Assignment 1: Review of CM
Due: October 6, 2008

PHYX412-1 Fall 2008 : Quantum Mechanics I

Homework Assignment 1 : Review of Classical Mechanics

1. Particle moving in a gravitational field

A particle of mass m moves in an external gravitational potential,

$$V(x,y,z) \;=\; -\frac{GMm}{\left(x^2+y^2+z^2\right)^{1/2}} \quad .$$

A. Write down the Lagrangian in spherical coordinates (r,θ,ϕ). Use the Euler-Lagrange equations to write down the three equations of motion. (Don't solve them).

B. Using the recipe we followed in class, move to the Hamiltonian formulation in terms of $(r,\theta,\phi,p_r,p_\theta,p_\phi)$. Is the Hamiltonian the total energy of this system? Show that p_ϕ is a conserved quantity. Derive (but don't solve) the equations of motion.

2. Invariance of the Poisson bracket

Prove that the Poisson bracket of two functions $\omega(q,p)$ and $\sigma(q,p)$ of a set of canonical variables q_i and p_i takes the same numerical value when expressed in terms of another set of canonical variables $\overline{q}(q,p)$ and $\overline{p}(q,p)$. (Hint: Use the chain rule to change derivatives with respect to q/p into derivatives on $\overline{q}/\overline{p}$.

3. Polar coordinates are canonical

Prove that polar coordinates given in terms of x and y and their canonical momenta p_x and p_y,

$$r \equiv \left(x^2+y^2\right)^{1/2} \qquad \phi \equiv \mathrm{Tan}^{-1}\left(\frac{y}{x}\right)$$

$$p_r \equiv \frac{xp_x+yp_y}{\left(x^2+y^2\right)^{1/2}} \qquad p_\phi \equiv xp_y - yp_x$$

form a set of canonical variables.