

PHYX412-1 Fall 2008 : Quantum Mechanics I

Homework Assignment 8 : Dilations and Hamiltonians

1. Passive Dilation

Compute the effect of a passive dilation on the operators for position and momentum, \hat{x} and \hat{p} . You may find it convenient to use the Baker-Hausdorff formula we discussed in class (or you may find it written in section 2.3 of *Sakurai*). Explain why the result for the transformed \hat{p} is sensible.

2. Spin One in a Magnetic Field

A spin one particle has magnetic moment $\vec{\mu} = \mu_0 \vec{S}$ where S is the particle's spin. It is subject to an external magnetic field in the z direction whose magnitude varies with time,

$$B(t) = B_0 \cos \omega t$$

A. Write down the time evolution operator from time $t = 0$ to t .

B. A spin one particle is in a state at $t = 0$ given by,

$$|\psi, t = 0\rangle = \alpha|m = 1\rangle + \beta|m = 0\rangle + \gamma|m = -1\rangle$$

where α , β , and γ are complex numbers with $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$. Using the results of part A, write down the state at time t , $|\psi, t = 0; t\rangle$.

C. An ensemble is prepared for which at $t = 0$, the particle's J_z is $m = 1$. Find the expectation values $\langle J_x \rangle$ and $\langle J_y \rangle$ as a function of time t .

D. An ensemble is prepared for which at $t = 0$, the particle's J_z is $m = 0$. Find the expectation values $\langle J_x \rangle$ and $\langle J_y \rangle$ as a function of time t .