Understanding the physics of heavy quark bound states from QCD

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Outline of the talk

- Effective Field Theories

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- The heavy quarkonium system

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- Conclusions

$$
\begin{array}{c}
\underbrace{\begin{array}{cc}
\text{Classical mechanics} & \text{Classical e.m.} \\
\downarrow & \downarrow \\
\text{Quantum mechanics} & \text{Relativity}
\end{array}} \\
\downarrow \\
\text{Quantum Field Theory}
\end{array}
\qquad
\begin{array}{c}
\text{Newtonian gravity} \\
\\
\downarrow \\
\\
\text{General Relativity}
\end{array}
$$

Classical mechanics Classical e.m. Newtonian gravity

$\downarrow$ $\downarrow$

$\underbrace{\text{Quantum mechanics} \quad \text{Relativity}}$ $\downarrow$

$\downarrow$

Quantum Field Theory General Relativity

$$\text{Standard Model}\begin{cases}\text{strong interactions (QCD)}\\ \text{weak interactions}\\ \text{electromagnetism (QED)}\end{cases}$$

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Gravity

SM

strong interactions (QCD)

weak interactions

electromagnetism (QED)

electro-weak sector

Gravity

QCD

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Quantum ChromoDynamics $\rightarrow$ Fundamental theory of the strong interactions

- Spectrum consists of color singlet states
- Asymptotic freedom. Coupling constant increases at low energy and decreases at high energy
- Develops an intrinsic scale (mass gap) at low energies Λ_{QCD}

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  - Symmetries
  - Hierarchy of energy scales
- The EFT gives equivalent physical results in its region of validity

To describe physics in a particular energy region we do not need to know the detailed dynamics of other energy regions

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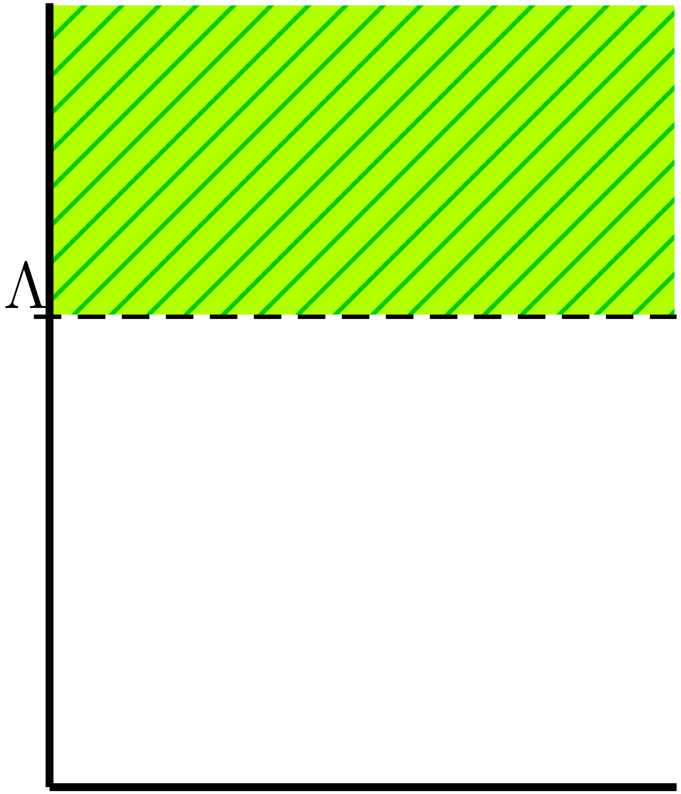

Examples of EFT

- *Fermi Theory of the weak interactions*
- *Chiral perturbation theory*. To describe low energy interactions among pions and kaons
- *Heavy Quark Effective Theory*. To describe mesons containing one heavy quark
- $\cdots$

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We will concentrate in the heavy quarkonium system

Heavy quarkonium

$$\left.\begin{array}{c} u \\ d \\ s \end{array}\right\} \text{light}(m < \Lambda_{\text{QCD}})\text{quarks} \quad \left.\begin{array}{c} c \\ b \\ t \end{array}\right\} \text{heavy}(m > \Lambda_{\text{QCD}})\text{quarks}$$



Pictures from:
en.wikipedia.org/wiki/Image:Twarog.jpg
www.startrek.com/startrek/view/series/DS9/character/1112445.html

www.bbc.co.uk/doctorwho/classic/gallery/monsters1/quark.shtml

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Relevant scales in heavy quarkonium

$$\left.\begin{array}{ll} m & \text{mass of the heavy quark} \\ mv & \text{typical 3-momentum} \\ mv^2 & \text{binding energy} \end{array}\right\}$$

Well separated scales

$$m \gg mv \gg mv^2$$

Relevant scales in heavy quarkonium

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$$\Lambda_{\text{QCD}}$$

Integrating out the scale m

(use the fact that

$m \gg$ *any other scale*)

Integrating out the scale m $\longrightarrow$ Non-Relativistic QCD

NRQCD

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- Gluons and light quarks $p \sim m$
- Relativistic heavy quarks $\longrightarrow$

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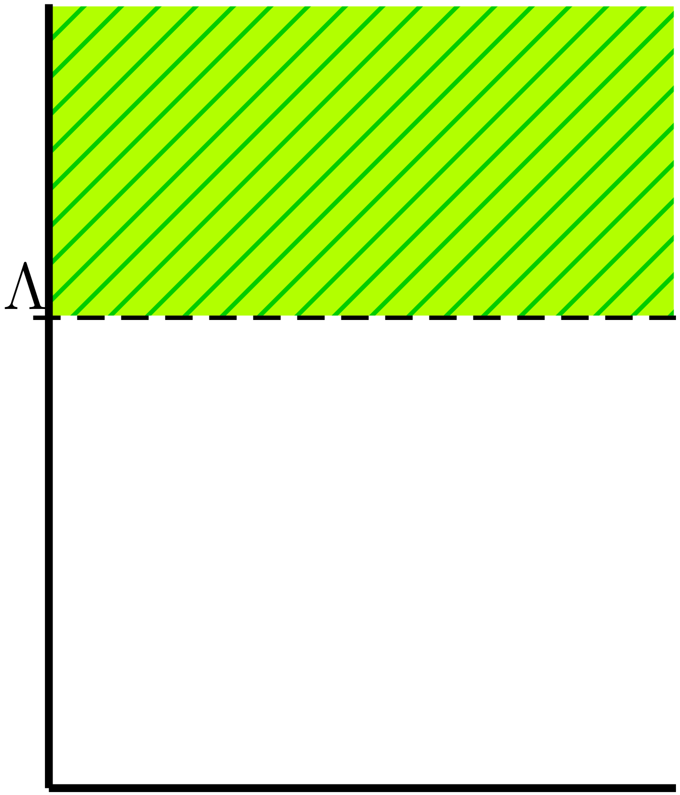
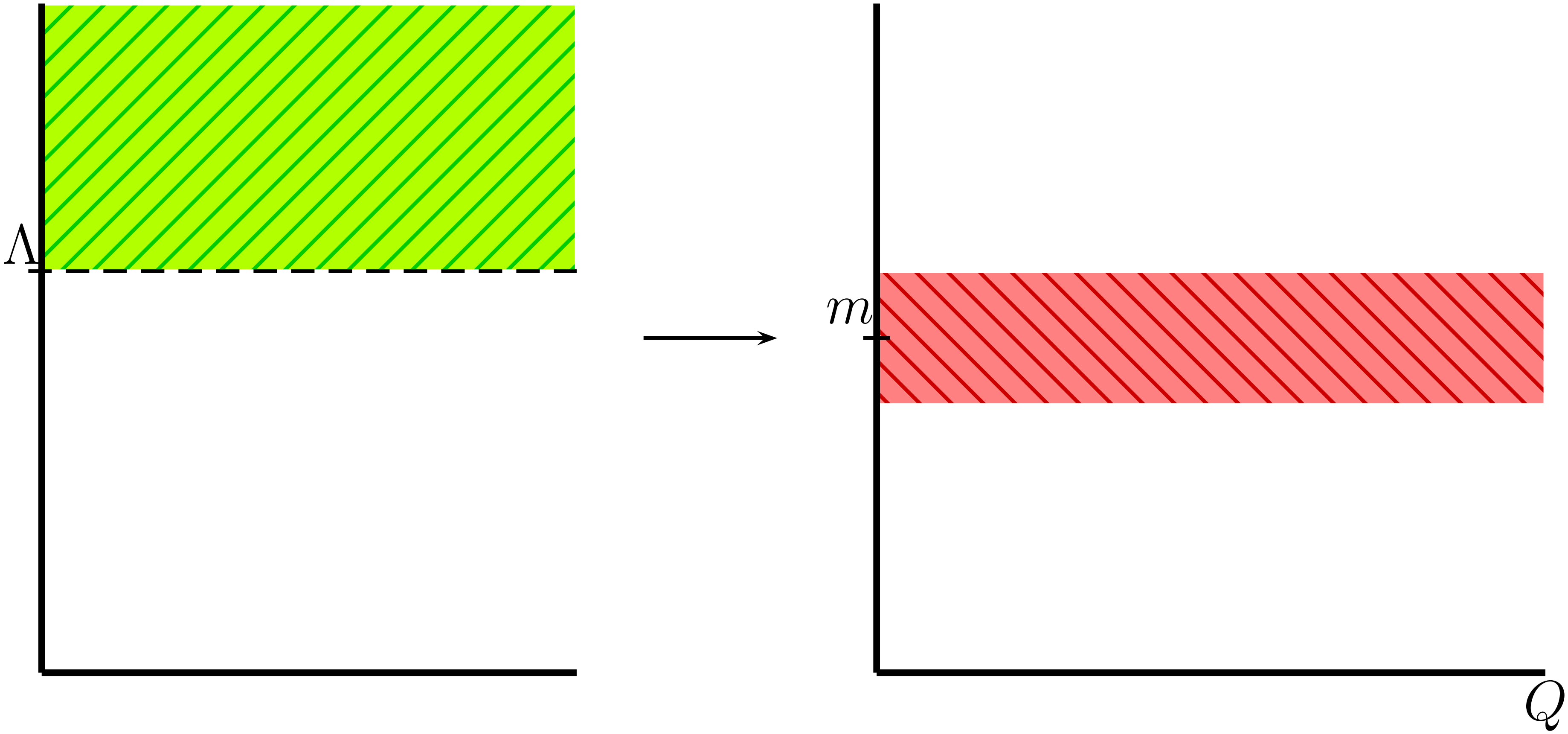
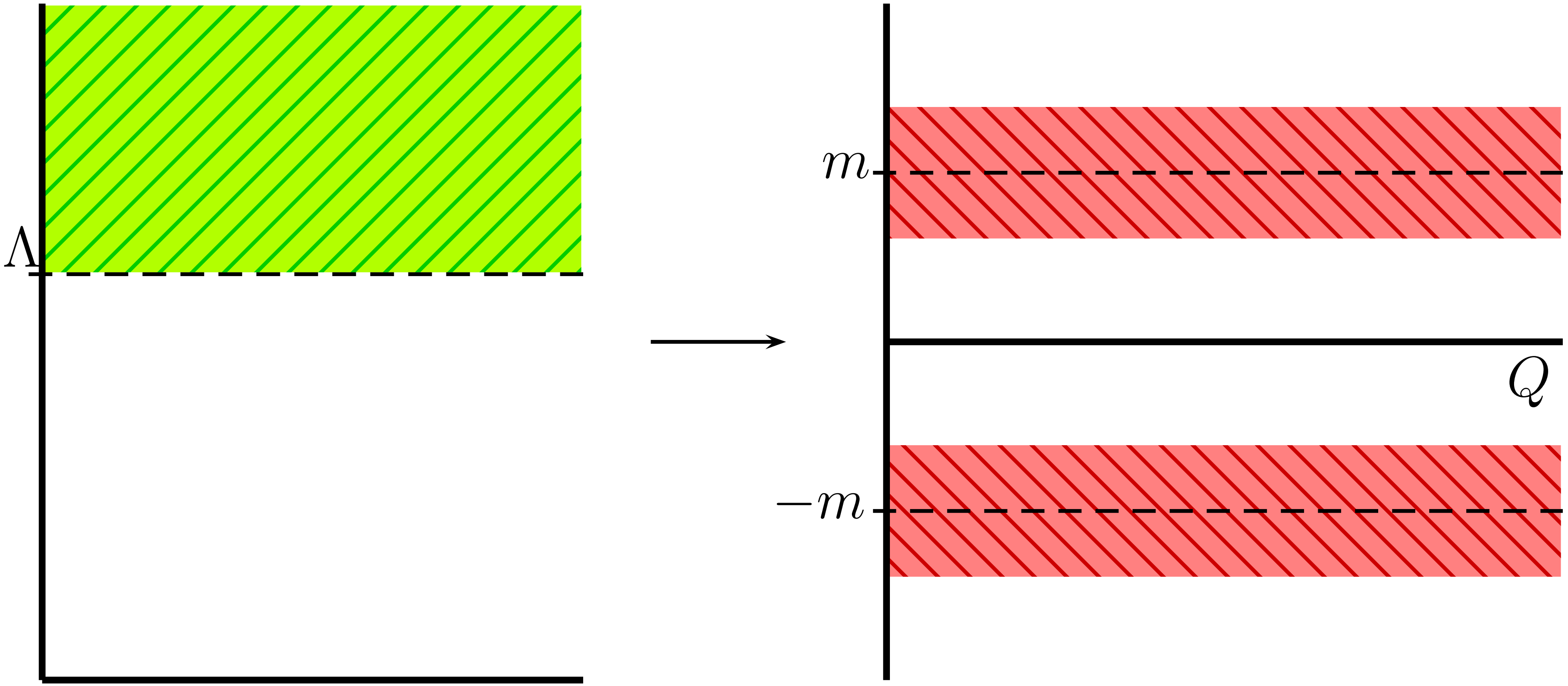
NRQCD

- ~~Gluons and light quarks $p \sim m$~~
- ~~Relativistic heavy quarks~~ $\rightarrow$ Non relativistic treatment of heavy quarks (separate 2 component spinors for Q and $\bar{Q}$)


Λ


Λ
m
Q


Λ
m
Q
$-m$

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- Short distance effects (at the scale m) are factorized
- Spectroscopy, production processes, decay processes... can be (and has been) rigorously studied

Annihilation of a $Q\bar{Q}$ pair $\longrightarrow$ Light quarks and gluons with energies of order m

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How are these processes incorporated in NRQCD?

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Annihilation processes incorporated in NRQCD through local 4 fermion operators

$Q\bar{Q}$ annihilation rate $\longrightarrow$ Imaginary parts of $Q\bar{Q} \to Q\bar{Q}$ scattering amplitudes

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$$\Downarrow$$

Coefficients of 4-fermion operators have imaginary parts

This allows for the study of inclusive decays

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The four fermion interactions have the general form

$$\delta\mathcal{L}_{4-f} = \sum_n \frac{f_n(\Lambda)}{m^{d_n-4}} \mathcal{O}_n(\Lambda)$$

for example

$$\mathcal{O}_1\left({}^1S_0\right) = \psi^\dagger \chi \chi^\dagger \psi$$

$$\mathcal{O}_8\left({}^1S_0\right) = \psi^\dagger T^a \chi \chi^\dagger T^a \psi$$

This allows for the study of inclusive decays

Which results in the following expression for the anhiliation rate

$$\Gamma\left(H \to LH\right) = \sum_n \frac{2\mathrm{Im}\, f_n(\Lambda)}{m^{d_n-4}} \langle H|\mathcal{O}_n(\Lambda)|H\rangle$$

power counting rules in NRQCD organizes the expansion

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\not{D} - m_i) q_i - \frac{1}{4} G^{\mu\nu\, a} G^{a}_{\mu\nu}$$

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$$\mathcal{L}_{NRQCD} = \psi^\dagger \left(iD_0 + \frac{1}{2m}\mathbf{D}^2 \right) \psi + \chi^\dagger \left(iD_0 - \frac{1}{2m}\mathbf{D}^2 \right) \chi +$$

$$+ \sum_{i=1}^{n_f} \bar{q}_i \, i\not{D} \, q_i - \frac{1}{4} G^{\mu\nu\, a} G^a_{\mu\nu} + \delta\mathcal{L}_{4f} + \cdots$$

Integrating out the scale $m\alpha_s$

(exploit further the non-relativistic hierarchy

$m \gg mv \gg mv^2$)

Integrating out the scale $m\alpha_s \quad \longrightarrow \quad$ potential Non-Relativistic QCD

pNRQCD

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weak coupling regime **strong coupling regime**

$\Lambda_{QCD} \lesssim mv^2$ $mv^2 \ll \Lambda_{QCD} \lesssim mv$

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- The Lagrangian has, roughly, the structure

$$\left.\begin{aligned}\mathcal{L} = \Phi(\mathbf{r})^\dagger \quad &\Bigg(i\partial_0 - \frac{\mathbf{p}^2}{2m} - V^{(0)}(r) \\ &+\text{corrections to the potential} \\ &+\text{int. with other low-energy d.o.f.} \Bigg)\Phi(\mathbf{r})\end{aligned}\right\}\text{pNRQCD}$$

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- Potentials appear as (non-local in r) matching coefficients of the theory

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Radiative Υ decays

$$\Upsilon(b\bar{b}) \to X\gamma$$

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Direct contributions:	e.m. coupling to the heavy quarks
Fragmentation contributions:	e.m. coupling to light quarks

Radiative Υ decays

$$\Upsilon(b\bar{b}) \to X\gamma$$

The NRQCD formalism organizes the decay as:

$$\frac{d\Gamma}{dz} = \sum_i C_i(M,z)\,\langle \Upsilon | \mathcal{O}_i | \Upsilon \rangle$$

where $z = \frac{E_\gamma}{M/2}$

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Expansion in powers of v:

- Leading Order (LO) operator

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- Leading Order (LO) operator

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- Next-to-Leading Order (NLO) (v^4 supressed)

$$\mathcal{O}_8\left({}^1S_0\right), \mathcal{O}_8\left({}^3P_J\right) \;\alpha\; \delta(1-z)$$

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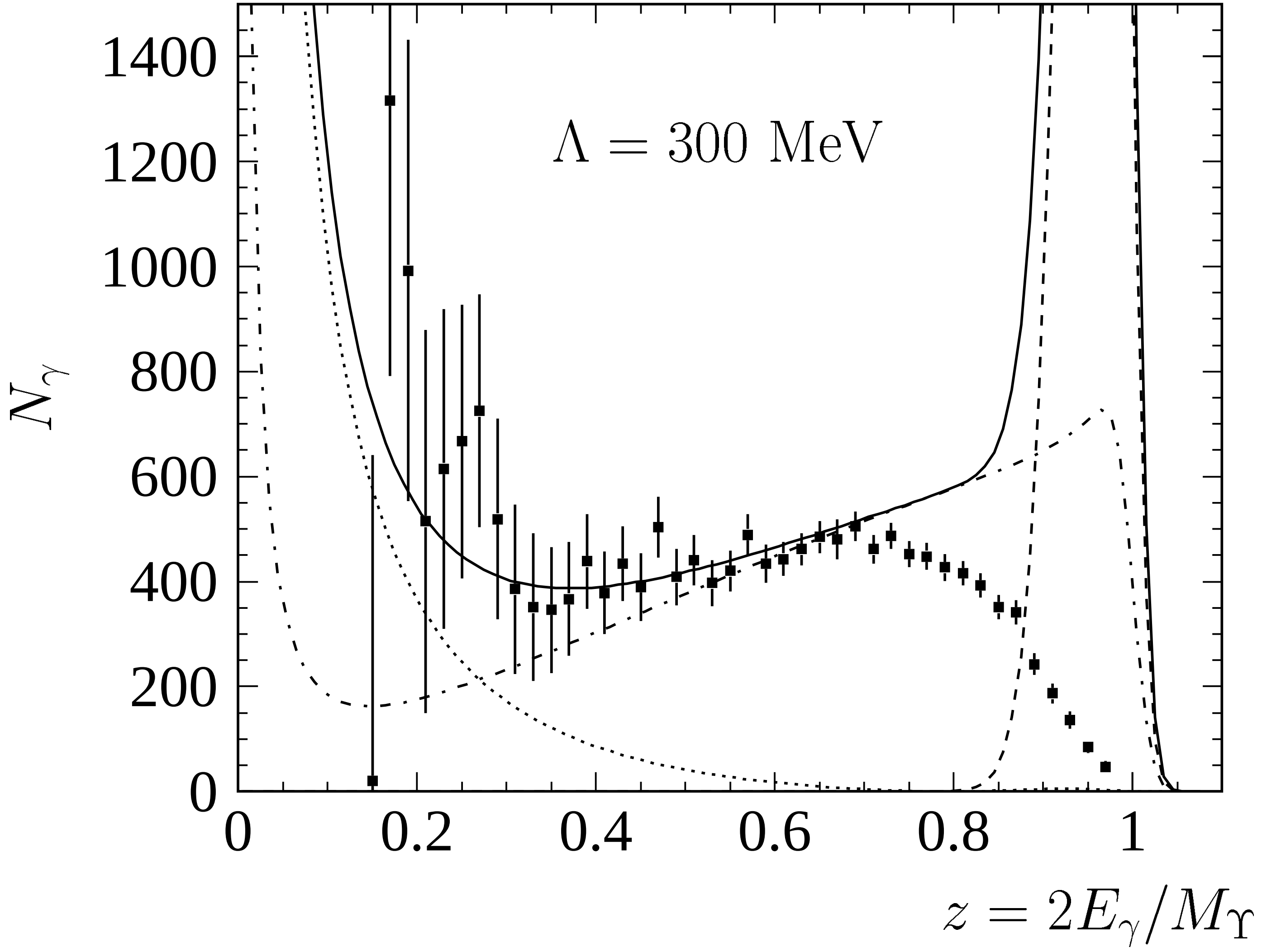
- NRQCD Operator Product Expansion breaks down
  - Shape functions resumming a certain class of operators must be introduced

    Rothstein and Wise '97



From S. Wolf, Phys. Rev. D **63** (2001) 074020 (arXiv:hep-ph/0010217)

What's happening?

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$$(M, M, 0) = (Mz, 0, 0) + (p_+, p_-, p_\perp) + \left(p'_+, p'_-, p'_\perp\right)$$

$$p_-, p'_- \sim M \quad p_+, p'_+ \sim M(1-z) \longrightarrow$$

$$\longrightarrow p^2 \approx 0 \sim p_+ \cdot p_- - p_\perp^2 \sim M^2(1-z) - p_\perp^2$$

$$p_\perp \sim M\sqrt{1-z}$$

$$p_+ = p_0 + p_3$$

$$p_- = p_0 - p_3$$

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$$p \sim (p_+, p_-, p_\perp) \quad p_+, p_\perp \ll M \quad p^2 \sim M^2\lambda^2 \ll M^2$$

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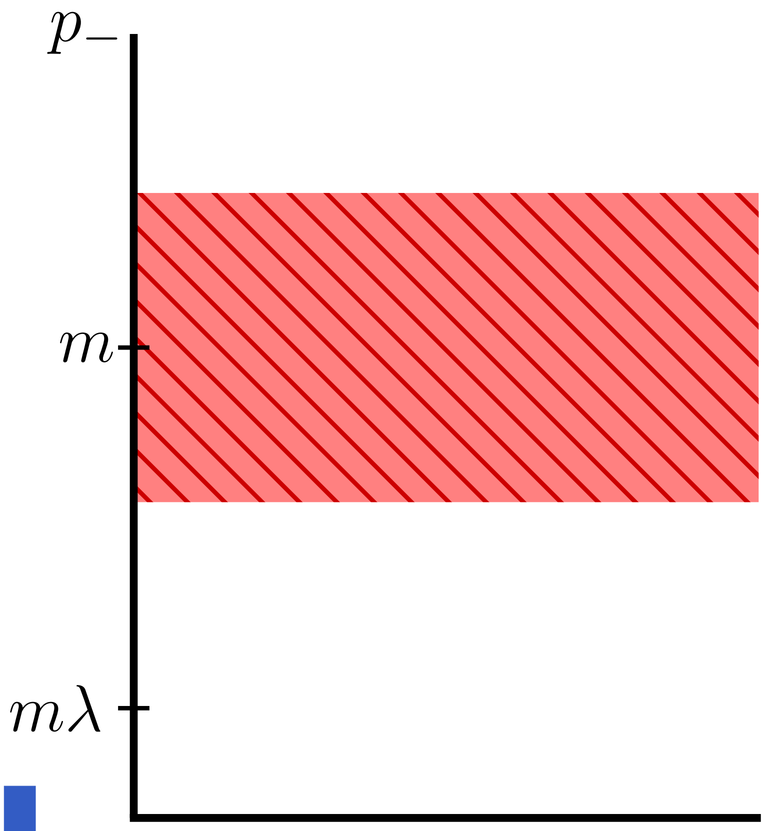

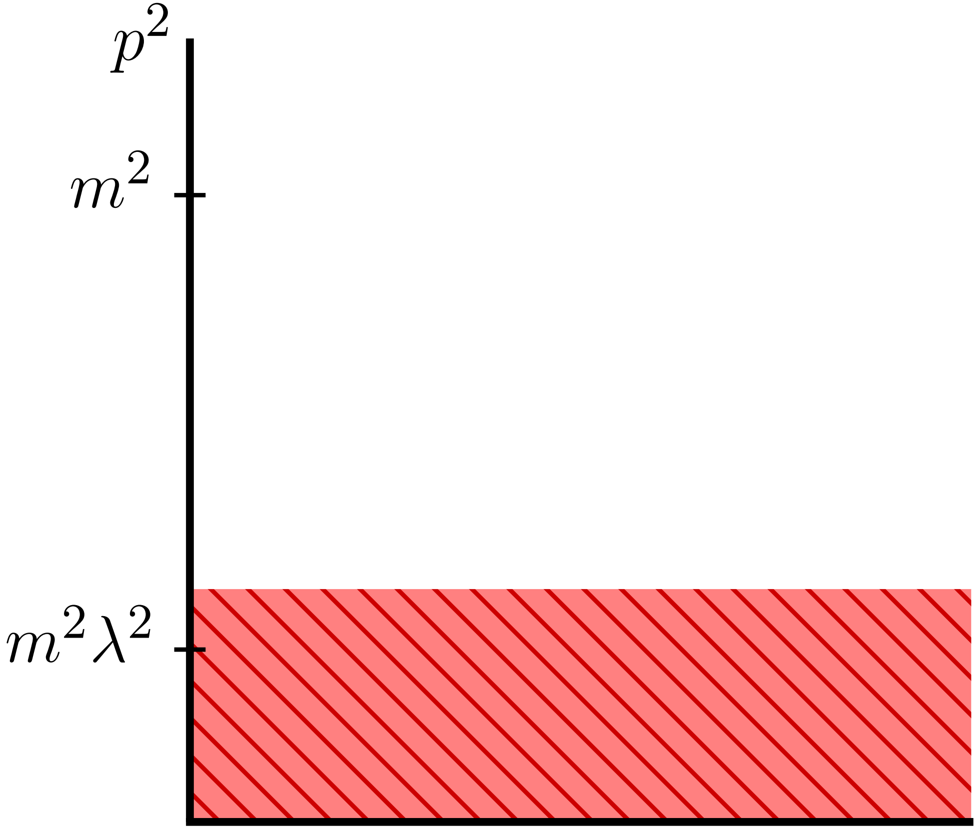

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- SCET describe the interactions of collinear modes with (ultra)soft modes
- A combination of NRQCD and SCET is needed to study the end-point region of the photon spectrum

- The decay rate has been expressed in the factorized form:

$$\frac{d\Gamma}{dz} = \sum_{\omega} H(M,\omega,\mu) \int dk^{+} S(k^{+},\mu) \mathrm{Im} J_{\omega}(k^{+}+M(1-z),\mu)$$

Fleming and Leibovich '02

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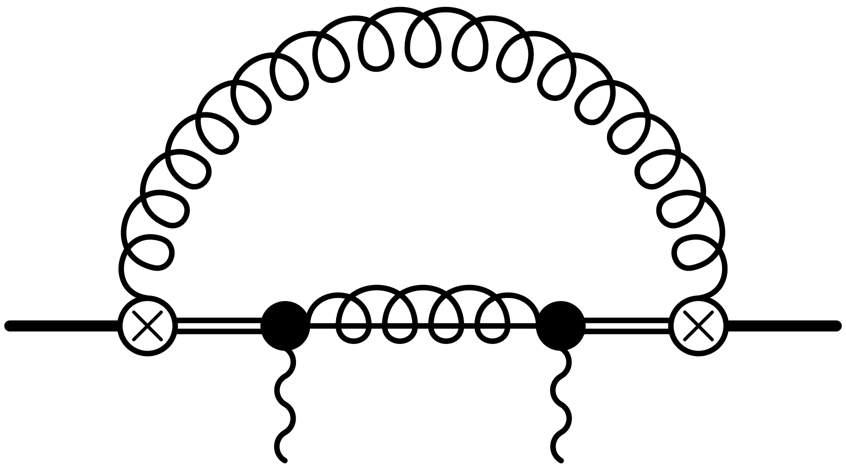
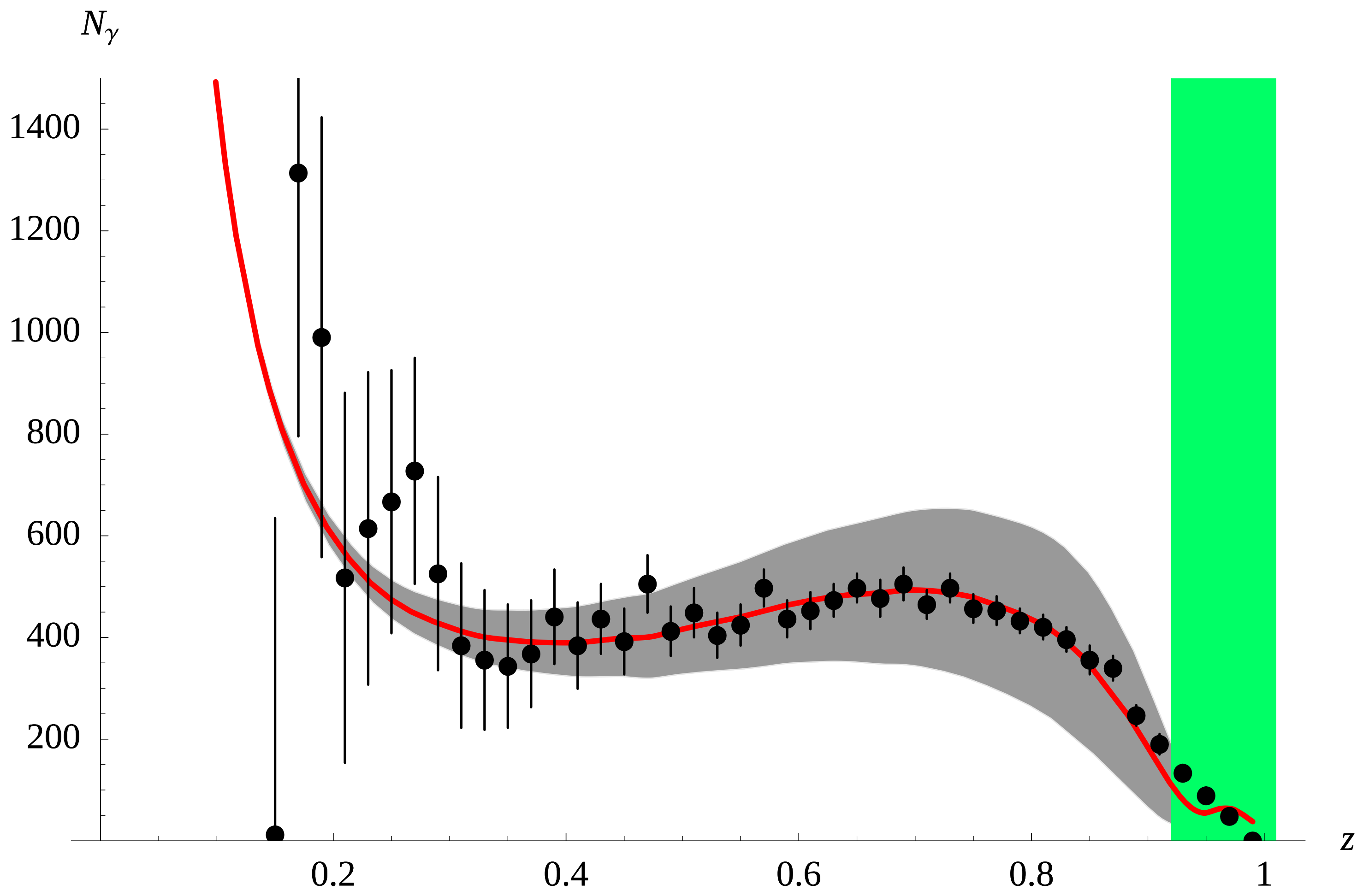
- Assuming $\Upsilon(1S)$ in the weak coupling regime the octet shape functions can be calculated

X.G.T. and Soto '04





X.G.T. and Soto '05

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- NRQCD is the adequate Effective Field Theory for describing heavy-quark bound states in QCD
- To explain the photon spectrum in radiatvie Υ decays we required the combination of NRQCD and SCET
- There are still many processes that need to be well understood (specially involving production of heavy quarkonium)