Top-Antitop Production Close to Threshold at NNLL Order in vNRQCD

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based on collaboration with

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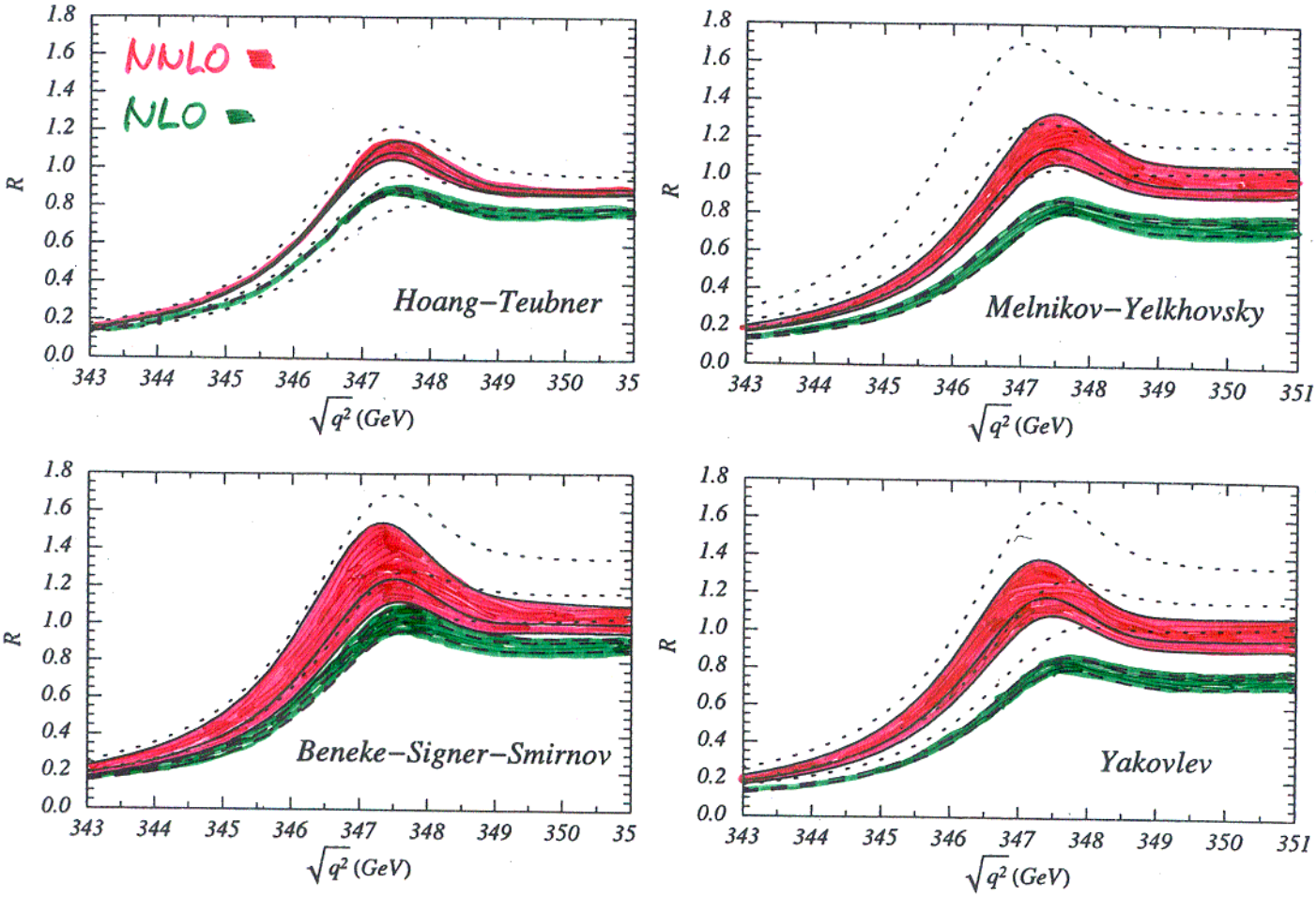


hep-ph/0001286

NNLO:

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \times \left\{ 1 \ (LO) \ ; \ \alpha_s, v \ (NLO) ; \alpha_s^2, \alpha_s v, v^2 \ (NNLO) \right\}$$

→ results obtained using concept of EFT's ($m \gg mv \gg mv^2$)

Conclusions:

☺ ⊗ good prospects for m_t : δm_t for threshold masses ~ 100 MeV

⊗ large uncertainties in the normalization :

☹ $\frac{\delta\sigma}{\sigma} \sim 20\%$ ⇒ bad prospects for measurement of α_s, g_{ttH}, Γ_{top}, formfactors

$\Delta\sigma_{NLO} \sim \Delta\sigma_{NNLO}$

But: $\boxed{m_{top} \gg \underbrace{m_{top} v}_{\langle p_{top}\rangle} \gg \underbrace{m_{top} v^2}_{\langle E_{top}\rangle} > \Gamma_{top} \gg \Lambda_{QCD}}$

$$v = \left(\frac{\sqrt{s} - 2m_t + i\Gamma}{m_t}\right)^{1/2}$$

$$|v|^2 > \frac{\Gamma_t}{m_t}$$

and

non-perturbative effects small

$$\left(\frac{\Delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right)_{\langle G^2\rangle} \ll 1\% \qquad \text{(Fadin, Yakovlev)}$$

$\Rightarrow$ $t\bar{t}$ dynamics should be entirely perturbative

What is wrong?

$\hookrightarrow$ $v \ll 1$, possible large higher order log-terms not taken into account

$\sim (\alpha_s \ln v)^n$ $\Rightarrow$ e.g. Lamb shift $\delta E = m\alpha^5 \ln\alpha$ ($\sim$NNLL!)

$\Rightarrow$ need (real) consistent EFT that can resum $\ln v$'s through RGE's that run from m down to non-relativistic scales

problem: mv, mv^2 are correlated scales

$\Rightarrow$ addressed systematically by Luke, Manohar, Rothstein, Stewart

"vNRQCD"

"Standard Program for EFT calculations"

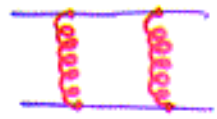
- Identify relevant momentum regions + d.o.f's
- Write down most general Lagrangian with d.o.f's that are on-shell + match to full QCD at $\mu = m$
- Determine RGE's of Operators + evolve Wilson coefficients to non-relativistic scales

non-trivial because non-rel. dynamics involves mv, mv^2 at the same time

$\oplus$ correlation $mv^2 = \frac{(mv)^2}{m}$

- Calculate matrixelements at the low scales (Solve Schrödinger equation)

Practical Requirement: use dimensional regularization



Momentum Regions (Beneke, Smirnov)

			on-shell d.o.f's
"hard"	$(k^0, \vec{k}) \sim$	(m, m)	
"soft"		(mv, mv)	Gluons $\longrightarrow A^\mu_q(x)$
"potential"		(mv^2, mv)	Quarks $\longrightarrow \psi_q(x)$, $\chi_q(x)$
"ultrasoft"		(mv^2, mv^2)	Gluons $\longrightarrow A^\mu(x)$

energy/momentum $\sim mv \longrightarrow$ index

energy/momentum $\sim mv^2 \longrightarrow$ dynamical

$\Longrightarrow$ write down most general Lagrangian from A^μ_q's, Q_q's, A^μ
"vNRQCD"

$\oplus$ loops that involve A^μ_q's or only Q_q's $\longrightarrow \mu_s = mv$
loops that involve only A's and Q_q's $\longrightarrow \mu_u = mv^2$

$\Big\}$ $\mu_u = \frac{\mu_s^2}{m}$ Correlation

$\Longrightarrow$ a) match vNRQCD to full QCD at $v = 1$ $(\mu = m)$
$\hat{=}$ "integrate out hard effects, soft quarks"

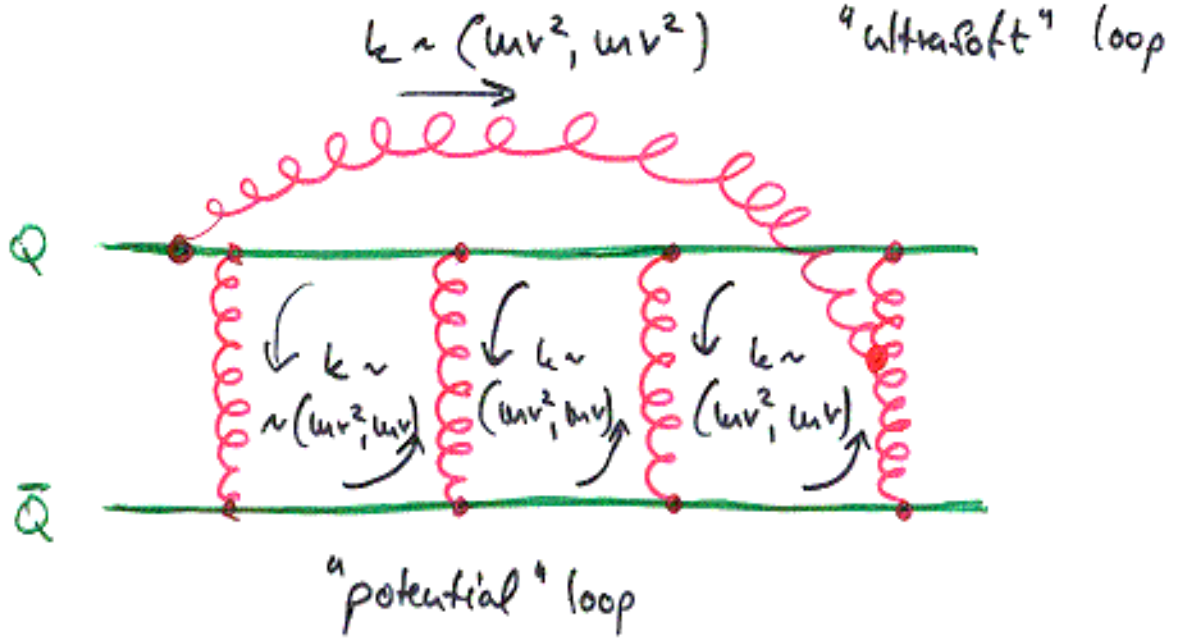
b) calculate RGE's of all vNRQCD operators (that are needed)
(use power counting)

c) scale down from $v = 1$ to $v \sim \alpha_s \sim \langle v \rangle$
(soft logs: $\longrightarrow mv$
ultrasoft logs: $\longrightarrow mv^2$)

d) integrate out soft gluons (cannot be on-shell in non-relativistic $Q\bar{Q}$ systems)

$\longrightarrow$ "Schrödinger equation"

→ Why cannot we integrate out the scale mv while keeping mv^2 dynamic?



→ the energy component in the loops responsible for the Coulombic binding is <u>ultrasoft</u> (mv^2)

⇒ the potential loops cannot be integrated out while keeping ultrasoft loops dynamic

→ through the heavy quark equation of motion: energy and momentum are correlated

$$\Rightarrow \quad E \sim \frac{p^2}{m} \qquad mv^2 = \frac{(mv)^2}{m}$$

the soft scale and the ultrasoft scale depend on each other

$$\mu_u = \frac{\mu_s^2}{m}$$

NNLLO:

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i$$

$$\times \left\{ 1\ (\mathrm{LL})\,;\ \alpha_s, v\ (\mathrm{NLL})\,;\ \alpha_s^2, \alpha_s v, v^2\ (\mathrm{NNLL}) \right\}$$

Schrödinger equation in D dimensions

$n = D-1$

$$\left[\frac{p^2}{m} - \frac{p^4}{4m^3} - (\sqrt{s} - 2m)\right] \tilde{G}(p,p') + \int \frac{d^n k}{(2\pi)^n} V(p,k)\, \tilde{G}(k,p') = (2\pi)^n \delta^{(n)}(p-p')$$

Potentials for S-waves ($\gamma \to t\bar{t}$)

$(k = p - p')$

$$V_c(p,p') = \frac{-4\pi V_c(v)}{|p-p'|^2} \left\{ 1 + \frac{\alpha_s}{4\pi}\left(-\beta_0 \ln\left(\frac{k^2}{\mu_s}\right) + \dots\right) + \left(\frac{\alpha_s}{4\pi}\right)^2 \dots \right\}$$

$$V_\delta(p,p') = \frac{4\pi}{m^2} V_\delta(v)$$

$$V_r(p,p') = \frac{4\pi (p^2 + p'^2)}{2m^2 |p-p'|^2} V_r(v)$$

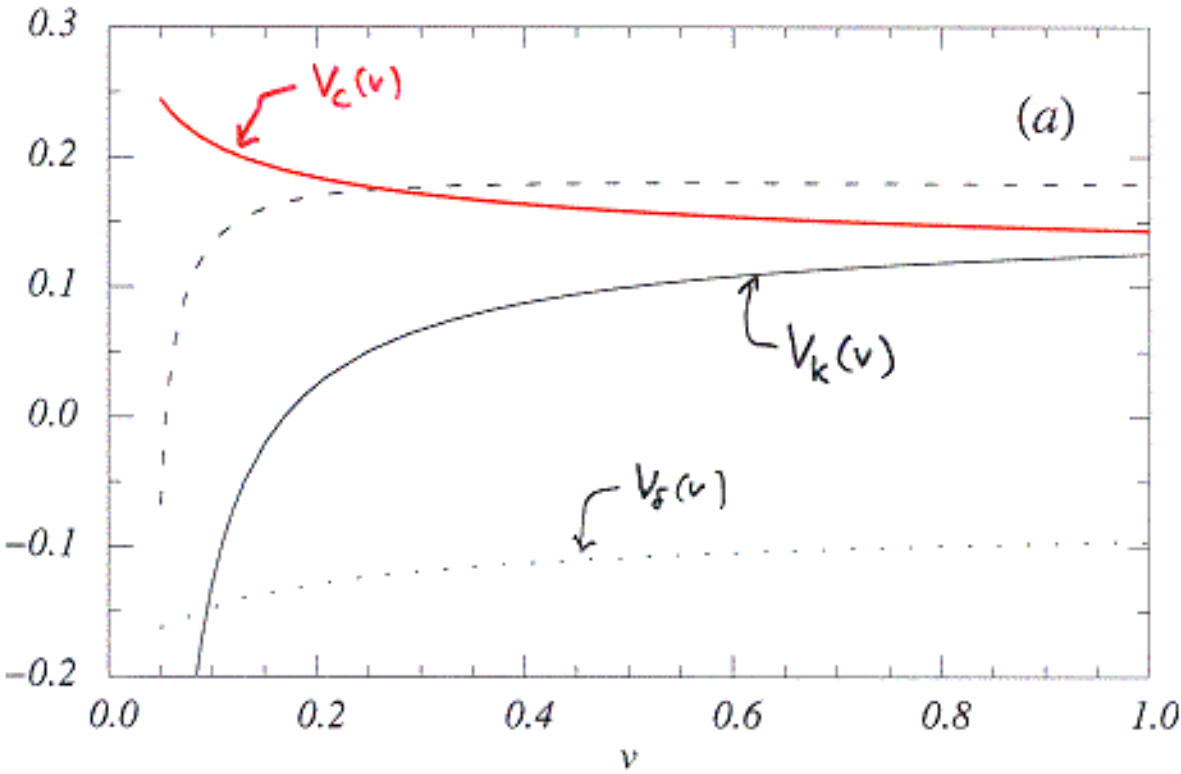
$$V_k(p,p') = \frac{\pi^2}{m|p-p'|} V_k(v)$$

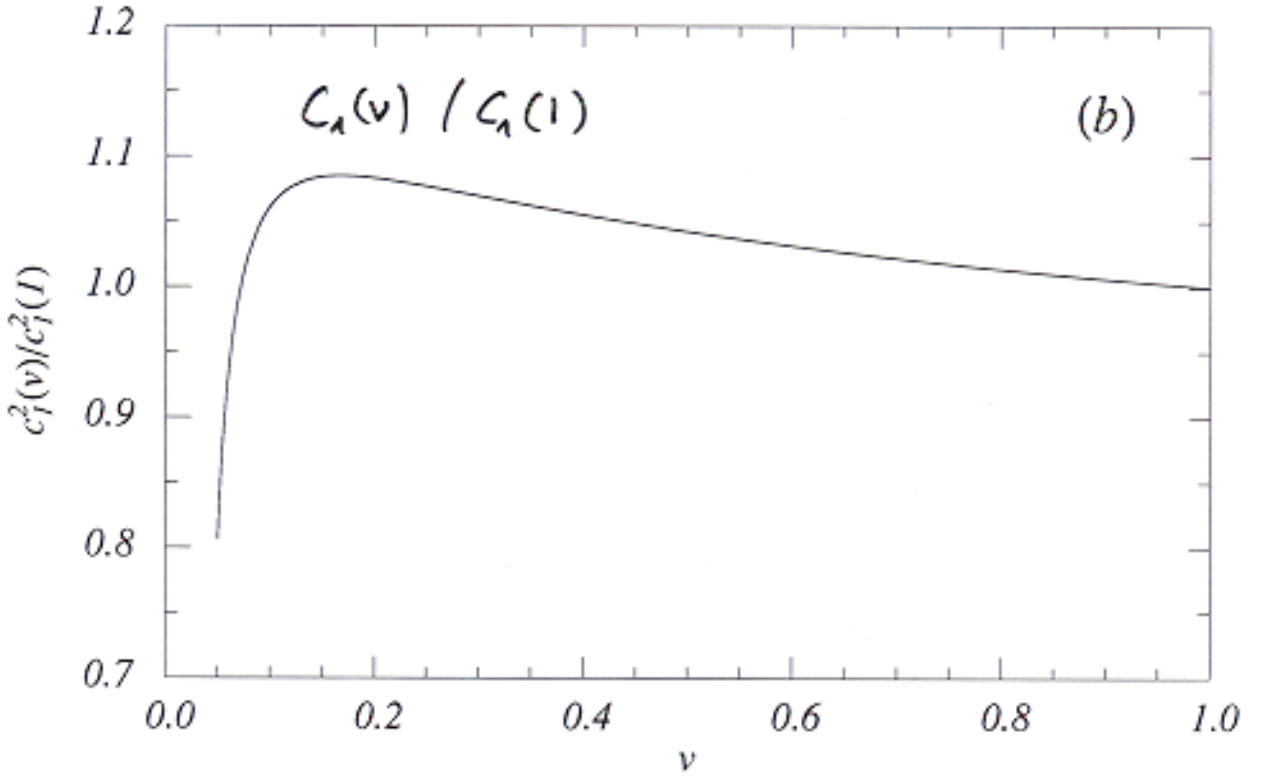
Cross Section

$$R \sim C_1(v)\ \mathrm{Im}\, G(0,0)$$

- Solve $\left[\frac{p^2}{m} - (\sqrt{s} - 2m)\right] \tilde{G} + \int \frac{d^n k}{(2\pi)^n} \underbrace{V_c}_{\text{Coulomb}} \tilde{G} = (2\pi)^n \delta^{(n)}$ exactly
- first order time-independent pert. theory for the other corrections

$\rightarrow$ resummation of energy poles not necessary!
(effect of higher orders $\lesssim 1\%$)


0.3
0.2
0.1
0.0
−0.1
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
v
(a)
$V_c(v)$
$V_k(v)$
$V_f(v)$



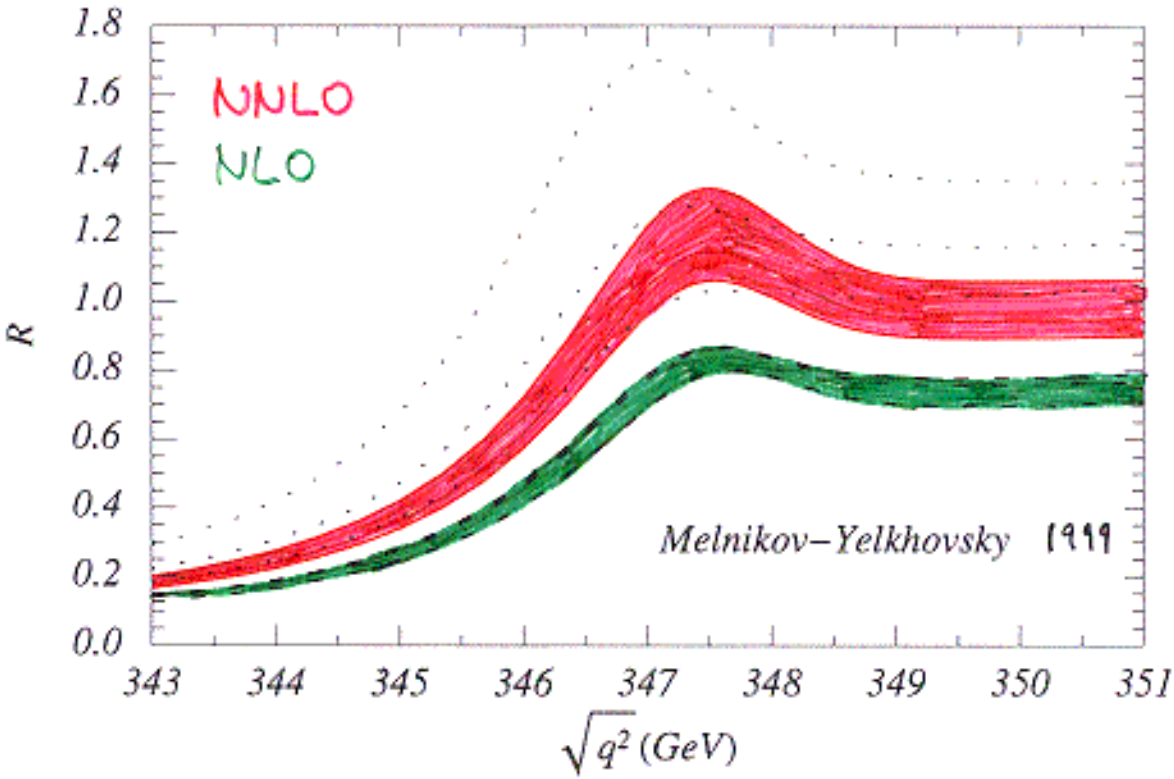
→ 2-loop matching conditions ✓

NLL evolution ✓

NNLL evolution w.ip.

but is not expected to give considerable changes

(should be comparable to using 2-loop vs. 3-loop running of α_s)

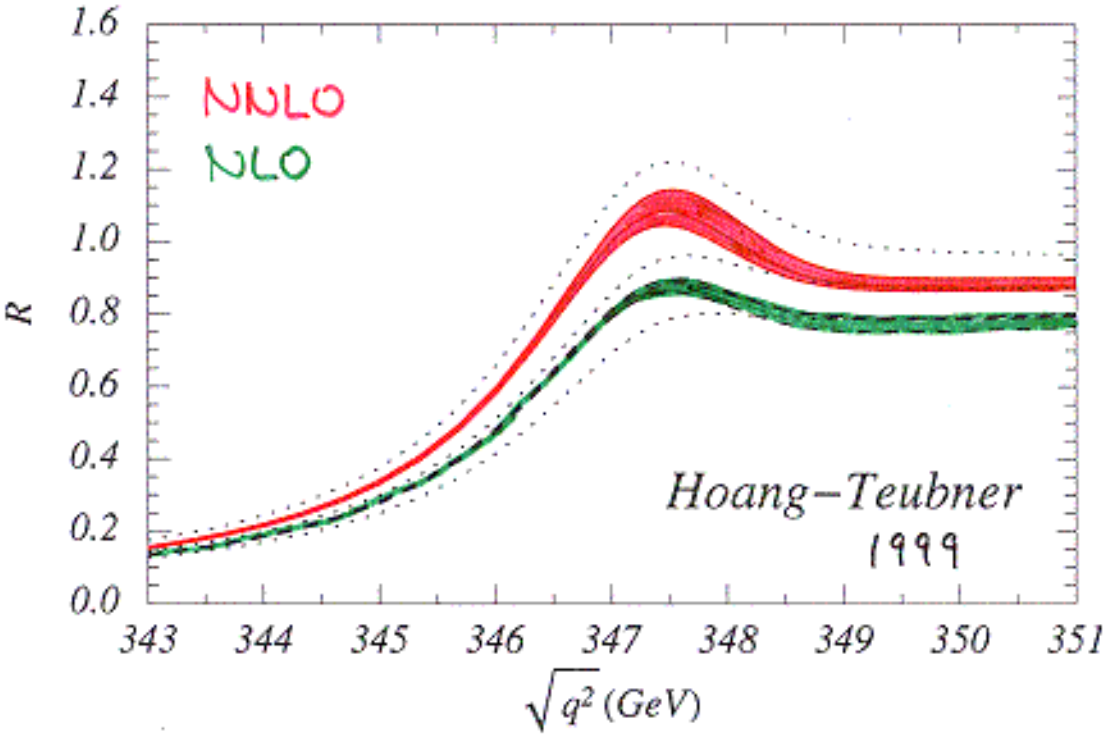


NNLO: $\mu = 15-60$ GeV
NLO

$m_t^{kin} = 173.1$ GeV

$\Gamma_t = 1.43$ GeV

$\alpha_s(m_Z) = 0.119$

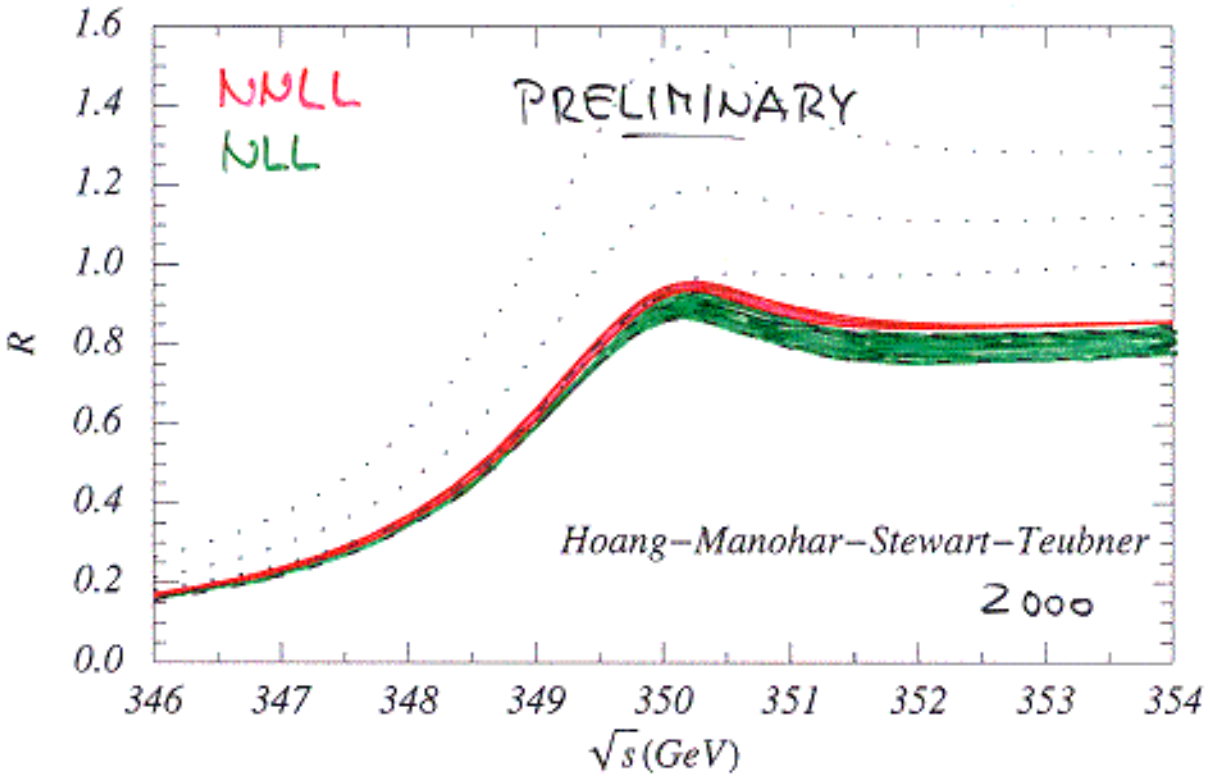


NNLO $\mu = 15 - 60$ GeV
NLO

$m_t^{1S} = 173.3$ GeV

$\Gamma_t = 1.43$ GeV

$\alpha_s(M_Z) = 0.119$



NNLL: $\mu_s = m_t v$, $\mu_u = m_t v^2$ } $v = 0.1 - 0.4$ LL, NLL; $(0.15 - 0.4)$ NNLL

$m_t^{1S} = 175$ GeV

$\Gamma_t = 1.43$ GeV

$\alpha_s(M_Z) = 0.118$

$\frac{d\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \sim 2\%$

check: multiple interactions of V_J: $< 1\%$
ultrasoft corrections (Kniehl, Penin): $< 2\%$

$\Rightarrow$ NORMALIZATION UNCERTAINTY REDUCED BY FACTOR 10 !

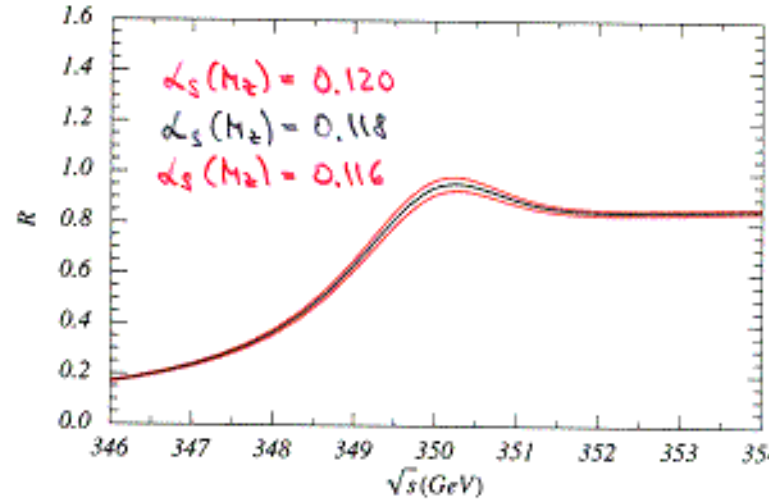
Sensitivity of σ_{tot} on small changes in $\alpha_s(M_Z)$, g_{ttH}, Γ_{top}

α_s larger

$\Rightarrow$ binding force larger

$$V(r) = -\frac{C_F \alpha_s}{r}$$

$\Rightarrow$ σ_{tot} enhanced

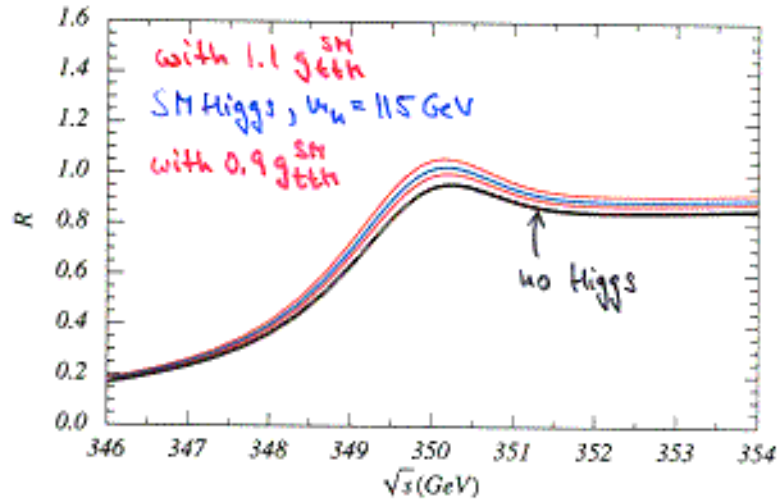


top Yukawa coupling larger

$\Rightarrow$ binding force larger

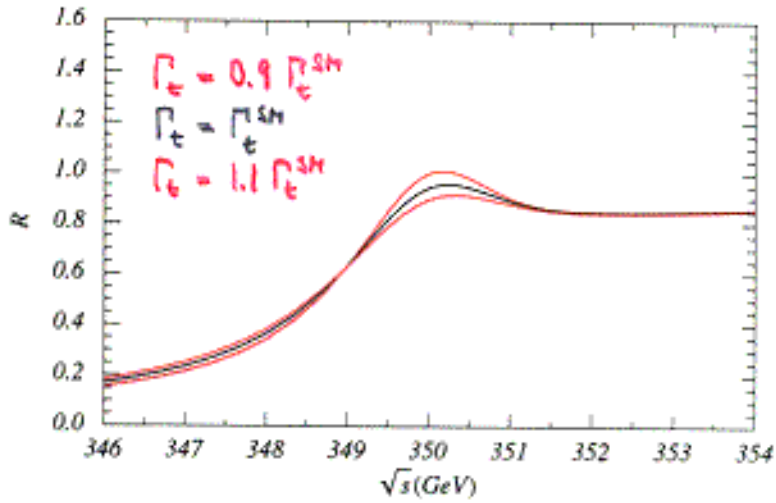
$$V^{Higgs}(r) = -\frac{g^2_{ttH}}{r} e^{-m_H r}$$

$\Rightarrow$ σ_{tot} enhanced



top width smaller

$\Rightarrow$ peak more pronounced



$\rightarrow$ the changes in σ_{tot} caused by the variations of α_s, g_{ttH}, Γ_t shown here are $\gtrsim$ the normalization uncertainties of the NNLL total cross section

$\rightarrow$ luminosity spectrum !!! ☺

$\mathcal{L} \sim 100\,\mathrm{fb}^{-1}$

What can be measured? (personal estimates)

$\delta m_{top} \approx 100\text{-}200$ MeV (NNLO) $\Longrightarrow$ "$\ll 100\text{-}200$ MeV" (NNLL)

$\delta \alpha_s(M_Z) \approx 0.01 \Longrightarrow \sim 0.002$

$\delta g_{ttH}/g_{ttH}$? $\Longrightarrow \sim 5\,\%$

$\delta \Gamma_t/\Gamma_t$? $\Longrightarrow \sim 5\,\%$

- light higgses (e.g. SUSY)

maybe better if optimization procedures are applied

What remains to be done?

$\Longrightarrow$ Many subleading ($\delta\sigma/\sigma \sim 1\%$) effects need to be worked out that have been considered irrelevant in view of large NNLO corrections.

- electroweak effects

  theory of unstable particles $\rightarrow$ distributions

  non-factorizable corrections $\rightarrow$ —"—

  hard ew. corrections

  background

  non-SM (light Higgses)

- more QCD corrections (depends on luminosity)

  status: $\delta_{theory} \approx \delta_{exp}$ ($100\,\mathrm{fb}^{-1}$)

  now! (doable / + done)