

PHYX412-1 Fall 2008 : Quantum Mechanics I

Homework Assignment 4 : Position and Momentum

1. Commutators of functions of \hat{x} and \hat{p}

Given functions of position and momentum $F(\hat{x})$ and $G(\hat{p})$, work out the commutators:

$$[\hat{x}, G(\hat{p})] \quad \text{and} \quad [\hat{p}, F(\hat{x})]$$

2. Translations of expectation values

A particle moves in one dimension, x . It is prepared in a state $|\alpha\rangle$ which has wave function,

$$\langle x|\alpha\rangle = \begin{cases} 0 & x < -\frac{L}{2} \\ \mathcal{N} \cos\left(\frac{\pi x}{L}\right) & -\frac{L}{2} \leq x \leq \frac{L}{2} \\ 0 & x > \frac{L}{2} \end{cases}$$

A. Normalize the state, determining \mathcal{N} .

B. Find the probability to measure the particle with momentum p' . Find $\langle \hat{p} \rangle$ and the dispersion of \hat{p} .

C. Determine the expectation value and dispersion of \hat{x} .

D. Apply a finite translation of distance d to the state $|\alpha\rangle$. Determine $\langle \hat{x} \rangle$ and $\langle \hat{p} \rangle$ and the dispersions of \hat{x} and \hat{p} of the translated state.

3. Meaning of an operator

Consider the operator defined in terms of a real number w ,

$$\hat{O}(w) \equiv \text{Exp} \left[\frac{iw\hat{x}}{\hbar} \right]$$

A. What units does w carry?

B. What is the significance of this operator? Be sure to fully explain your reasoning.