The QCD static potential: ultrasoft effects

Xavier Garcia i Tormo
Argonne National Laboratory
(based on work done with Nora Brambilla, Joan Soto and Antonio Vairo)

Outline of the talk

- Introduction: the static potential

Outline of the talk

- Introduction: the static potential

- Calculation in perturbation theory

Outline of the talk

- Introduction: the static potential
- Calculation in perturbation theory
- Renormalon singularity

Outline of the talk

- Introduction: the static potential
- Calculation in perturbation theory
- Renormalon singularity
- Renormalon effects in the static potential

Outline of the talk

- Introduction: the static potential
- Calculation in perturbation theory
- Renormalon singularity
- Renormalon effects in the static potential
- Conclusions

Introduction

We want to study the potential of a static quark and antiquark separated by a distance r (*the QCD static potential*).

Introduction

We want to study the potential of a static quark and antiquark separated by a distance r (*the QCD static potential*).

It is basic ingredient in a Schrödinger-like formulation of heavy quark bound states.

Introduction

We want to study the potential of a static quark and antiquark separated by a distance r (*the QCD static potential*).

It is basic ingredient in a Schrödinger-like formulation of heavy quark bound states.

A linear behavior at long distances is a signal for confinement.

Introduction

We want to study the potential of a static quark and antiquark separated by a distance r (*the QCD static potential*).

It is basic ingredient in a Schrödinger-like formulation of heavy quark bound states.

A linear behavior at long distances is a signal for confinement.

Here we are interested in the short distance region ($r \ll 1/\Lambda_{QCD}$), where perturbative (weak coupling) calculations are reliable.

$$V_s = -C_F \frac{\alpha_{\rm s}(1/r)}{r}\left(1 + \tilde{a}_1 \frac{\alpha_{\rm s}(1/r)}{4\pi} + \tilde{a}_2 \left(\frac{\alpha_{\rm s}(1/r)}{4\pi}\right)^2 + \cdots\right)$$

Introduction

We want to study the potential of a static quark and antiquark separated by a distance r (*the QCD static potential*).

It is basic ingredient in a Schrödinger-like formulation of heavy quark bound states.

A linear behavior at long distances is a signal for confinement.

Here we are interested in the short distance region ($r \ll 1/\Lambda_{QCD}$), where perturbative (weak coupling) calculations are reliable.

(in this talk I will always refer to the weak coupling regime)

Introduction

We want to study the potential of a static quark and antiquark separated by a distance r (*the QCD static potential*).

It is basic ingredient in a Schrödinger-like formulation of heavy quark bound states.

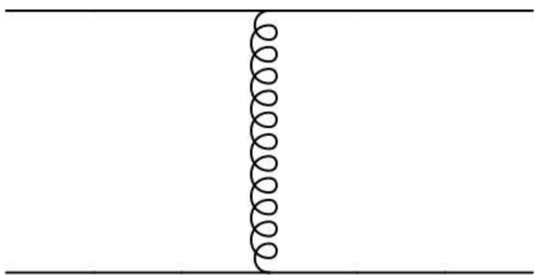
A linear behavior at long distances is a signal for confinement.

Here we are interested in the short distance region ($r \ll 1/\Lambda_{QCD}$), where perturbative (weak coupling) calculations are reliable.

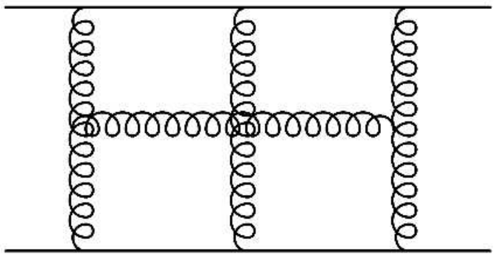
With the additional help of lattice data we can study the interplay of the perturbative and the non-perturbative aspects of QCD.

When calculated in perturbation theory infrared divergences are found, starting at three loops Appelquist, Dine, Muzinich '78

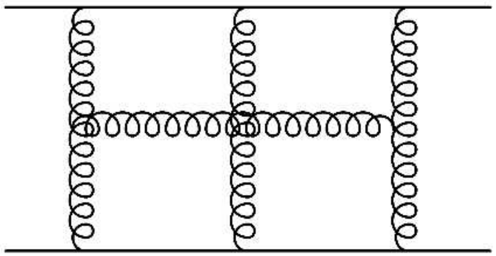
When calculated in perturbation theory infrared divergences are found, starting at three loops Appelquist, Dine, Muzinich '78



When calculated in perturbation theory infrared divergences are found, starting at three loops Appelquist, Dine, Muzinich ’78



When calculated in perturbation theory infrared divergences are found, starting at three loops Appelquist, Dine, Muzinich '78



Selective resummation of a certain type of diagrams is needed.

Consider a non-relativistic bound state

Consider a non-relativistic bound state

$$\begin{array}{rc} m\,(\gg \Lambda_{QCD}) & \text{hard scale} \\ p \sim mv & \text{soft scale} \\ E \sim mv^2 & \text{ultrasoft scale} \end{array}$$

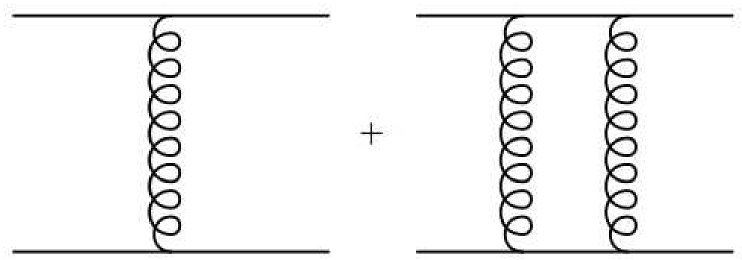
Consider a non-relativistic bound state

$$
\begin{array}{rc}
m\,(\gg \Lambda_{QCD}) & \text{hard scale} \\
p \sim mv & \text{soft scale} \\
E \sim mv^2 & \text{ultrasoft scale} \\
v \ll 1 & (\alpha_{\rm s}(mv) \sim v)
\end{array}
$$

Argonne NATIONAL LABORATORY

Consider a non-relativistic bound state

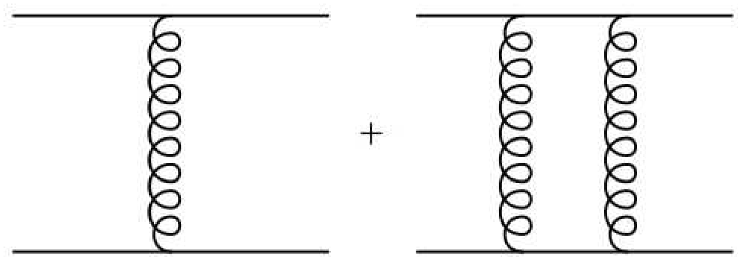
$m\,(\gg \Lambda_{QCD})$	hard scale
$p \sim mv$	soft scale
$E \sim mv^2$	ultrasoft scale
$v \ll 1$	$(\alpha_s(mv) \sim v)$



$$+ \cdots \simeq \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

Consider a non-relativistic bound state

$m\,(\gg \Lambda_{QCD})$	hard scale
$p \sim mv$	soft scale
$E \sim mv^2$	ultrasoft scale
$v \ll 1$	$(\alpha_s(mv) \sim v)$

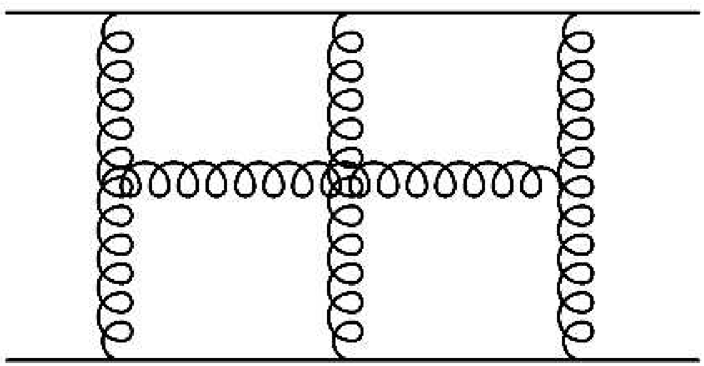


$$+ \cdots \simeq \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

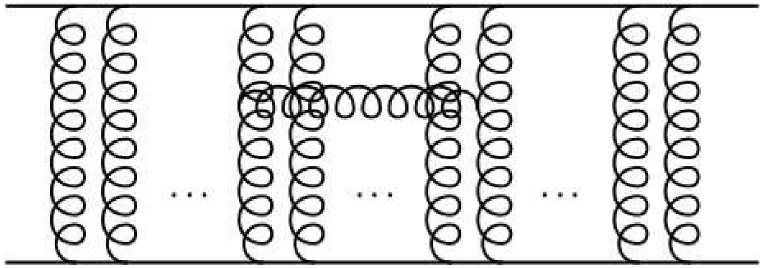
We are organizing the expansion around the Coulombic state

Then the diagram

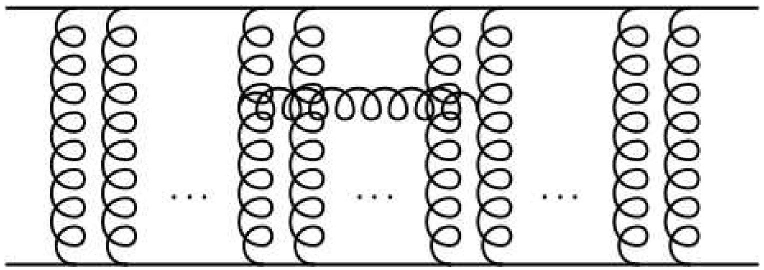
Then the diagram



Then the diagram



Then the diagram

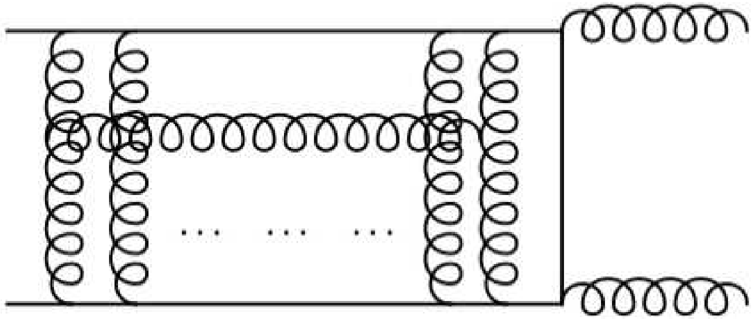


Will give rise to contributions

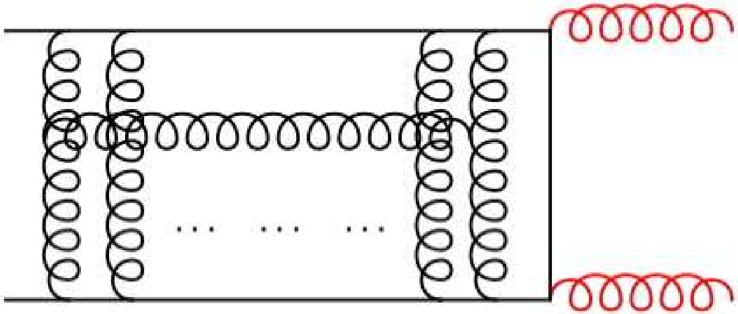
$$\sim \alpha_s^4 \log \alpha_s$$

The difficulty is that all those bound state scales (hard, soft, ultrasoft) get entangled in typical amplitudes

The difficulty is that all those bound state scales (hard, soft, ultrasoft) get entangled in typical amplitudes

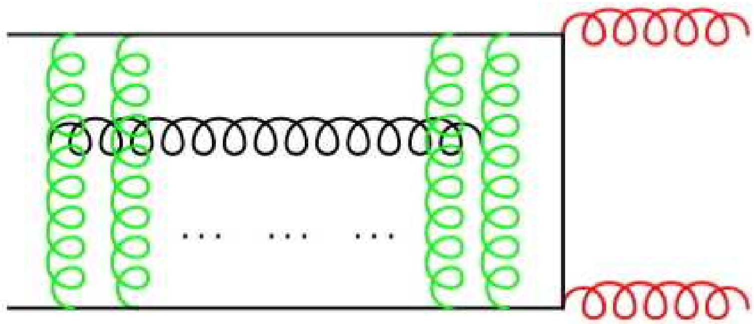


The difficulty is that all those bound state scales (hard, soft, ultrasoft) get entangled in typical amplitudes



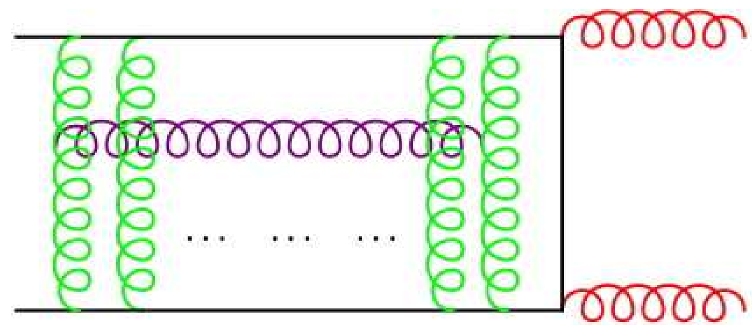
hard

The difficulty is that all those bound state scales (hard, soft, ultrasoft) get entangled in typical amplitudes



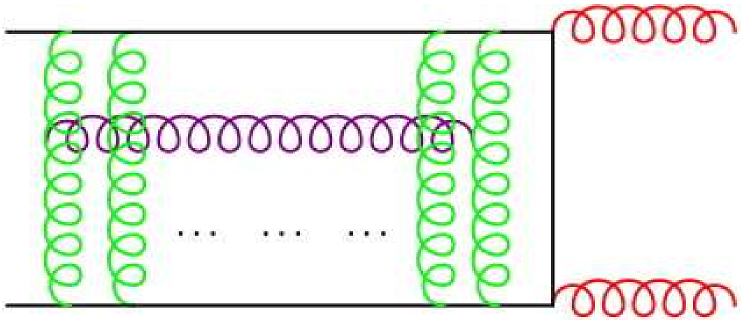
hard soft

The difficulty is that all those bound state scales (hard, soft, ultrasoft) get entangled in typical amplitudes



hard soft ultrasoft

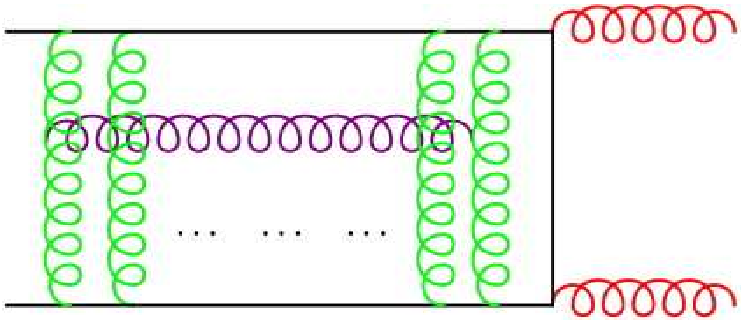
The difficulty is that all those bound state scales (hard, soft, ultrasoft) get entangled in typical amplitudes



hard soft ultrasoft

We can construct Effective Field Theories to disentangle the effects from those scales

The difficulty is that all those bound state scales (hard, soft, ultrasoft) get entangled in typical amplitudes

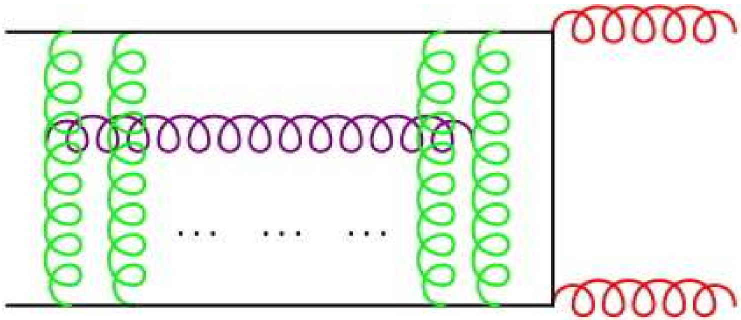


hard soft ultrasoft

We can construct Effective Field Theories to disentangle the effects from those scales

QCD

The difficulty is that all those bound state scales (hard, soft, ultrasoft) get entangled in typical amplitudes

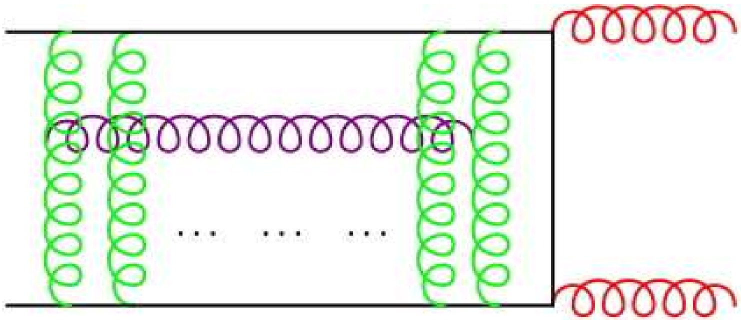


hard soft ultrasoft

We can construct Effective Field Theories to disentangle the effects from those scales

$$\text{QCD} \stackrel{m \gg mv, mv^2}{\longrightarrow} \text{NRQCD}$$

The difficulty is that all those bound state scales (hard, soft, ultrasoft) get entangled in typical amplitudes



hard soft ultrasoft

We can construct Effective Field Theories to disentangle the effects from those scales

$$\mathrm{QCD} \stackrel{m \gg mv,\, mv^2}{\longrightarrow} \mathrm{NRQCD} \stackrel{m \gg mv \gg mv^2}{\longrightarrow} \mathrm{pNRQCD}$$

potential Non-Relativistic QCD

pNRQCD can be organized as an expansion in r (multipole expansion) and $1/m$

potential Non-Relativistic QCD

pNRQCD can be organized as an expansion in r (multipole expansion) and $1/m$

$$\mathcal{L} = \int d^3\mathbf{r}\ \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left[i\partial_0 - V_s(r;\mu) \right] \mathrm{S} + \mathrm{O}^\dagger \left[iD_0 - V_o(r;\mu) \right] \mathrm{O} \right\} -$$

$$-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a}$$

potential Non-Relativistic QCD

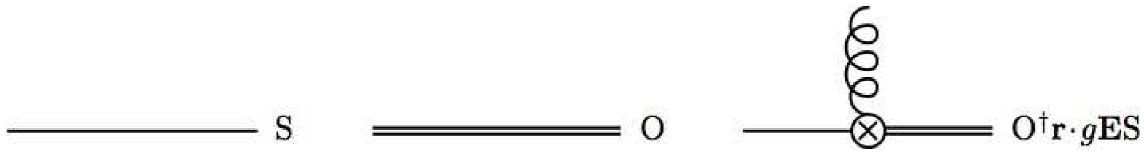
pNRQCD can be organized as an expansion in r (multipole expansion) and $1/m$

$$\mathcal{L} = \int d^3\mathbf{r}\ \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left[i\partial_0 - V_s(r;\mu) \right] \mathrm{S} + \mathrm{O}^\dagger \left[iD_0 - V_o(r;\mu) \right] \mathrm{O} \right\} +$$

$$+ V_A(r;\mu) \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\mathrm{S} + \mathrm{S}^\dagger \mathbf{r}\cdot g\mathbf{E}\mathrm{O} \right\} +$$

$$+ \frac{V_B(r;\mu)}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\mathrm{O} + \mathrm{O}^\dagger \mathrm{O}\mathbf{r}\cdot g\mathbf{E} \right\} -$$

$$- \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a}$$



potential Non-Relativistic QCD

pNRQCD can be organized as an expansion in r (multipole expansion) and $1/m$

$$\mathcal{L} = \int d^3\mathbf{r}\ \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left[i\partial_0 - V_s(r;\mu) \right] \mathrm{S} + \mathrm{O}^\dagger \left[iD_0 - V_o(r;\mu) \right] \mathrm{O} \right\} +$$

$$+ V_A(r;\mu) \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\mathrm{S} + \mathrm{S}^\dagger \mathbf{r}\cdot g\mathbf{E}\mathrm{O} \right\} +$$

$$+ \frac{V_B(r;\mu)}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\mathrm{O} + \mathrm{O}^\dagger \mathrm{O}\mathbf{r}\cdot g\mathbf{E} \right\} -$$

$$- \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a}$$

Potentials appear as Wilson coefficients in the EFT.

potential Non-Relativistic QCD

pNRQCD can be organized as an expansion in r (multipole expansion) and $1/m$

$$\mathcal{L} = \int d^3\mathbf{r}\ \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left[i\partial_0 - V_s(r;\mu) \right] \mathrm{S} + \mathrm{O}^\dagger \left[iD_0 - V_o(r;\mu) \right] \mathrm{O} \right\} +$$

$$+V_A(r;\mu)\mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\mathrm{S} + \mathrm{S}^\dagger \mathbf{r}\cdot g\mathbf{E}\mathrm{O} \right\} +$$

$$+\frac{V_B(r;\mu)}{2}\mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\mathrm{O} + \mathrm{O}^\dagger \mathrm{O}\mathbf{r}\cdot g\mathbf{E} \right\} -$$

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\, a}$$

Potentials appear as Wilson coefficients in the EFT.
Scale dependence of the potentials satisfies renormalization group equations, which can be used to resum logarithms

Calculation in perturbation theory

Calculation in perturbation theory

We obtain the potential by matching NRQCD to pNRQCD. Schematically (at order r^2)

Calculation in perturbation theory

We obtain the potential by matching NRQCD to pNRQCD. Schematically (at order r^2)

$$E_0(r) = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle P \exp\left\{-ig \oint_{r\times T} dz^\mu A_\mu(z)\right\}\right\rangle = V_s(r;\mu) - i\frac{g^2}{N_c}\int_0^\infty dt\, e^{-it(V_o - V_s)} \langle \mathbf{r}\cdot\mathbf{E}\ \mathbf{r}\cdot\mathbf{E}\rangle(\mu)$$

Calculation in perturbation theory

We obtain the potential by matching NRQCD to pNRQCD. Schematically (at order r^2)

$$E_0(r) = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle P \, \exp\left\{ -ig \oint_{r\times T} dz^\mu A_\mu(z) \right\} \right\rangle = V_s(r;\mu) - i \frac{g^2}{N_c} V_A^2 \int_0^\infty dt \, e^{-it(V_o - V_s)} \langle \mathbf{r}\cdot\mathbf{E} \; \mathbf{r}\cdot\mathbf{E} \rangle (\mu)$$

Expectation value of Wilson loop operator

Calculation in perturbation theory

We obtain the potential by matching NRQCD to pNRQCD. Schematically (at order r^2)

$$E_0(r) = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle P \; \exp\left\{ -ig \oint_{r\times T} dz^\mu A_\mu(z) \right\} \right\rangle = V_s(r;\mu) - i\frac{g^2}{N_c} V_A^2 \int_0^\infty dt \, e^{-it(V_o - V_s)} \left\langle \mathbf{r}\cdot\mathbf{E} \; \mathbf{r}\cdot\mathbf{E} \right\rangle(\mu)$$

Expectation value of Wilson loop operator | Matching coefficient

Calculation in perturbation theory

We obtain the potential by matching NRQCD to pNRQCD. Schematically (at order r^2)

$$E_0(r) = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle P \exp\left\{-ig \oint_{r\times T} dz^\mu A_\mu(z)\right\}\right\rangle = V_s(r;\mu) - i\frac{g^2}{N_c} V_A^2 \int_0^\infty dt\, e^{-it(V_o - V_s)} \langle \mathbf{r}\cdot\mathbf{E}\ \mathbf{r}\cdot\mathbf{E}\rangle(\mu)$$

Expectation value of Wilson loop operator — Matching coefficient — Ultrasoft contribution (retardation effects)

Calculation in perturbation theory

We obtain the potential by matching NRQCD to pNRQCD. Schematically (at order r^2)

$$E_0(r) = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle P \, \exp\left\{-ig \oint_{r\times T} dz^\mu A_\mu(z)\right\}\right\rangle = V_s(r;\mu) - i\frac{g^2}{N_c} V_A^2 \int_0^\infty dt\, e^{-it(V_o - V_s)} \langle \mathbf{r}\cdot\mathbf{E}\ \mathbf{r}\cdot\mathbf{E}\rangle(\mu)$$

Expectation value of Wilson loop operator   Matching coefficient   Ultrasoft contribution (retardation effects)

- Left hand side must be μ independent (static energy)

Calculation in perturbation theory

We obtain the potential by matching NRQCD to pNRQCD. Schematically (at order r^2)

$$E_0(r) = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle P \, \exp\left\{ -ig \oint_{r\times T} dz^\mu A_\mu(z) \right\} \right\rangle = V_s(r;\mu) - i\frac{g^2}{N_c} V_A^2 \int_0^\infty dt \, e^{-it(V_o - V_s)} \langle \mathbf{r}\cdot\mathbf{E} \; \mathbf{r}\cdot\mathbf{E}\rangle(\mu)$$

Expectation value of Wilson loop operator   Matching coefficient   Ultrasoft contribution (retardation effects)

- Left hand side must be μ independent (static energy)
- The logarithmic contribution at three loops can be deduced from the leading ultrasoft contribution, the logarithmic terms at four loops from the sub-leading contribution

The Wilson loop is known at two loops

The Wilson loop is known at two loops

$$\lim_{T\to\infty} \frac{i}{T} \ln \langle W_\Box \rangle = -C_F \frac{\alpha_s(1/r)}{r} \left(1 + \tilde{a}_1 \frac{\alpha_s(1/r)}{4\pi} + \tilde{a}_2 \left(\frac{\alpha_s(1/r)}{4\pi} \right)^2 + \cdots \right)$$

$$\tilde{a}_1 = \frac{31}{9} C_A - \frac{20}{9} T_F n_f + 2\gamma_E \beta_0$$

Billoire'80

$$\begin{aligned} \tilde{a}_2 &= \left(\frac{4343}{162} + 4\pi^2 - \frac{\pi^4}{4} + \frac{22}{3}\zeta(3) \right) C_A^2 - \left(\frac{1798}{81} + \frac{56}{3}\zeta(3) \right) C_A T_F n_f - \\ &- \left(\frac{55}{3} - 16\zeta(3) \right) C_F T_F n_f + \left(\frac{20}{9} T_F n_f \right)^2 + \left(\frac{\pi^2}{3} + 4\gamma_E^2 \right) \beta_0^2 + \\ &+ \gamma_E \left(4a_1\beta_0 + 2\beta_1 \right) \end{aligned}$$

Peter'97 Schröder'98

We need to calculate the ultrasoft contribution

We need to calculate the ultrasoft contribution

$$\delta_{\rm US} = -i\frac{g^2}{N_c}\,T_F\,V_A^2\,\frac{r^2}{d-1}\,\int_0^\infty dt\,e^{-it(V_o-V_s)}\langle 0|\mathbf{E}^a(t)\phi(t,0)^{\rm adj}_{ab}\mathbf{E}^b(0)|0\rangle$$

We need to calculate the ultrasoft contribution

$$\delta_{\mathrm{US}} = -i\frac{g^2}{N_c}\,T_F\,V_A^2\,\frac{r^2}{d-1}\,\int_0^\infty dt\,e^{-it(V_o-V_s)}\langle 0|\mathbf{E}^a(t)\phi(t,0)^{\mathrm{adj}}_{ab}\mathbf{E}^b(0)|0\rangle$$

up to next-to-leading order in α_{s}.

We need to calculate the ultrasoft contribution

$$\delta_{\rm US} = -i\frac{g^2}{N_c}\,T_F\,V_A^2\,\frac{r^2}{d-1}\int_0^\infty dt\,e^{-it(V_o-V_s)}\langle 0|\mathbf{E}^a(t)\phi(t,0)^{\rm adj}_{ab}\mathbf{E}^b(0)|0\rangle$$

up to next-to-leading order in $\alpha_{\rm s}$.

We need to calculate the ultrasoft contribution

$$\delta_{\mathrm{US}} = -i\frac{g^2}{N_c}\,T_F\,V_A^2\,\frac{r^2}{d-1}\int_0^\infty dt\,e^{-it(V_o-V_s)}\langle 0|\mathbf{E}^a(t)\phi(t,0)^{\mathrm{adj}}_{ab}\mathbf{E}^b(0)|0\rangle$$

up to next-to-leading order in α_{s}.

V_A is the matching coefficient of the singlet-octet vertex

We need to calculate the ultrasoft contribution

$$\delta_{\rm US} = -i\frac{g^2}{N_c} T_F\, V_A^2\, \frac{r^2}{d-1} \int_0^\infty dt\, e^{-it(V_o-V_s)} \langle 0|\mathbf{E}^a(t)\phi(t,0)^{\rm adj}_{ab}\mathbf{E}^b(0)|0\rangle$$

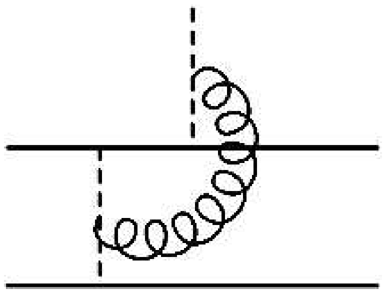
up to next-to-leading order in $\alpha_{\rm s}$.
V_A is the matching coefficient of the singlet-octet vertex

We need to calculate the ultrasoft contribution

$$\delta_{\mathrm{US}} = -i\frac{g^2}{N_c} T_F V_A^2 \frac{r^2}{d-1} \int_0^\infty dt\, e^{-it(V_o - V_s)} \langle 0|\mathbf{E}^a(t)\phi(t,0)^{\mathrm{adj}}_{ab}\mathbf{E}^b(0)|0\rangle$$

up to next-to-leading order in α_{s}.

V_A is the matching coefficient of the singlet-octet vertex



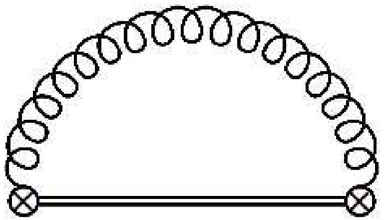
Then we have

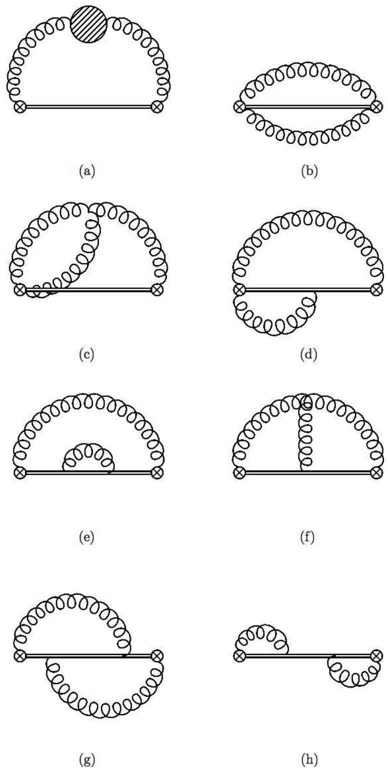
$$V_A = 1 + \mathcal{O}(\alpha_{\mathrm{s}}^2)$$

The chromoelectric field correlator is known at NLO Eidemüller, Jamin '97

The chromoelectric field correlator is known at NLO

Eidemüller, Jamin '97





We can then obtain ($V_s = -C_F \alpha_{V_s}/r$)

We can then obtain ($V_s = -C_F \alpha_{V_s}/r$)

$$\alpha_{V_s}(r,\mu) = \alpha_s(1/r)\Bigg\{1 + (a_1 + 2\gamma_E\beta_0)\,\frac{\alpha_s(1/r)}{4\pi}$$

$$+\left[a_2 + \left(\frac{\pi^2}{3} + 4\gamma_E^2\right)\beta_0^2 + \gamma_E\left(4a_1\beta_0 + 2\beta_1\right)\right]\left(\frac{\alpha_s(1/r)}{4\pi}\right)^2$$

$$+\left[\frac{16\,\pi^2}{3}C_A^3\,\ln r\mu + \tilde{a}_3\right]\left(\frac{\alpha_s(1/r)}{4\pi}\right)^3$$

$$+\left[a_4^{L2}\ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\,C_A^3\beta_0(-5+6\ln 2)\right)\ln r\mu + \tilde{a}_4\right]\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\Bigg\}$$

$$a_4^{L2} = \frac{16\pi^2}{3}C_A^3\left(-\frac{11}{3}C_A + \frac{4}{3}T_F n_f\right)$$

$$a_4^L = 16\pi^2 C_A^3\left[a_1 + 2\gamma_E\beta_0 + T_F n_f\left(-\frac{40}{27} + \frac{8}{9}\log 2\right) + C_A\left(\frac{149}{27} - \frac{22}{9}\log 2 + \frac{4}{9}\pi^2\right)\right]$$

We can then obtain ($V_s = -C_F \alpha_{V_s}/r$)

$$\alpha_{V_s}(r,\mu) = \alpha_{\rm s}(1/r)\left\{1 + (a_1 + 2\gamma_E\beta_0)\,\frac{\alpha_{\rm s}(1/r)}{4\pi}\right.$$
$$+\left[a_2 + \left(\frac{\pi^2}{3} + 4\gamma_E^2\right)\beta_0^2 + \gamma_E\left(4a_1\beta_0 + 2\beta_1\right)\right]\left(\frac{\alpha_{\rm s}(1/r)}{4\pi}\right)^2$$
$$+\left[\frac{16\,\pi^2}{3}C_A^3\,\ln r\mu + \tilde{a}_3\right]\left(\frac{\alpha_{\rm s}(1/r)}{4\pi}\right)^3$$
$$\left.+\left[a_4^{L2}\ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\,C_A^3\beta_0(-5+6\ln 2)\right)\ln r\mu + \tilde{a}_4\right]\left(\frac{\alpha_{\rm s}(1/r)}{4\pi}\right)^4\right\}$$

From leading ultrasoft contribution   From sub-leading ultrasoft contribution

Brambilla Pineda Soto Vairo '99   Brambilla X.G.T. Soto Vairo '06

$$
\begin{aligned}
E_0(r) \quad = \quad & -\frac{C_F \alpha_s(1/r)}{r} \Bigg\{ 1 + \frac{\alpha_s(1/r)}{4\pi} \left[a_1 + 2\gamma_E \beta_0 \right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^2 \left[a_2 + \left(\frac{\pi^2}{3} + 4\gamma_E^2 \right) \beta_0^2 + \gamma_E \left(4 a_1 \beta_0 + 2\beta_1 \right) \right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^3 \left[\frac{16\pi^2}{3} C_A^3 \log \frac{C_A \alpha_s(1/r)}{2} + \tilde{a}_3 \right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^4 \left[a_4^{L2} \log^2 \frac{C_A \alpha_s(1/r)}{2} + a_4^L \log \frac{C_A \alpha_s(1/r)}{2} + \tilde{a}_4 \right] \Bigg\}
\end{aligned}
$$

As mentioned before the three loop coefficient a_3 is not known. Padé approximation methods has been used to estimate it. Chishtie Elias '01

As mentioned before the three loop coefficient a_3 is not known. Padé approximation methods has been used to estimate it. Chishtie Elias '01

The first unknown coefficient of a perturbative series is obtained by approximating the series by a quotient of polynomials involving the known terms.

As mentioned before the three loop coefficient a_3 is not known. Padé approximation methods has been used to estimate it. Chishtie Elias '01

The first unknown coefficient of a perturbative series is obtained by approximating the series by a quotient of polynomials involving the known terms.

The result

$$a_3 = 6240$$

is obtained ($n_f = 4$).

We will use this value in the plots for illustration.

The ultrasoft logarithms can be resumed by solving the renormalization group equations

The ultrasoft logarithms can be resumed by solving the renormalization group equations

Pineda Soto '00 Brambilla X.G.T. Soto Vairo '08 (preliminary)

$$\begin{cases} \mu \frac{d}{d\mu}\alpha_{V_s} = \frac{2}{3}\frac{\alpha_s}{\pi}V_A^2\left(\frac{\alpha_{V_o}}{2N_c} + C_F\alpha_{V_s}\right)^3\left(1+6\,\frac{\alpha_s}{\pi}\,B\right) \\ \mu \frac{d}{d\mu}\alpha_{V_o} = \frac{2}{3}\frac{\alpha_s}{\pi}V_A^2\left(\frac{\alpha_{V_o}}{2N_c} + C_F\alpha_{V_s}\right)^3\left(1+6\,\frac{\alpha_s}{\pi}\,B\right) \\ \mu \frac{d}{d\mu}\alpha_s = \alpha_s\beta(\alpha_s) \\ \mu \frac{d}{d\mu}V_A = 0 \\ \mu \frac{d}{d\mu}V_B = 0 \end{cases}$$

$(B = \frac{-5n_f + C_A(6\pi^2 + 47)}{108})$

The solution to the equations gives

Brambilla X.G.T. Soto Vairo '08 (preliminary)

$$\alpha_{V_s}(\mu) = \alpha_{V_s}(1/r) + \frac{C_A^3}{6\beta_0}\alpha_s^3(1/r)\left\{\left(1+\frac{3}{4}\frac{\alpha_s(1/r)}{\pi}a_1\right)\ln\frac{\alpha_s(1/r)}{\alpha_s(\mu)} + \left(\frac{\beta_1}{4\beta_0}-6B\right)\left[\frac{\alpha_s(\mu)}{\pi}-\frac{\alpha_s(1/r)}{\pi}\right]\right\}$$

Let's recapitulate

Let's recapitulate

- We have obtained the infrared behavior of the potential up to $\mathrm{N}^4\mathrm{LO}$

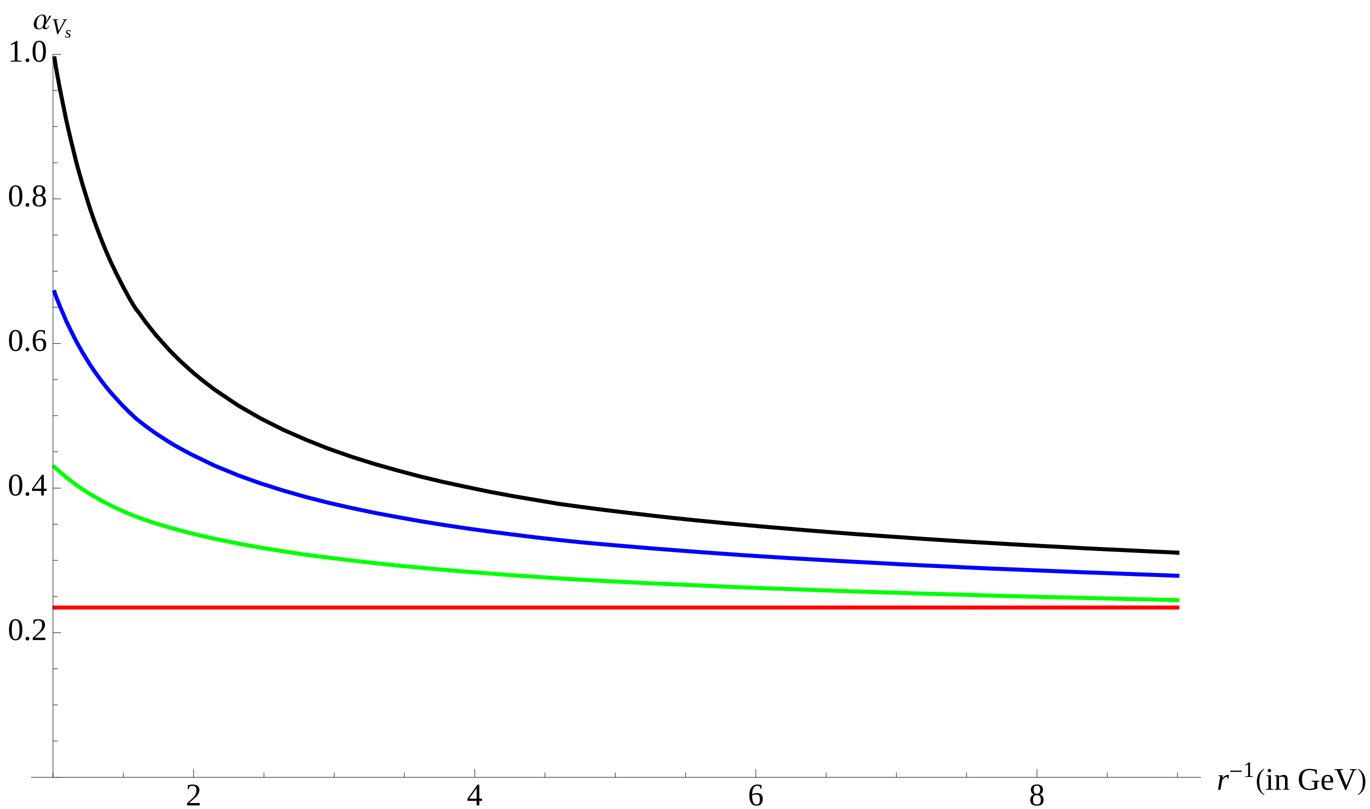
Let's recapitulate

- We have obtained the infrared behavior of the potential up to N^{4}LO

- The infrared logs have been resummed at sub-leading order

Let's recapitulate

- We have obtained the infrared behavior of the potential up to N^4LO
- The infrared logs have been resummed at sub-leading order
- The use of effective field theories is crucial in simplifying these computations



LO NLO N^2LO N^3LO

$(n_f = 4 \quad \mu = \alpha_s(r)/r \quad \nu = 1/r)$

$$V_s = -C_F \frac{\alpha_{V_s}}{r}$$

Clearly the perturbative expansion does not seem to be converging.

Clearly the perturbative expansion does not seem to be converging.

This is conventionally believed to be due to the renormalon singularity. Therefore if one subtracts the renormalon one would expect an improved behavior regarding the convergence of the series.

Renormalon singularity

Typically perturbative series in quantum field theory

$$R = \sum_n r_n \alpha^n$$

present divergent coefficients

$$r_n \overset{n\to\infty}{\sim} K a^n n! n^b$$

Renormalon singularity

Typically perturbative series in quantum field theory

$$R = \sum_n r_n \alpha^n$$

present divergent coefficients

$$r_n \overset{n\to\infty}{\sim} K a^n n! n^b$$

The divergent behavior can be analyzed using the Borel transform of the series

$$R = \sum_{n=0}^{\infty} r_n \alpha^{n+1} \quad \rightarrow \quad B[R](t) = \sum_{n=0}^{\infty} r_n \frac{t^n}{n!}$$

If it exists, the integral $\tilde{R}$ gives the Borel sum of the original series

$$\tilde{R} = \int_0^{\infty} dt\, e^{-t/\alpha} B[R](t)$$

In any case the Borel transform $B[R](t)$ is a *generating function* for the coefficients r_n. Singularities in t in the Borel transform encode divergent behavior of the original coefficients r_n.

In any case the Borel transform $B[R](t)$ is a *generating function* for the coefficients r_n. Singularities in t in the Borel transform encode divergent behavior of the original coefficients r_n.

The singularity closest to the origin in the t plane will give the most important (fastest divergent) contribution to the coefficients r_n.

In any case the Borel transform $B[R](t)$ is a *generating function* for the coefficients r_n. Singularities in t in the Borel transform encode divergent behavior of the original coefficients r_n.

The singularity closest to the origin in the t plane will give the most important (fastest divergent) contribution to the coefficients r_n.

A certain kind of singularities of the Borel plane related to large or small loop momentum are called *renormalons*.

In any case the Borel transform $B[R](t)$ is a *generating function* for the coefficients r_n. Singularities in t in the Borel transform encode divergent behavior of the original coefficients r_n.

The singularity closest to the origin in the t plane will give the most important (fastest divergent) contribution to the coefficients r_n.

A certain kind of singularities of the Borel plane related to large or small loop momentum are called *renormalons*.

There are several methods (diagrammatic and based on the renormalization group) to obtain the nature of the renormalon singularity.

For the heavy quark pole mass

$$m_{pole} = m_{\overline{\mathrm{MS}}} + \sum_n r_n \alpha^{n+1}$$

the closest singularity to the origin is located at $t = 2\pi/\beta_0$.

For the heavy quark pole mass

$$m_{pole} = m_{\overline{\mathrm{MS}}} + \sum_n r_n \alpha^{n+1}$$

the closest singularity to the origin is located at $t = 2\pi/\beta_0$. One obtains

$$B[m_{pole}] = N\nu \frac{1}{(1-2u)^{1+b}} \left(1 + c_1(1-2u) + c_2(1-2u)^2 + \cdots\right) + \cdots$$

$$(u = \frac{\beta_0 t}{4\pi})$$

The normalization N is the only parameter that cannot be determined exactly (it receives unsuppressed contributions from all orders in the perturbative expansion)

To improve the perturbative behavior the idea is then to define a scheme that cancels the renormalons (singularities in the Borel plane) in the matching coefficients of the effective theory.

To improve the perturbative behavior the idea is then to define a scheme that cancels the renormalons (singularities in the Borel plane) in the matching coefficients of the effective theory.

Several different implementations of this idea has been developed during the years. Bigi et.al. '94, Beneke '98, Hoang et.al. '99, Yakovlev et.al. '01, ...

To improve the perturbative behavior the idea is then to define a scheme that cancels the renormalons (singularities in the Borel plane) in the matching coefficients of the effective theory.

Several different implementations of this idea has been developed during the years. Bigi et.al. '94, Beneke '98, Hoang et.al. '99, Yakovlev et.al. '01, ...

In the following we use the analysis in the so called RS scheme. Pineda '01

Renormalon effects in the static potential

The quantity

$$2m + V_s + \delta_{US}$$

is an observable and, therefore, free of renormalons. We can see that the renormalons in the static potential are related to those of the mass.

Renormalon effects in the static potential

The quantity

$$2m + V_s + \delta_{US}$$

is an observable and, therefore, free of renormalons. We can see that the renormalons in the static potential are related to those of the mass.

At order r^0 in the multipole expansion (no ultrasoft effects) the renormalon structure of the potential is analogous to that of the mass.

Renormalon effects in the static potential

The quantity

$$2m + V_s + \delta_{US}$$

is an observable and, therefore, free of renormalons. We can see that the renormalons in the static potential are related to those of the mass.

At order r^0 in the multipole expansion (no ultrasoft effects) the renormalon structure of the potential is analogous to that of the mass.

$$B[V_s] = N_V \nu \frac{1}{(1-2u)^{1+b}} \left(1 + c_1(1-2u) + c_2(1-2u)^2 + \cdots\right) + \cdots$$

Renormalon effects in the static potential

The quantity

$$2m + V_s + \delta_{US}$$

is an observable and, therefore, free of renormalons. We can see that the renormalons in the static potential are related to those of the mass.

At order r^0 in the multipole expansion (no ultrasoft effects) the renormalon structure of the potential is analogous to that of the mass. The potential in the RS scheme is defined

$$V_{s,RS} = V_s + 2\delta m_{RS}$$

such that δm_{RS} comes from the contributions of the first renormalon at $u = 1/2$ Pineda '02

Renormalon effects in the static potential

The quantity

$$2m + V_s + \delta_{US}$$

is an observable and, therefore, free of renormalons. We can see that the renormalons in the static potential are related to those of the mass.

At order r^0 in the multipole expansion (no ultrasoft effects) the renormalon structure of the potential is analogous to that of the mass. The potential in the RS scheme is defined

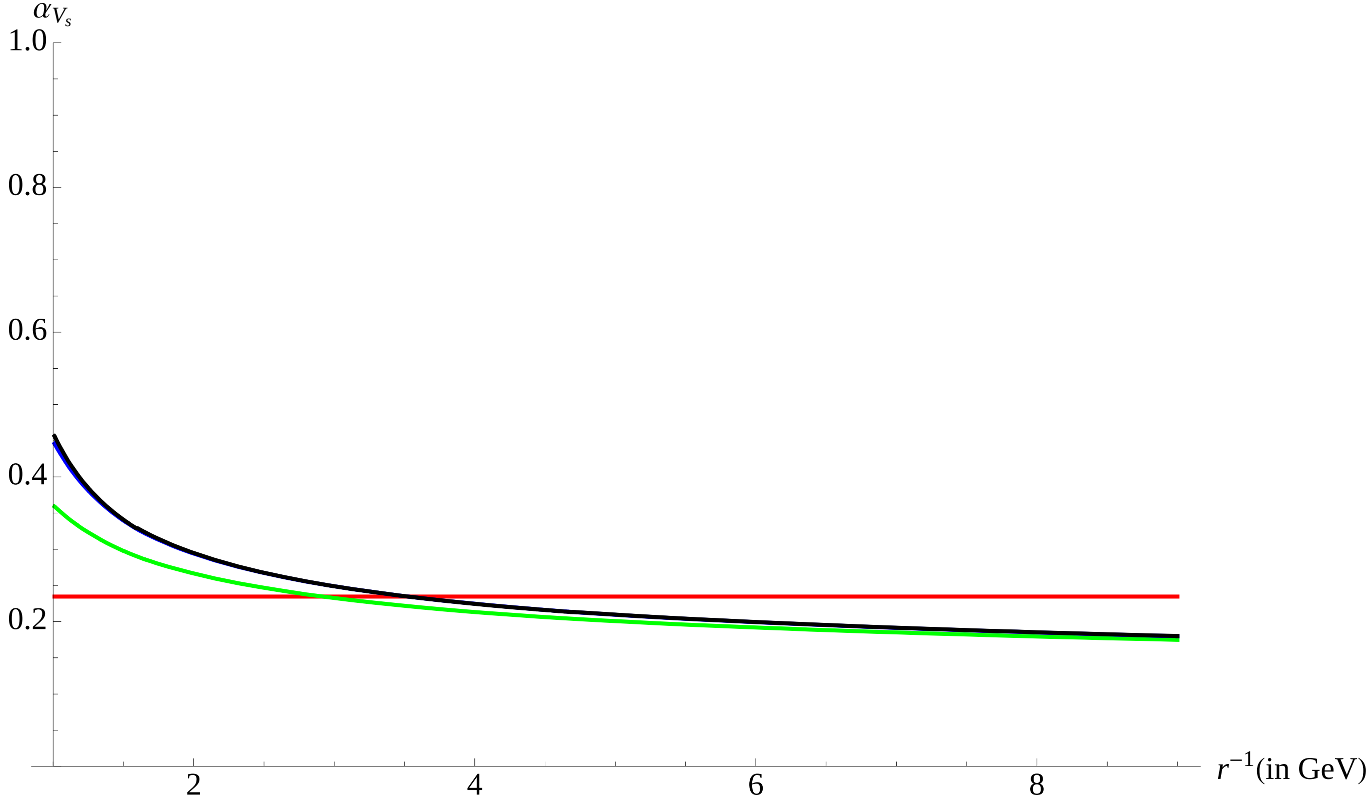
$$V_{s,RS} = V_s + 2\delta m_{RS}$$

such that δm_{RS} comes from the contributions of the first renormalon at $u = 1/2$ Pineda '02

$$\delta m_{RS} = \sum_{n=1}^{\infty} N\nu_f \left(\frac{\beta_0}{2\pi}\right)^n \alpha_{\mathrm{s}}^{n+1}(\nu_f) \sum_{k=0}^{\infty} c_k \frac{\Gamma(n+1+b-k)}{\Gamma(1+b-k)}$$

With this definition of the potential we now obtain

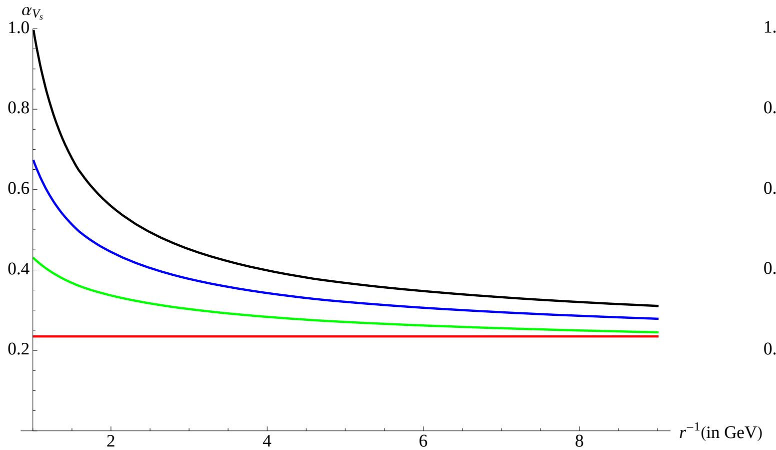
With this definition of the potential we now obtain

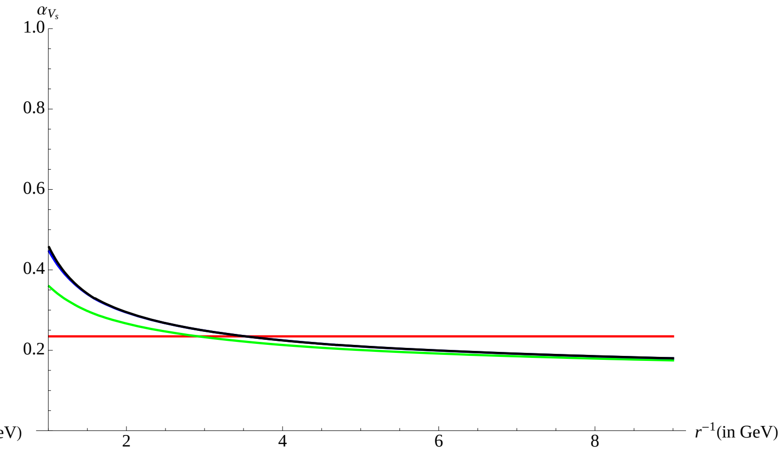


LO NLO N^2LO N^3LO

$(n_f = 4 \quad \mu = \alpha_s(1/r)/r \quad \nu_f = \nu = 1/r)$

Clearly the behavior of the perturbative series is improved





So far we have ignored the ultrasoft effects when analyzing the renormalon.

So far we have ignored the ultrasoft effects when analyzing the renormalon.

The analysis of the renormalons at order r^2 is in progress.

So far we have ignored the ultrasoft effects when analyzing the renormalon.

The analysis of the renormalons at order r^2 is in progress. The ultrasoft effects will modify the non-analytic structure of the Borel transform

So far we have ignored the ultrasoft effects when analyzing the renormalon.

The analysis of the renormalons at order r^2 is in progress. The ultrasoft effects will modify the non-analytic structure of the Borel transform

$$N_{V_s}\nu\frac{1}{(1-2u)^{1+b}}\left(1+c_1(1-2u)+c_2(1-2u)^2+\cdots\right)+\cdots$$

So far we have ignored the ultrasoft effects when analyzing the renormalon.

The analysis of the renormalons at order r^2 is in progress. The ultrasoft effects will modify the non-analytic structure of the Borel transform

$$N_{V_s}\nu\frac{1}{(1-2u)^{1+b}}\left(1+c_1(1-2u)+c_2(1-2u)^2+\right.$$

$$\left.+d_1(1-2u)^2\ln(1-2u)\right)+\cdots$$

$$d_1=\frac{C_F C_A^2}{\beta_0}\frac{N_{V_o}-N_{V_s}}{N_{V_s}}\frac{1}{b(b-1)}\left(\frac{2\pi}{\beta_0}\right)^2$$

Brambilla X.G.T. Soto Vairo '08 (preliminary)

Conclusions

- The static QCD potential presents infrared divergences starting at three loops. That infrared behavior can be concisely analyzed within pNRQCD

Conclusions

- The static QCD potential presents infrared divergences starting at three loops. That infrared behavior can be concisely analyzed within pNRQCD
- We have obtained the leading and sub-leading infrared behavior from the effective field theory (it corresponds to 4 loops terms in the potential!)

Conclusions

- The static QCD potential presents infrared divergences starting at three loops. That infrared behavior can be concisely analyzed within pNRQCD
- We have obtained the leading and sub-leading infrared behavior from the effective field theory (it corresponds to 4 loops terms in the potential!)
- The infrared logs can also be resumed using the ultrasoft RG equations in the effective theory

Conclusions

- The static QCD potential presents infrared divergences starting at three loops. That infrared behavior can be concisely analyzed within pNRQCD
- We have obtained the leading and sub-leading infrared behavior from the effective field theory (it corresponds to 4 loops terms in the potential!)
- The infrared logs can also be resumed using the ultrasoft RG equations in the effective theory
- The apparent bad behavior of the perturbative series seems to be explained by the presence of renormalon singularities, which can then be removed in the EFT

Conclusions

- The static QCD potential presents infrared divergences starting at three loops. That infrared behavior can be concisely analyzed within pNRQCD
- We have obtained the leading and sub-leading infrared behavior from the effective field theory (it corresponds to 4 loops terms in the potential!)
- The infrared logs can also be resumed using the ultrasoft RG equations in the effective theory
- The apparent bad behavior of the perturbative series seems to be explained by the presence of renormalon singularities, which can then be removed in the EFT
- Refined understanding of the static potential is crucial for comparisons with lattice data, precise analysis of the heavy quarkonium spectrum and top quark pair production at the ILC