PHYX412-1 Fall 2008 : Quantum Mechanics I

Homework Assignment 9 : Time Evolution

1. Neutral Kaons

A neutral kaon is a spin zero bound state (held together by the strong nuclear force) of a strange quark with an anti-down quark, $K^0 = s\bar{d}$. It carries a quantum number called *strangeness* with value $+1$. It has an anti-particle is made from an anti-strange quark bound to a down quark, $\overline{K^0} = d\bar{s}$ which has strangeness -1. For this problem, work in the $\left\{|K^0\rangle, |\overline{K^0}\rangle\right\}$ basis.

A. Write down the Hermitean operator that measures strangeness number. The *strong* nuclear force conserves strangeness number. Use that fact to determine the most general form of the strong nuclear force Hamiltonian in this basis. Further constrain the strong force Hamiltonian by using the fact that the strong force does not distinguish between strange and down quarks, and write its matrix elements in terms of $\langle K^0|\hat{H}_s|K^0\rangle = \Lambda$.

B. The *weak* nuclear force violates conservation of strangeness number. Write down the most general matrix elements of $\hat{H}_{tot} = \hat{H}_s + \hat{H}_w$, making use of the fact that the weak force has much, much smaller matrix elements than the strong force and so we can neglect it in any matrix element to which the strong force has already made a contribution. How many new parameters do we need to specify the additional matrix elements of $\hat{H}_{tot}$?

C. Find the energy eigenstates. The less energetic one is called the K_S and the more energetic one the K_L (because one lives a "short" and the other a "long" time before decaying).

D. Let's say at $t = 0$ we have created a kaon with strangeness $= +1$. As a function of elapsed time, t, determine the survival probability, the probability that we will again measure strangeness $+1$.

2. Free-moving Gaussian

A non-relativistic particle of mass m moves freely in one dimension. At time $t = 0$ the particle is in a state whose wave function is given by a Gaussian centered at x_0 with width σ,

$$\langle x|\psi, t_0\rangle = \frac{1}{\pi^{1/4}\sqrt{\sigma}} \quad \mathrm{Exp}\left[ikx - \frac{(x-x_0)^2}{2\sigma^2}\right]$$

A. Working in the Heisenberg picture, determine $\hat{x}(t)$ and $\hat{p}(t)$ in terms of $\hat{x}(0)$ and $\hat{p}(0)$. Determine the commutator $[x(0), x(t)]$.

B. Compute $\langle x\rangle(t)$, $\langle p\rangle(t)$, $\Delta x(t)$, and $\Delta p(t)$ for an ensemble of particles in the same Gaussian state at $t = 0$.

3. Two Harmonic Oscillators

A particle of mass m moving in two dimension feels a harmonic oscillator potential,

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{k_1}{2}\hat{x}^2 + \frac{k_2}{2}\hat{y}^2 \quad .$$

A. Derive the conditions under which $\hat{H}$ commutes with $\hat{L}_z$.

B. Under the conditions of part A, use the operator techniques discussed in class (or in Sakurai) to write the position space wave functions for the $|i, j\rangle$ states, where $i, j = 0, 1, 2$. Classify each state by its L_z, taking linear combinations to get eigenstates as necessary.