Dispersive approach to power correct. in QCD hard processes

Yu. Dokshitzer, B. Webber, G.M. NPB469 (96)

Marchesini

QED Redman Uretsky 58

Bogoliubov Lugunov Shirkov 60

QCD Shirkov Solovtsov 96

Dokshitzer Shirkov 95

Zakharov, Ball Braun Beneke (95)

....

Perturbative running coupling

Di Remo Onofri GM

Grunberg Dokshitzer

I) $k \gg \Lambda_{QCD}$ quark-gluon, perturbat. QCD

$$\alpha_s(k) \simeq 4\pi / \beta_0 \ln \frac{k^2}{\Lambda^2_{QCD}}$$

II) $k \lesssim \Lambda_{QCD}$ hadrons, confinement ?

Use pert. QCD in I to constraint II

Hadron-parton duality $\Rightarrow$ final hadron emission

Power-corrections $\Rightarrow$ $\alpha_s(k)$ for $k^2 \to \Lambda^2_{QCD}$

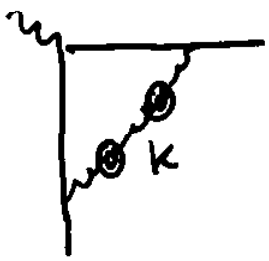
Dispersive approach: try to introduce/use

$$\alpha_s(k)$$

at any scale.

Working strategy in QED and $O(N)$ sigma mod.

Running coupling in QED



from Thompson cross-section

$$\alpha(k^2) = \alpha(0)\, Z_3(k^2)$$

photon w.f.

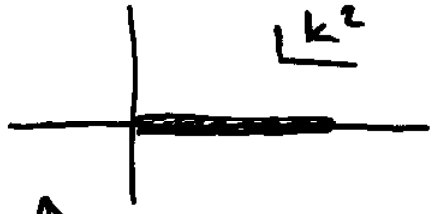
$Z_3(0) = 1$

$Z_3(k^2) =$ $=$ $+ \cdots$

$\rho(k^2) \equiv \frac{-1}{2\pi i}\,\mathrm{Disc}\{\alpha(-k^2)\} =$ ≥ 0 Unitarity

spectral function

Implement causality



No Landau ghost

$$\alpha(k^2) = -\int_0^\infty \frac{d\mu^2}{k^2+\mu^2}\,\rho(\mu^2)$$

Running coupling in QCD

$\frac{1}{k^2}$ sing + $\frac{1}{k^2}$ regul $\qquad\qquad$ $\frac{1}{k^2}$ regul

(gauge depend. cancell)

Use only $\Rightarrow$ generalis. spect. function $\rho_s(k^2)$

gauge invariat at 1-loop

$\rho_s(k^2) =$

$>0 \qquad <0 \qquad >0$

$$\alpha_s(k^2) = -\int_0^\infty \frac{d\mu^2}{\mu^2+k^2}\,\rho_s(\mu^2)$$ implement causality

Unitarity

Causality

Asympt. Freed. $\rho_s < 0$

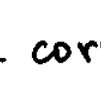
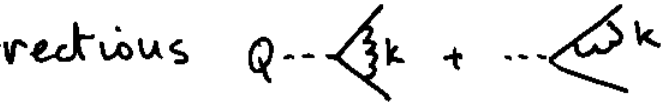
Gauge inv. at 1-loop

$\Rightarrow k_t$ as argument of the running coupl.

Aniti Barletta
Cefalonia Veronese
GM
Dokshitzer Khoze
Trayan

1-loop corrections in quark-induced processes

e^+e^- or DIS, or DY, ...

1) Virtual corrections Q  k +  k

$$F^V(Q) = \int_0^\infty dk^2 \left\{ \frac{\alpha_s(k^2)}{k^2} \right\} \cdot M^V(k^2/Q^2)$$

← massless gluon exch + running c.

Effective coupling from dispersive representation

$$\frac{\alpha_s(k^2)}{k^2} = -\int_0^\infty \frac{d\mu^2}{k^2(\mu^2+k^2)} \rho_s(\mu^2) = \frac{\partial}{\partial k^2} \int_0^\infty \frac{d\mu^2}{\mu^2+k^2} \alpha_{eff}(k^2)$$

$$\alpha_{eff}(k^2) = \alpha_s(k^2) - \frac{\beta_0^2}{48\pi} \alpha_s^3(k^2) + \dots$$

Characteristic function (virtual part)

$$F^V(Q^2) = \int_0^\infty \frac{d\mu^2}{\mu^2} \alpha_{eff}(\mu^2) \; \frac{-\mu^2 \partial}{\partial \mu^2} \underbrace{\int_0^\infty dk^2 \left\{ \frac{1}{k^2+\mu^2} \right\} M^V(k^2/Q^2)}_{\mathcal{F}^V(\mu^2/Q^2)}$$

amplitude for the 1-gluon exchange with massive gl.

μ = dispersive param. Gauge inv. OK 1-loop

$$F^V(Q^2) = \int_0^\infty \frac{d\mu^2}{\mu^2}\, \alpha_{eff}(\mu^2)\, \dot{\mathcal{J}}^V(\mu^2/Q^2) \qquad \dot{\mathcal{J}} = \frac{\partial}{\partial t}\mathcal{J}$$

$$t = \ln\left(\frac{Q^2}{\mu^2}\right)$$

2) Real emission (fully inclusive)

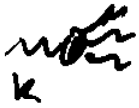
$$F^R(Q^2) = \int_0^\infty dk^2\, \mathcal{J}^R(k^2/Q^2) \cdot \frac{\mathrm{Disc}}{2\pi i}\left\{\frac{\alpha_s(k^2)}{k^2 - i\epsilon}\right\}$$

emission of gluon of mass k^2

Effective coupling $\quad \frac{\mathrm{Dis}}{2\pi i}\left\{\frac{\alpha_s(k^2)}{k^2 - i\epsilon}\right\} = -\frac{\partial}{\partial k^2}\left\{\alpha_{eff}(k^2)\right\}$

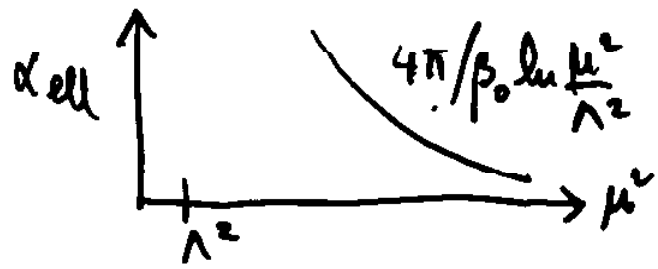
Characteristic function $\quad \mathcal{J} = \mathcal{J}^V + \mathcal{J}^R$

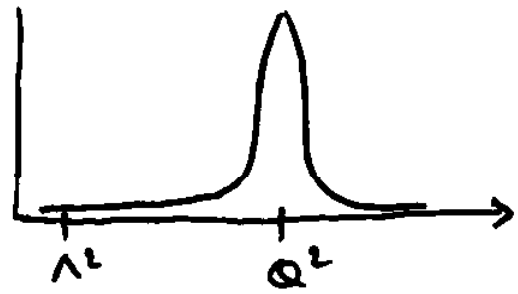
$$F^{V+R}(Q^2) = \int_0^\infty \frac{d\mu^2}{\mu^2}\, \alpha_{eff}(\mu^2)\left\{\dot{\mathcal{J}}^V(\mu^2/Q^2) + \dot{\mathcal{J}}^R(\mu^2/Q^2)\right\}$$

Inclusive distribution : no resolution in 

One-loop corrections to quark-induced proc

$$F(Q^2) = \int_0^\infty \frac{d\mu^2}{\mu^2}\, \alpha_{ell}(\mu^2) \cdot \dot{\mathcal{J}}(\mu^2/Q^2) \qquad \left(t = \ln\frac{Q^2}{\mu^2}\right.$$





ITEP: In general contrib. with low $k^2 \lesssim \Lambda^2$ are taken into account by dimensional operators in OPE

$$\alpha_s(k^2) = \alpha_s^{high}(k^2) + \alpha_s^{low}(k^2) = \alpha_s^{PT}(k^2) + \delta\alpha_s(k^2)$$

$$F(Q^2) = F^{PT}(Q^2) + F^{NP}(Q^2)$$

$$F^{NP}(Q^2) = \int_0^\infty \frac{d\mu^2}{\mu^2}\, \delta\alpha_{ell}(\mu^2) \cdot \dot{\mathcal{J}}(\mu^2/Q^2)$$

Universality: all distributions (time-, space-like) depend on the same α_{ell}, $\delta\alpha_{ell}$

$$F^{NP}(Q^2) = \int_0^\infty \frac{d\mu^2}{\mu^2}\, \delta\alpha_{ell}(\mu^2)\; \mathcal{J}(\mu^2/Q^2)$$

$$\delta\alpha_s(k^2) = \int_0^\infty \frac{k^2 d\mu^2}{(k^2+\mu^2)^2}\, \delta\alpha_{ell}(\mu^2)$$

bounded to low frequencies

then $\displaystyle\int_0^\infty \frac{d\mu^2}{\mu^2} (\mu^2)^p\, \delta\alpha_{ell}(\mu^2) = 0 \qquad p = 1, 2, \ldots$

Zakharov, Beneke Braun ...

Only non analytic terms in expans. of $\mathcal{J}(\mu^2/Q^2)$

$$\mathcal{J}(\mu^2/Q^2) \simeq C\sqrt{\frac{\mu^2}{Q^2}} \Rightarrow \quad F^{NP}(Q^2) = \frac{C}{Q}\int_0^\infty \frac{d\mu^2}{\mu^2}\, \mu\, \delta\alpha_{ell}(\mu^2)$$

eg. Thrust

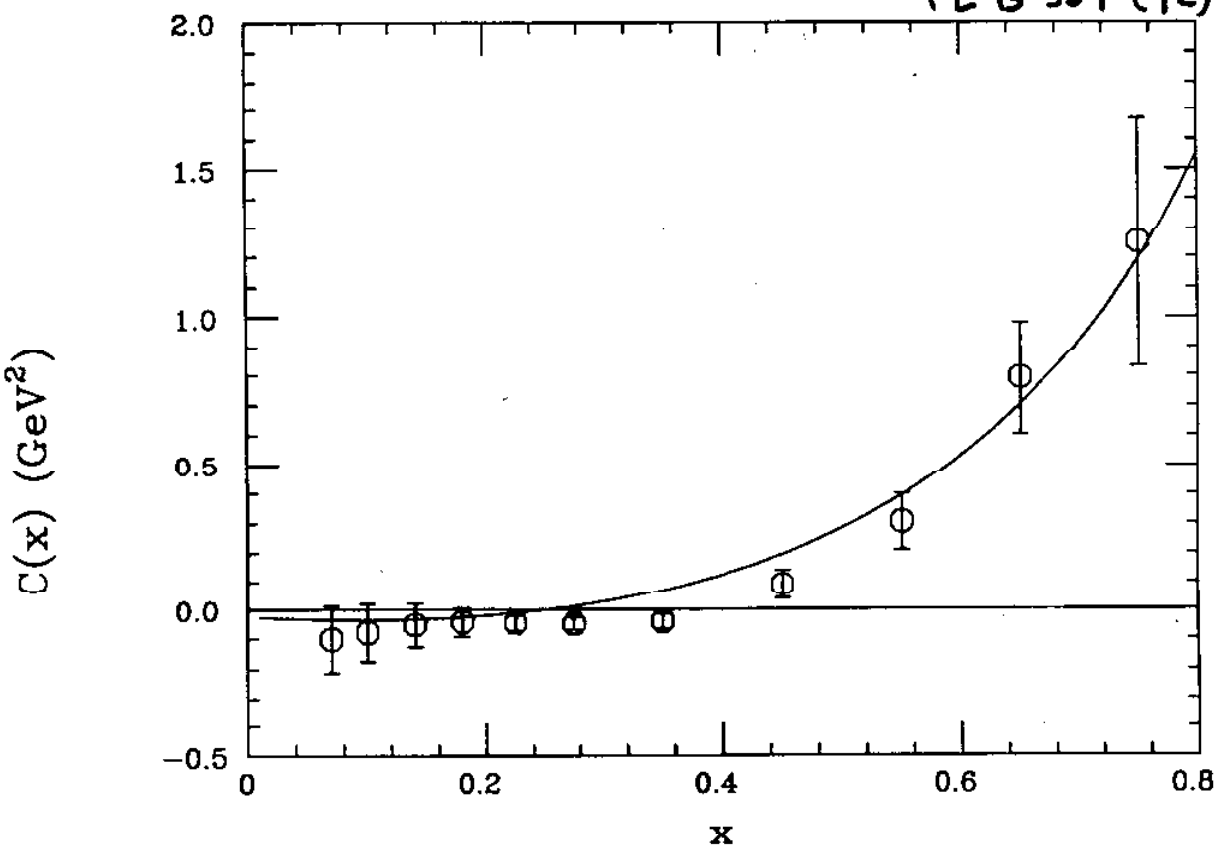
$$\mathcal{J}(\mu^2/Q^2) \simeq C\left(\frac{\mu^2}{Q^2}\right)^p \ln\frac{\mu^2}{Q^2} \Rightarrow$$

$$\Rightarrow F^{NP}(Q^2) = \frac{C}{Q^{2p}} \int_0^\infty \frac{d\mu^2}{\mu^2}\, \delta\alpha_{ell}(\mu^2)\, \mu^{2p} \ln \mu^2$$

eg in $F_2(xQ)$ $\quad C = C(x)$ singular for $x \to 1$

enhanced, detectable

Virchaux Milstajn
PL B 357 (92) 221



Dokshitzer, Webber G.M. Nucl. Phys. B (96)

MRSA

$$F_2(x,Q^2) = F_2^{PT}(x,Q^2)\left(1 + \frac{C(x)}{Q^2}\right)$$

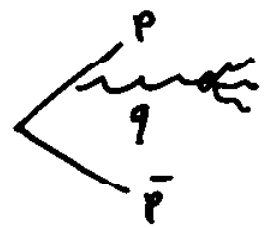
$$C(x) \simeq A_2' \frac{1}{1-x}$$

$$A_2' = \frac{C_F}{2\pi}\int_0^\infty d\mu^2 \ln \mu^2 \, \delta\alpha_{eff}(\mu^2) = -1\,\mathrm{GeV}^2$$

Thrust distribution Dokshitzer Webber

(1.) Possibility of soft gluon exponentiation

(2) Need in resolution of gluon branching



$$(1) \Rightarrow \quad \frac{1}{\sigma}\frac{d\sigma}{dT}(T) = \left(\frac{1}{\sigma}\frac{d\sigma}{dT}\right)^{pT}(T-\delta T)$$

Dispersive method

$$\delta T = \frac{A_1}{\varkappa Q} \qquad A_1 = \frac{C_F}{2\pi}\int_0 \frac{d\mu^2}{\mu^2}\,\delta\alpha_{eff}(\mu^2)\,\mu$$

$\varkappa > 1$ Kinematical requirement for gluon branching into p-hemisphere

$$\varkappa^2 pq < \bar{p}q$$

Effect sensitive to large angle gluon emission $\Rightarrow$ (2)

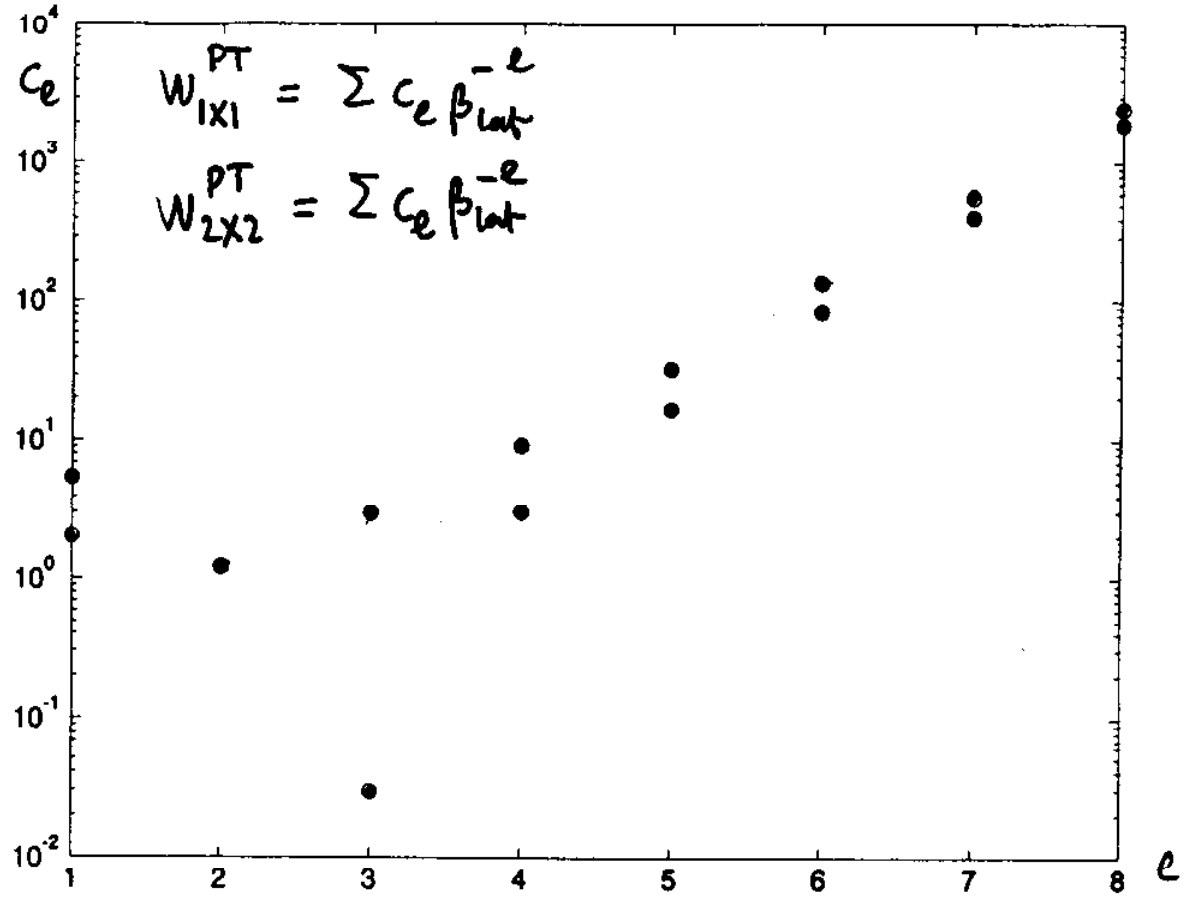
$$\alpha_s(k^2) = \alpha_s^{PT}(k^2) + \delta\alpha_s(k^2)$$

PT ≡ high frequency part $\quad k^2 \gg \Lambda^2$

$$\alpha_s(k^2) = \alpha_s^{PT}(k^2) = \frac{4\pi}{\beta \ln\left(\frac{k^2}{\Lambda^2}\right)} + \quad ?$$

$\uparrow$

Power correction present?



. 5. The values of $c_n^{\rm lat}$, $(n = 1 \ldots, 8)$ obtained by the stochastic method: elementary $W_{1\times1}(\beta)$ plaquette :cles) and double $W_{2\times2}(\beta)$ plaquette (crossed-circles).

compatible with the one of the continuum schemes and the second parameter r' is the same order of magnitude as the one in MS scheme. We also remark that we do t know the best physical coupling in which the renormalon expression (22) is best scribed, i.e. with smaller subleading corrections at finite order.

Considering all this we conclude that there are good indications from our lattice culation of the presence of the renormalon singularity in a continuum coupling for elementary plaquette expectation $W_{1\times1}(\beta)$.

n order to corroborate the indication that the perturbative expansions of the plaquette ectations are affected by the renormalon singularity we have computed also the fficients of the double plaquette (see Fig. 5). For large order the two coefficients w in a similar way as expected by the universality of the renormalon singularity.

Final considerations

Ve used a numerical method to obtain long perturbative expansions in four dimen- nal Yang–Mills theory with a lattice regularization. The expansion parameter is the

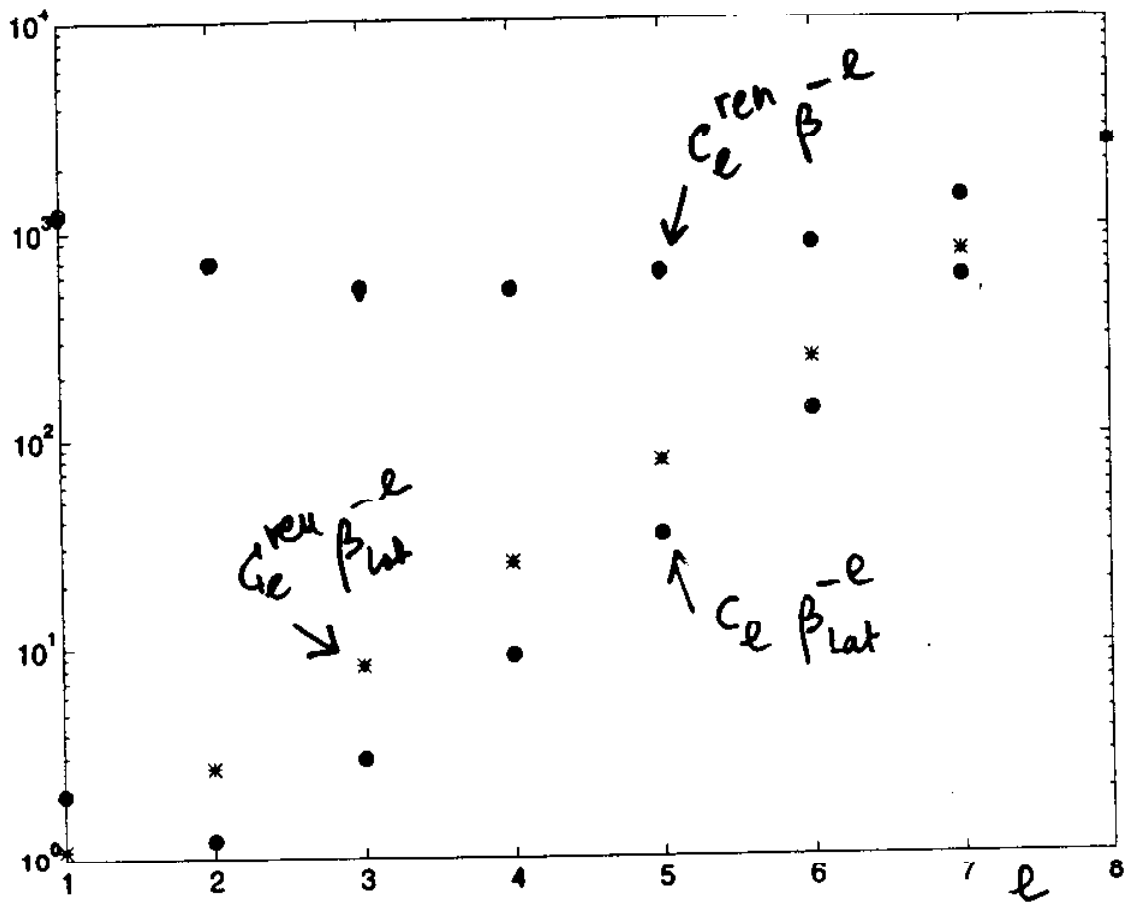


Fig. 4. Coefficients of the elementary plaquette $W_{1\times1}(\beta)$. The values of c_n^{lat}, $(n = 1\ldots,8)$ obtained by the stochastic method (circles) together with renormalon coefficients c_n^{ren} (crossed-circles) and C_n^{ren} with $r = 1.8545$, $r' = 1.667$ (stars).

the lattice and the $\overline{\text{MS}}$ coupling at the natural scale $q = \pi/a$ has been recently obtained at three-loop level [16]

$$r = 1.8545, \qquad r' = 1.667 . \tag{25}$$

In Fig. 4 we plot the coefficients C_n^{ren} in (24) with r and r' given in (25). The growth of C_n^{ren} is closer to the one of c_n^{lat}.

For the other two continuum schemes only the one-loop relation with the lattice coupling is known, i.e. only the parameter r is known. We tried to determine the two parameters r and r' by a fit. Namely by minimizing the quantity

$$\sum_{n=n_0}^{8} |C_n^{ren}(r,r') - c_n^{lat}|$$

with $n_0 = 4, 5$ we find $r = 2.4$ and $r' = 5.2$. The value of r is quite close to the value $r = 2.25$ obtained in the static potential scheme. The second parameter r' is larger

High frequency part of $\alpha_s(k^2)$

Large pert. order calculations in LGT

by parallel computers APE F. Di Renzo E. Onofri G.M. NP 457 (95)

Wilson action $S[U] = \frac{\beta}{6} \sum_p \mathrm{Tr}\left(U_p + U_p^+\right)$ SU(

$\beta/6 = \frac{1}{4\pi\alpha_s}$ $U_\mu(x) = \exp a A_\mu(x) \frac{1}{\sqrt{\beta}}$

a = lattice spac. $Q = \frac{\pi}{a}$ UV-cutoff

Measured $W \equiv 1 - \frac{1}{3}\langle \mathrm{tr}\, U_p \rangle = \langle \alpha_s \mathrm{tr}\, F^2 \rangle \frac{1}{Q^4}$

OPE $W = W_0 + \frac{\Lambda^4}{Q^4} W_4 + \cdots$ $\Lambda^2 \simeq Q^2 e^{-\beta/6b_0}$

W_4 "genuine" gluon cond. $\delta = 4$

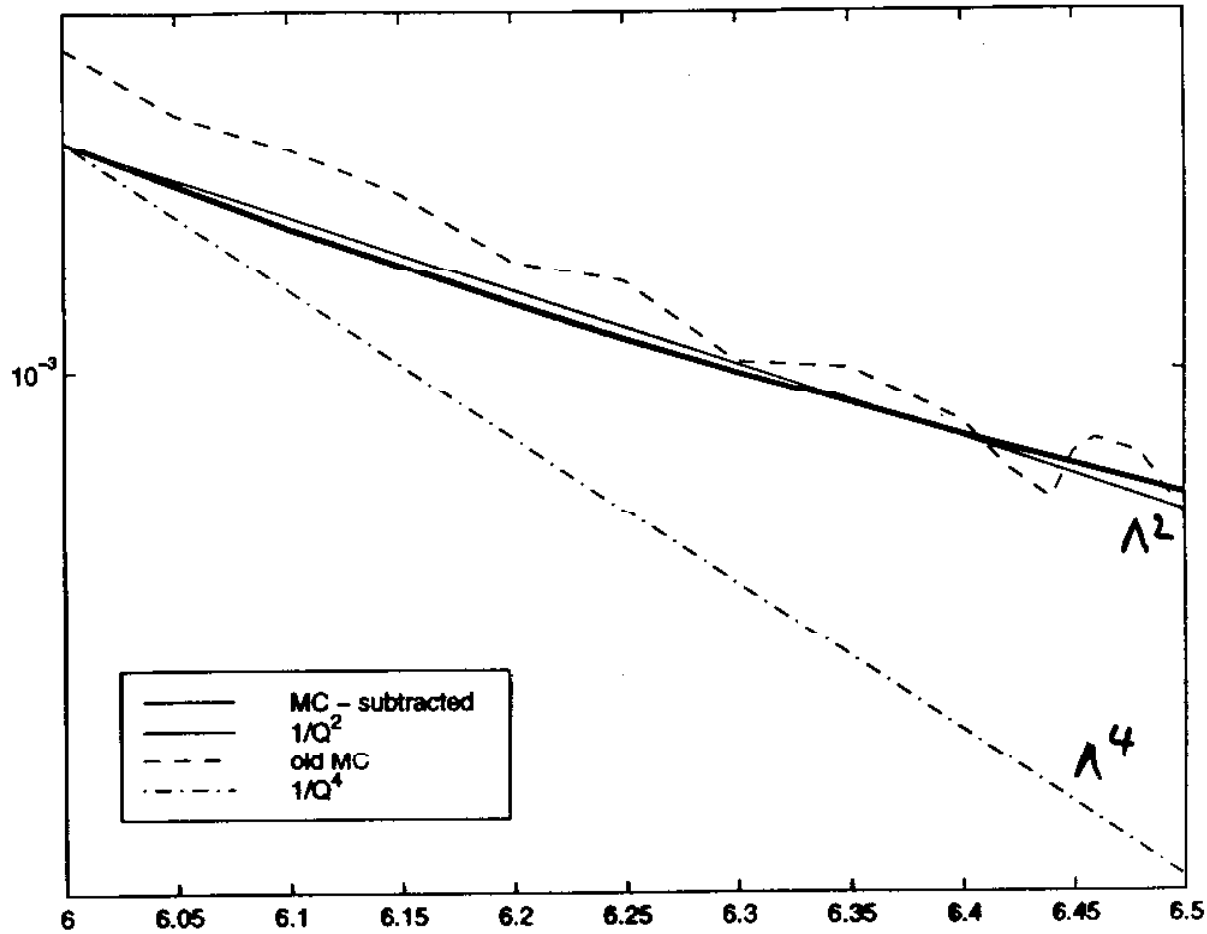
Λ^2 missing

W_0 contains pert. exp. UV diverg. $\delta = 4$

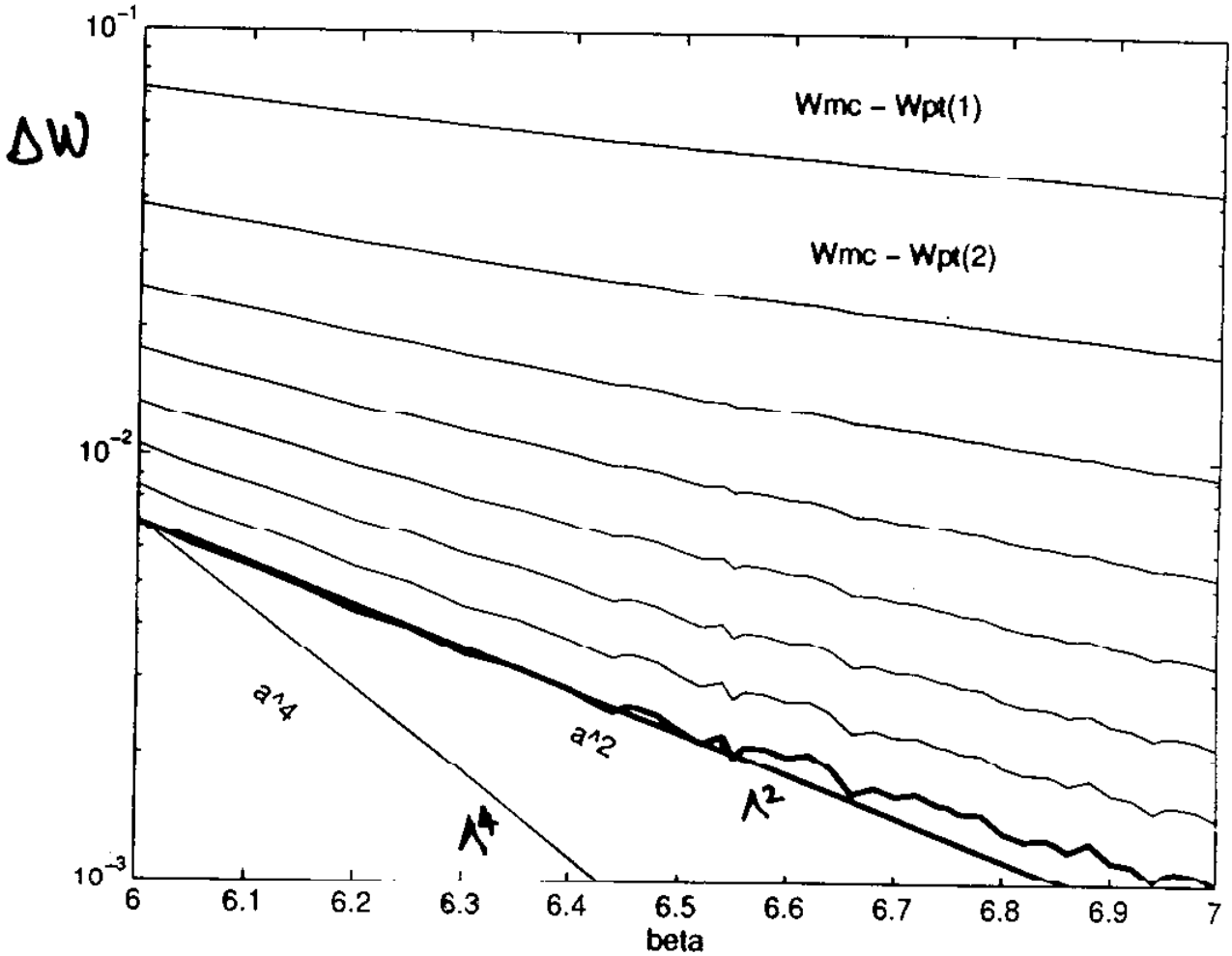
$W^{PT} = \sum_{\ell=1} c_\ell \beta^{-\ell}$ $\beta/6 = 1/4\pi\alpha_s$ $\alpha_s = \alpha_s(Q)$

c_ℓ computed for $\ell = 1, 2, 3, 4, 5, 6, 7, 8$

Non-convergent expansion $\Leftrightarrow$ IR renormalon $\delta = 4$


10^{-3}
6
6.05
6.1
6.15
6.2
6.25
6.3
6.35
6.4
6.45
6.5
MC – subtracted
$1/Q^2$
old MC
$1/Q^4$
Λ^2
Λ^4

$$\Delta W = W_{\text{Monte Carlo}} - \sum_{1}^{L} c_\ell \, \beta^{-\ell}$$



$$\Lambda^2 \sim Q^2 e^{-4\pi/\beta_0 \alpha_s}$$

$$W = W_0 + \Lambda^4/Q^4\, W_4 + \cdots$$

$$W \sim \int_0^{Q^2} \frac{k^2 dk^2}{Q^4} f(k^2/\Lambda^2) \qquad \text{ren. gr. inv.}$$

ITEP $k^2 \gg \Lambda^2 \Rightarrow W_0$

$$W_0 \sim \int_{r\Lambda^2}^{Q^2} \frac{k^2 dk^2}{Q^4} \left(\alpha_s(k^2) + \cdots \right) \qquad r \gg 1$$

Use $\alpha_s(k^2)$ at two loops $\quad 4\pi \alpha_s^{\text{2-loops}}(Q^2) = \dfrac{6}{\beta_{\text{cont.}}}$

$$W_0 \sim \sum_{\ell=1} \beta_{\text{cont}}^{-\ell} \left(c_\ell^{\text{ren}} + O(\Lambda^4/Q^4) \right)$$

$$c_\ell^{\text{ren}} = \mathcal{N}\, \Gamma(\ell+\gamma+1) \left(\frac{12 b_0}{4} \right)^\ell \qquad \gamma = \frac{102}{121}$$

Compare c_ℓ^{ren} with lattice c_ℓ^{lat}

- Effect of finite volume $\Rightarrow$ small for $\ell \lesssim 8$
  Di Renzo E Onofri G.M. NP (97)

- cont. coupling β_{cont}, lattice coupling β_{lat}

$$\beta_{\text{cont}} = \beta_{\text{lat}} - r - r'/\beta_{\text{lat}} \qquad \text{2-loops}$$

$\Rightarrow$ good fit with (r, r') $\quad \overline{\text{MS}} \leftrightarrow$ lattice reg.

$$W = W_0 + \Lambda^4/Q^4 W_4 + \cdots$$

Attempt to construct W_0

$$"W_0" = \sum_1^8 c_\ell \beta^{-\ell} + \delta W_0$$

$$\delta W_0 = N \int_{r\Lambda^2}^{Q^2} \frac{k^2 dk^2}{Q^4} \alpha_s^{(2\text{-loops})}(k^2) - \sum_1^8 c_\ell^{ren} \beta^{-\ell}$$

NB ambiguity in δW_0 is $O(\Lambda^4/Q^4)$

δW_0 negligeble for $\beta = 6 - 6.5$

<u>Results</u>

1) c_ℓ $\ell = 1, \cdots 8$ $\Longleftrightarrow$ IR renormalon $\delta = 4$

2) W_0 seems to contain Λ^2/Q^2 contrib.

Caveat: - finite volume effect
- finite order calcul.

Result 2) - compatible with OPE ?

1) $C_e \Leftrightarrow$ IR renorm. $\delta = 4$ $\quad \Lambda^4/Q^4$ (OPE)

2) W_0 seems to contain Λ^2/Q^2 terms (OPE?)

G. Grunberg (1997) has suggested explanation compatible with OPE

Ball Braun Beneke NP B452 (95)

suppose for $k^2 \gg \Lambda^2$ $\quad \alpha_s(k) \simeq \dfrac{1}{b_0 \ln \frac{k^2}{\Lambda^2}} + O\left(\dfrac{\Lambda^2}{k^2}\right)$

non-analytical beta function in LGT

$$\text{"}W_0\text{"} \sim \int_{r\Lambda^2}^{Q^2} \frac{k^2 dk^2}{Q^4} \alpha_s(k) \begin{cases} \text{1,2 loops} \Rightarrow \left(\frac{\beta_0}{4}\right)^2 \ell! + \frac{\Lambda^4}{Q^4} \\ \frac{\Lambda^2}{k^2} \Rightarrow \frac{\Lambda^2}{Q^2} - r\frac{\Lambda^4}{Q^4} \end{cases}$$

Λ^2/Q^2 scheme dependent, definition of α_s

OPE ok

Phenomenologically relevant? No for good α_s

CONCLUSIONS

Dispersive method to introduce

$$\alpha_s(k^2)$$

at any scale

$\alpha_s^{low}(k^2) \Rightarrow$ • OPE
• univ. power corrections

e^+e^-
DIS
DY

(hope) strong $\left(\frac{1}{Q^2}\right)^p$ variation $Q > \Lambda_{QCD}$

$$\alpha_s^{high}(k^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{k^2}{\Lambda^2}\right)} + \cdots \quad \text{powers?}$$

- scheme dependence
- non observable, smooth variation in Q^2 ?
- disturbing OPE connection