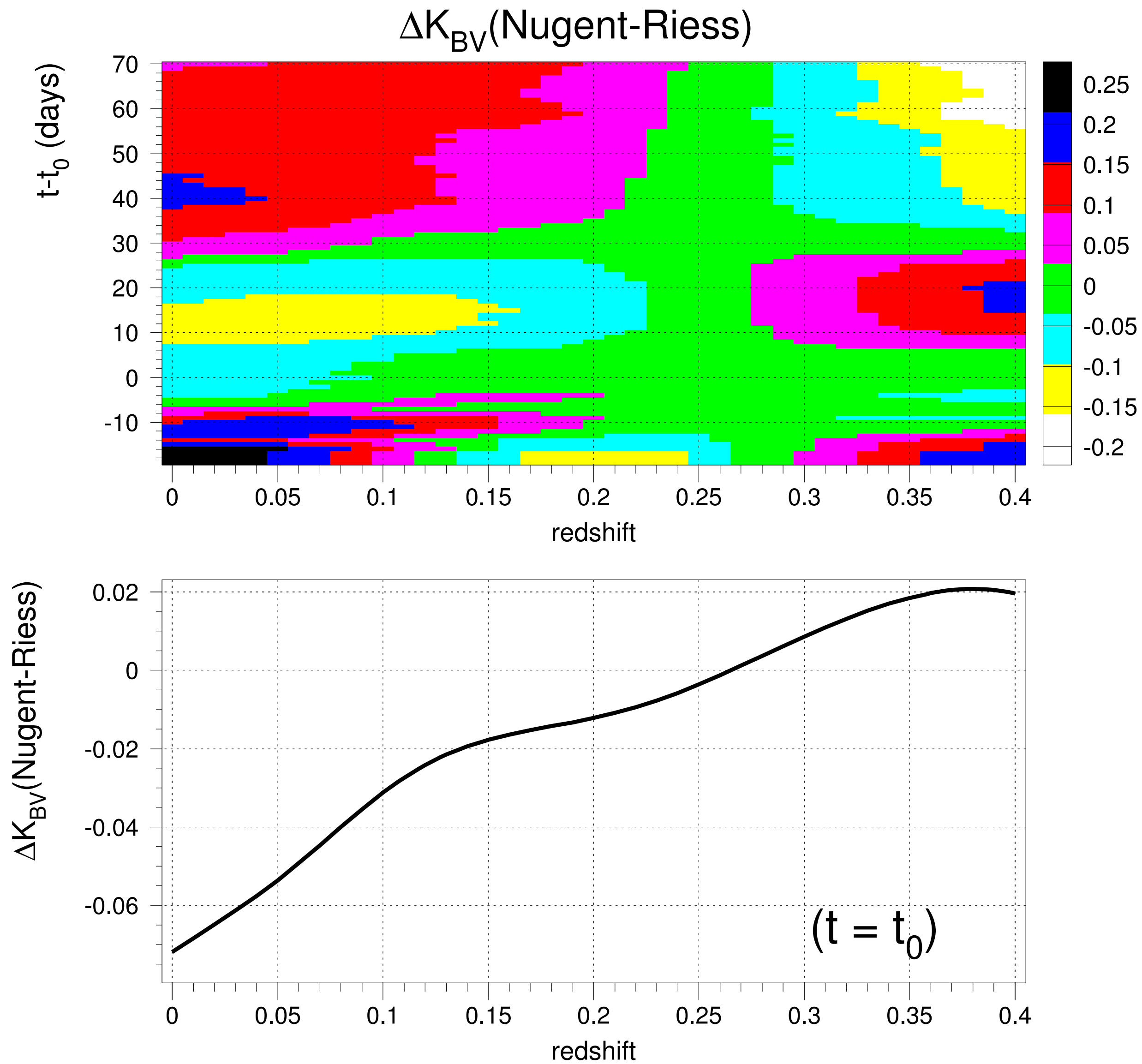

Figure 8: Top: difference in K_{BV} for Nugent and Riess SN Ia templates. Difference ΔK_V is shown in bins of redshift and epoch. Lower plot shows ΔK_V vs. redshift at the epoch corresponding to peak magnitude.

6 SDSS Filter Overlaps

Define filter overlap

$$\mathcal{O}_{ro} \equiv \frac{\int \left[f_\lambda(\lambda) \tilde{S}_o(\lambda(1+z))/S_r(\lambda) \right] d\lambda}{\int f_\lambda(\lambda) d\lambda}$$

where $\tilde{S}_o = \min(S_o, S_r)$.
Note $\mathcal{O}_{ro} = 1$ for perfectly matched filters in observer and rest frames.

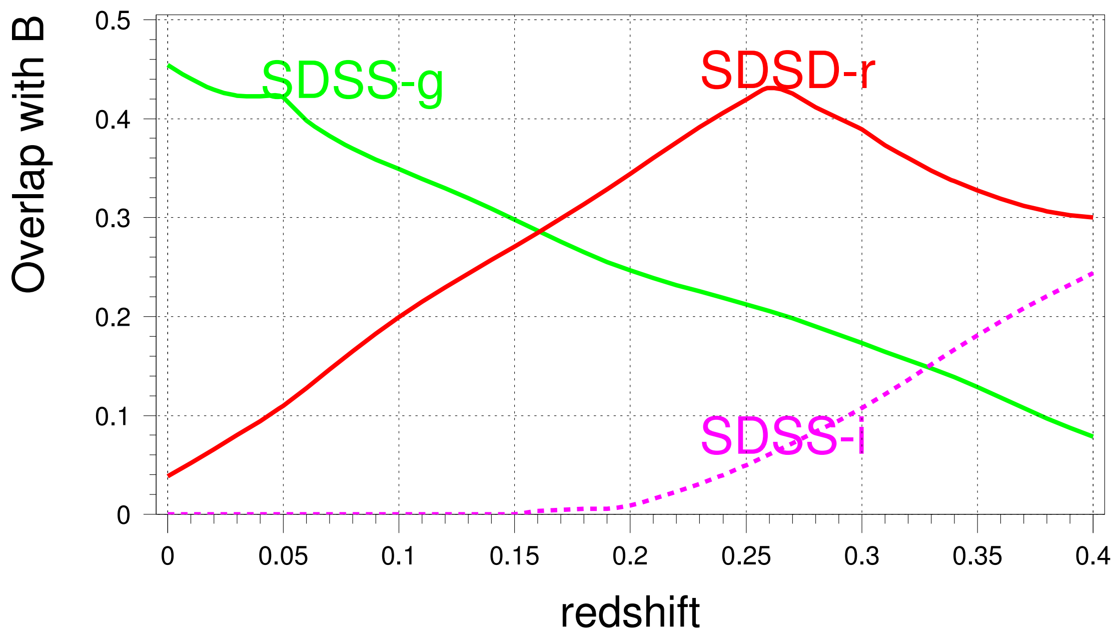

Figure 9: Filter overlaps $\mathcal{O}_{Bg}$, $\mathcal{O}_{Br}$, and $\mathcal{O}_{Bi}$ vs. redshift. "B" refers to Johnson-B filter.

7 Remaining Confusion

- Which magnitude system ???
  For example, Knopp 2003 (Sec 2.3 of Ap.J **598**, 102, 2003) suggests the Bessell system, while Nugent 2002 (Appendix A.3) states that Johnson-Cousins photon-based system "should and has been used in SCP analysis."
- RK-Nugent comparisons agree at few percent level ... discrepancies not understood.
- large differences between Nugent and Riess SN Ia templates ??
- We only have fudged templates ... individual spectra to make templates are not (yet ??) available.
  Need current spectra in order to:
  - add spectra from HET, ARC, etc .. to improve templates.
  - error analysis based on scatter of K-corrections.
  - re-evaluate which spectra to use.
  - test different fudging algorithms.
  - lightcurve fitting.