PHYX412-1 Fall 2008 : Quantum Mechanics I

Useful Formulae

Baker-Hausdorff Formula (real λ, Hermitean $\hat{G}$)

$$e^{i\hat{G}\lambda}\hat{A}e^{-i\hat{G}\lambda} = \hat{A} + i\lambda\left[\hat{G},\hat{A}\right] + \frac{(i\lambda)^2}{2!}\left[\hat{G},\left[\hat{G},\hat{A}\right]\right] + ... + \frac{(i\lambda)^n}{n!}\left[\hat{G},...\left[\hat{G},\left[\hat{G},\hat{A}\right]\right]...\right] + ...$$

Angular Momentum

$$\hat{J}_\pm|j,m\rangle = \sqrt{(j\mp m)(j\pm m+1)}\hbar|j,m\pm 1\rangle$$

Orbital Angular Momentum : Matrix Elements in Position Space

$$\begin{aligned}
\langle r,\theta,\phi|\hat{L}_z|\psi\rangle &= -i\hbar\frac{\partial}{\partial\phi}\langle r,\theta,\phi|\psi\rangle \\
\langle r,\theta,\phi|\hat{L}^2|\psi\rangle &= -\hbar^2\left\{\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\right\}\langle r,\theta,\phi|\psi\rangle \\
\langle r,\theta,\phi|\hat{L}_\pm|\psi\rangle &= -i\hbar e^{\pm i\phi}\left(\pm i\frac{\partial}{\partial\theta} - \cot\theta\frac{\partial}{\partial\phi}\right)\langle r,\theta,\phi|\psi\rangle
\end{aligned}$$

Simple Harmonic Oscillator

$$\begin{aligned}
\hat{H} &= \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2 = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \\
\hat{a} &\equiv \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right) \\
\hat{a}^\dagger &\equiv \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)
\end{aligned}$$