Assignment 7: Angular Momentum II
Due: November 17, 2008

PHYX412-1 Fall 2008 : Quantum Mechanics I

Homework Assignment 7 : Angular Momentum II

1. Addition of angular momentum

A bound state consists of two particles, one of which is spin 1 and the other of which is spin 1/2. They are in a state with no orbital angular momentum.

A. What values of j_{tot} are possible?

B. Construct the dictionary between the states that are eigenstates of $\hat{J}^2_{tot}$ and $\hat{J}^{tot}_z$ and the states that are eigenstates of $\hat{J}^{(1)}_z$ and $\hat{J}^{(2)}_z$ by writing one set as a linear combination of the other.

C. If we consider a different state (of the same spin 1 and spin 1/2 particle) with non-zero orbital angular momentum, and measure the total angular momentum to be $j_{tot} = 25/2$, what values of orbital angular momentum are possible?

2. Rotated Wave function

An electron (spin 1/2) is in a state $|\psi\rangle$ with wave function,

$$\langle \vec{x}|\psi\rangle \;=\; \frac{\left(z+\sqrt{2}iy\right)}{4a_0^2\sqrt{6\pi a_0}}\;\mathrm{Exp}\left[-\frac{r}{2a_0}\right]\;\otimes\;|+\rangle$$

where a_0 is a constant with dimensions of length whose value you shouldn't need, and $|+\rangle$ indicates the electron is spin-up along the z-axis as usual.

A. Verify this state is correctly normalized.

B. Apply a rotation by angle $\pi/4$ about the y-axis and find the new wave function. Don't forget to rotate both the spatial dependence *and* the electron's spin!

You may find the following d matrix from last week's homework useful:

$$d^{(1)}(\beta) = \begin{bmatrix} \frac{1}{2}(1+\cos\beta) & -\frac{1}{\sqrt{2}}\sin\beta & \frac{1}{2}(1-\cos\beta) \\ \frac{1}{\sqrt{2}}\sin\beta & \cos\beta & -\frac{1}{\sqrt{2}}\sin\beta \\ \frac{1}{2}(1-\cos\beta) & \frac{1}{\sqrt{2}}\sin\beta & \frac{1}{2}(1+\cos\beta) \end{bmatrix} ,$$

and this one we derived in class during the lecture:

$$d^{(1/2)}(\beta) \;=\; \begin{bmatrix} \cos\frac{\beta}{2} & -\sin\frac{\beta}{2} \\ \sin\frac{\beta}{2} & \cos\frac{\beta}{2} \end{bmatrix} .$$