Modeling Laser-Plasma Accelerators

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Acknowledgements

Collaborators

- G. M. Tarkenton, IAP
- E. Esarey, LBNL
- C. B. Schroeder, LBNL

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Support

- US DoE
- IAP

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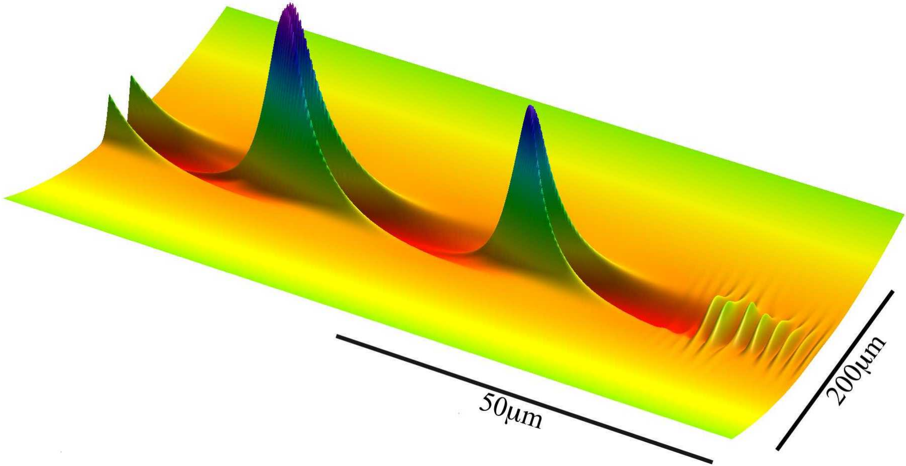

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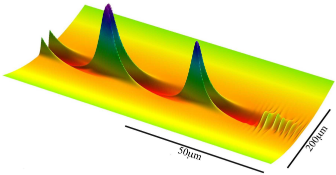

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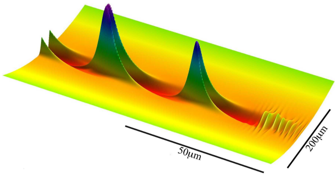

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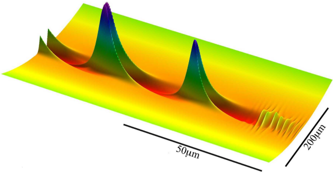

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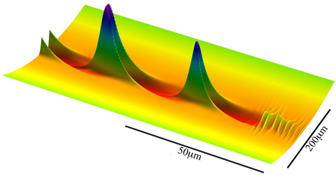

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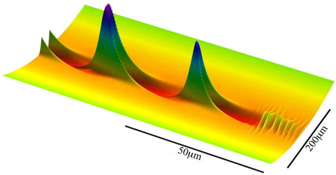

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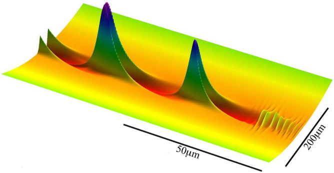

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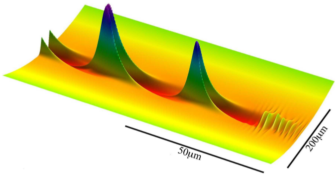

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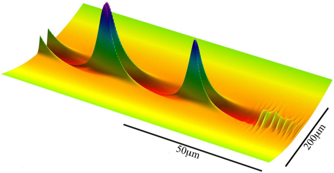

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  - In 2-D, 2×10^6 grid points, 4×10^6 time-steps.

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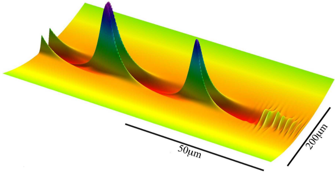
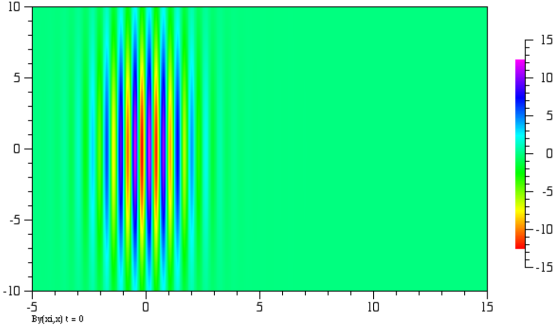


$B_y(\xi, x)$



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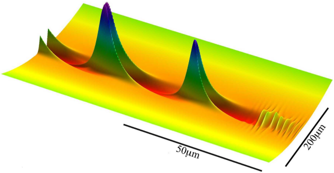
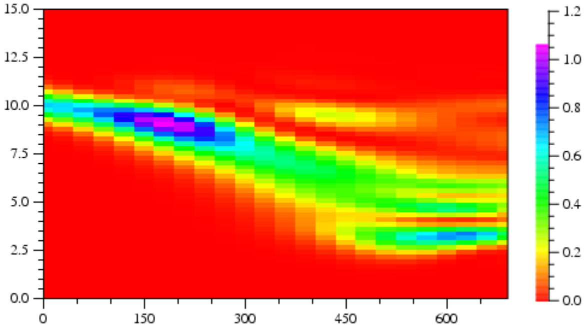


$$|B_y(k, x = 0, t)|^2$$



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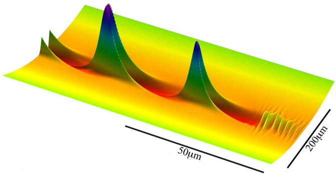

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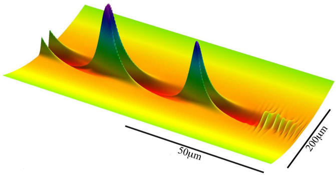

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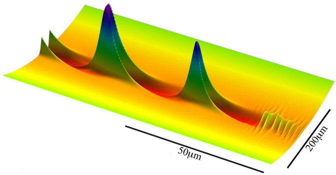

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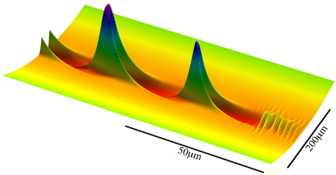

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- We can explore much of the phenomenology by studying 1-D.

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- Conclusions will be (mostly) independent of the plasma model.

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  - LBNL Fluid codes & Vlasov codes, QUICKPIC . . .

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- Numerical parameters:

$$
\begin{aligned}
k_0 \Delta\xi &= 0.0621 \quad (\lambda_0/\Delta\xi \approx 100) \\
\Delta t &= \tfrac{1}{4}\,\Delta\xi
\end{aligned}
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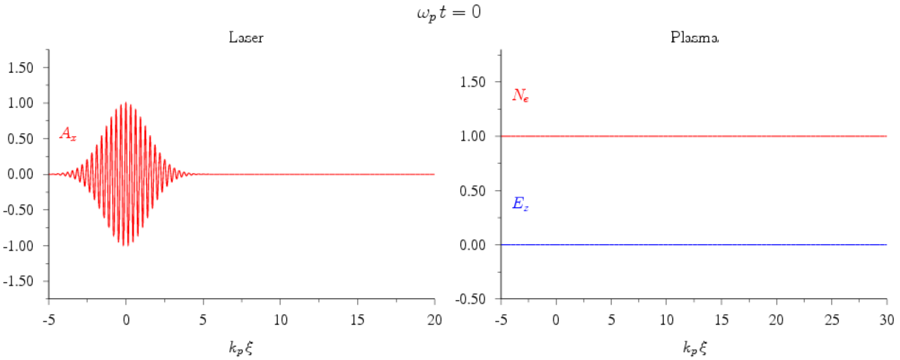

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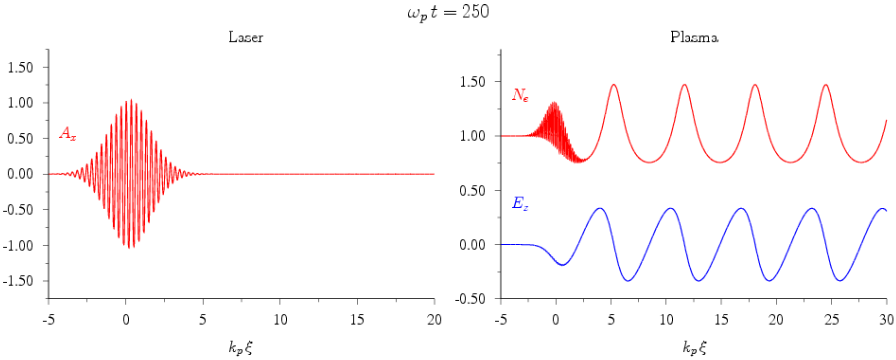

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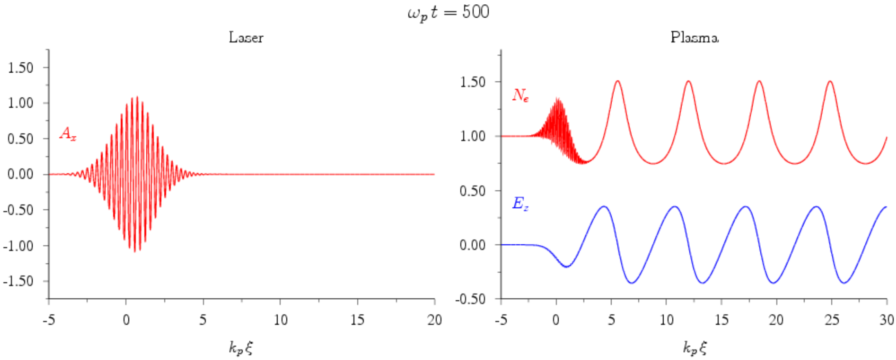

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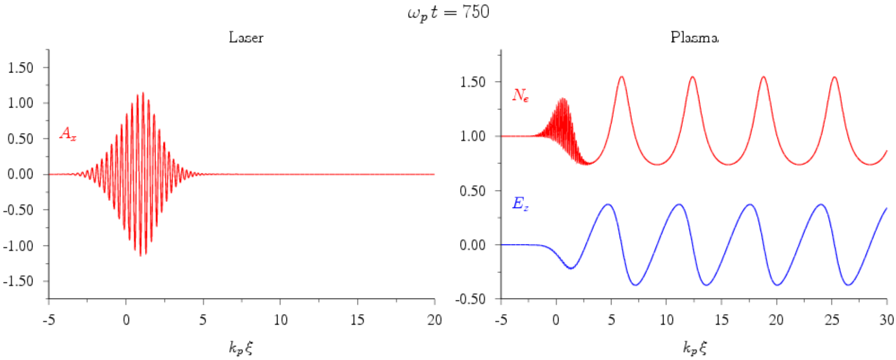

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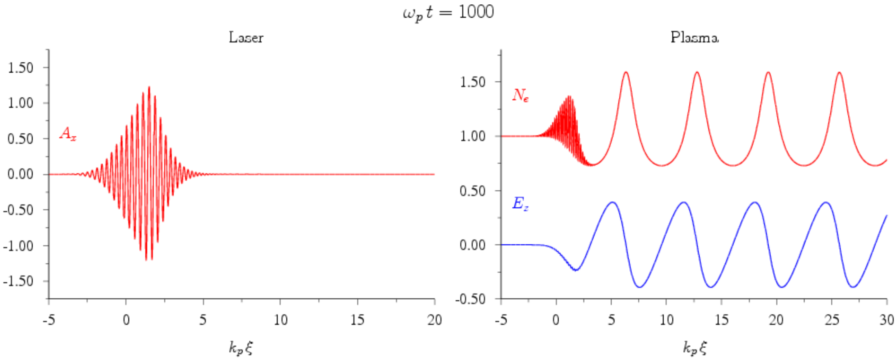

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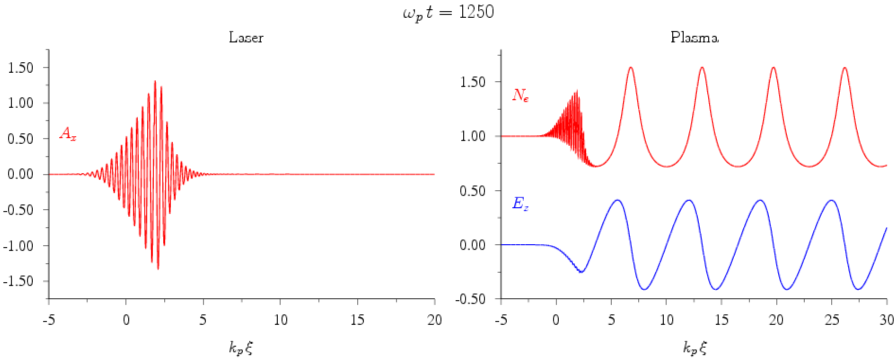

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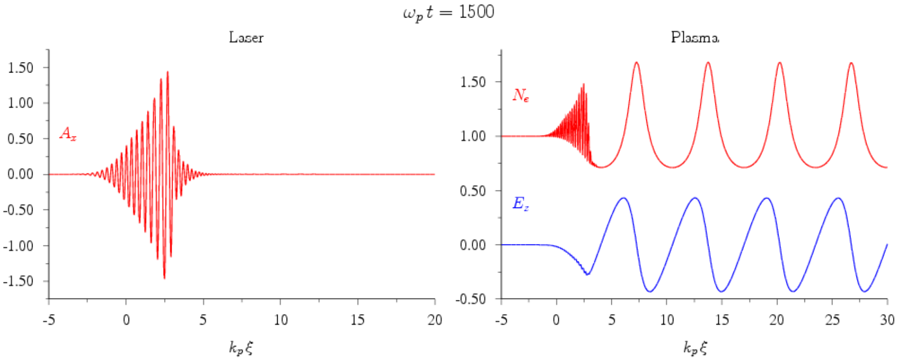

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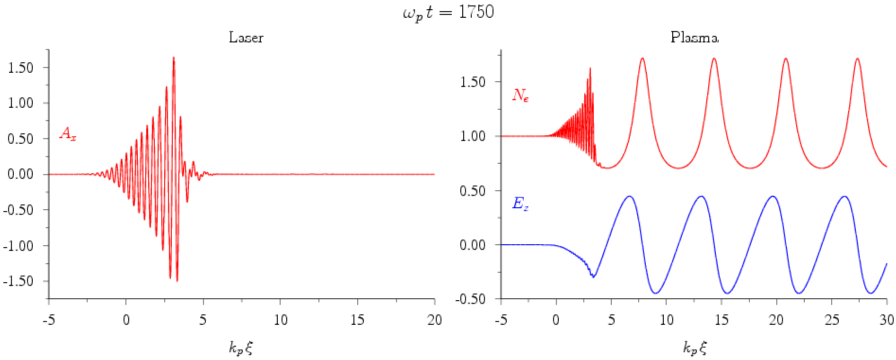

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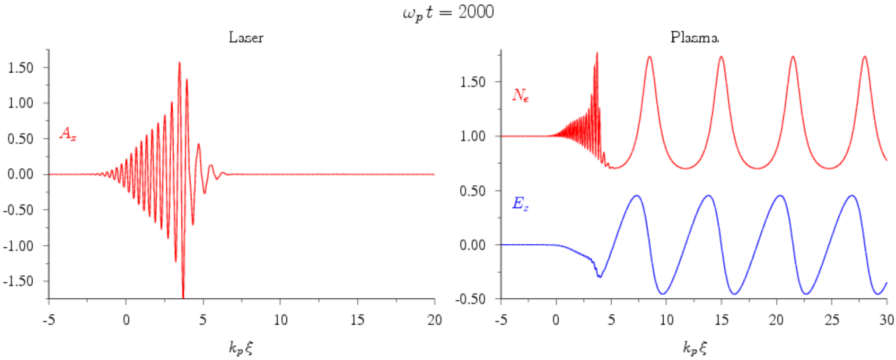

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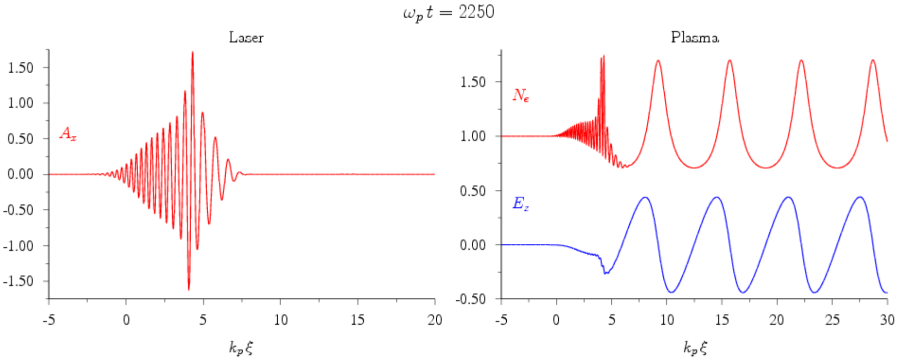

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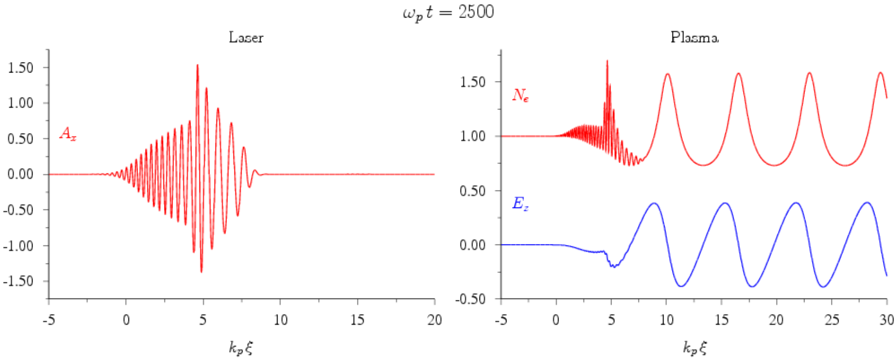

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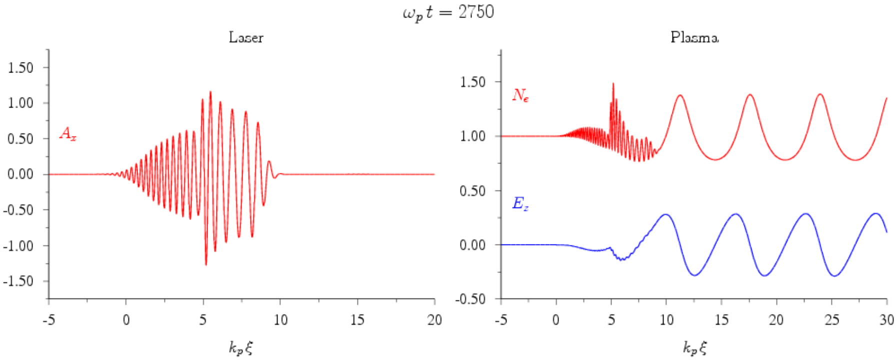

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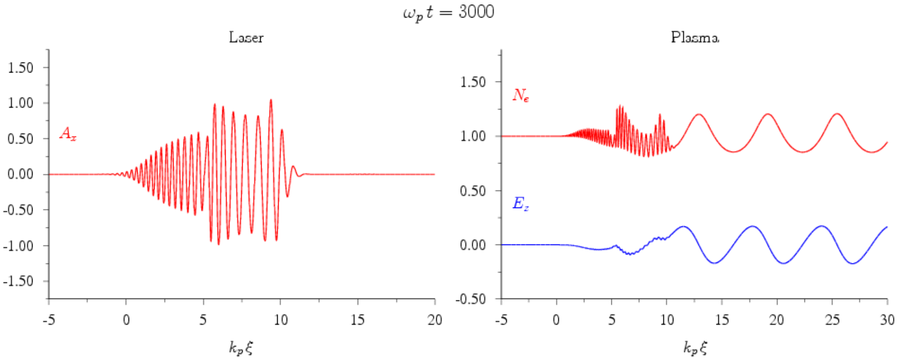

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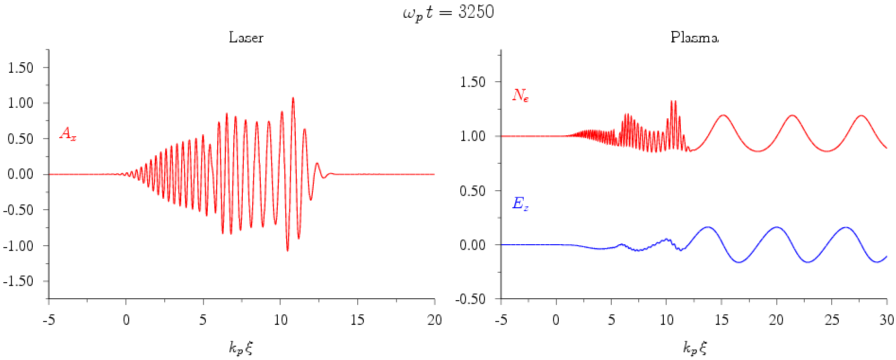

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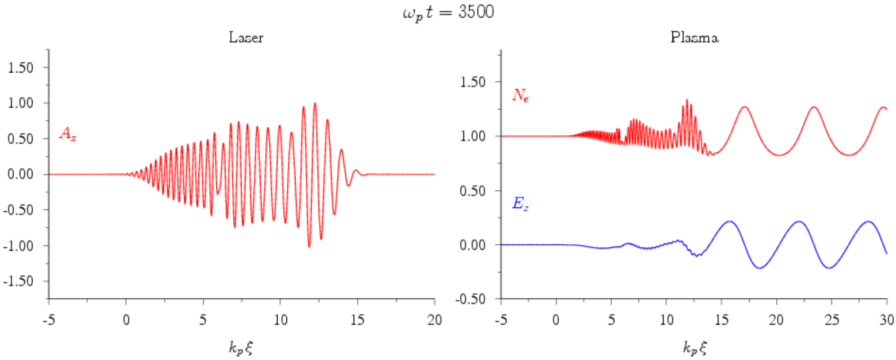

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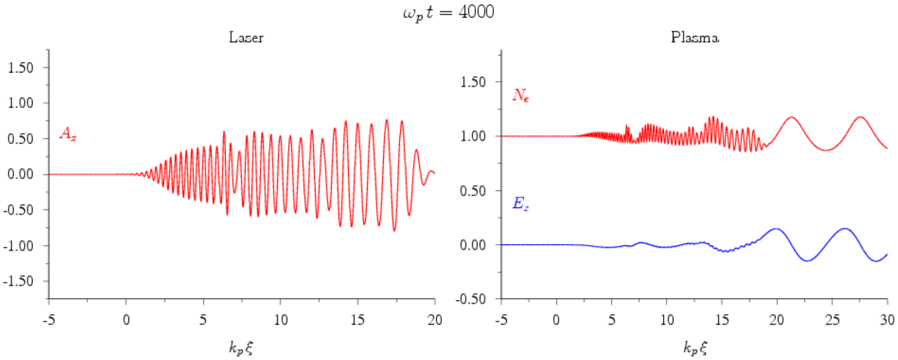

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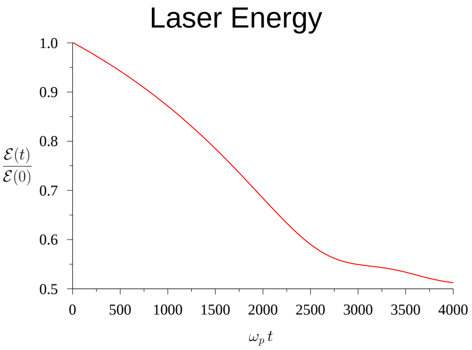

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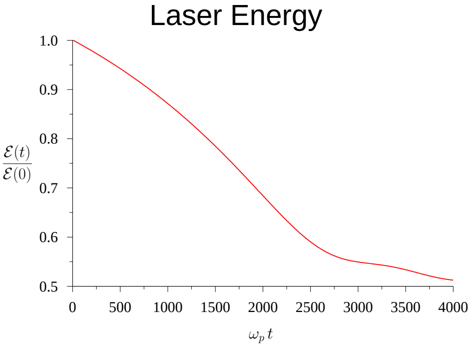

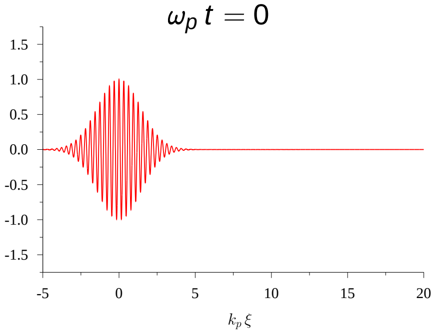

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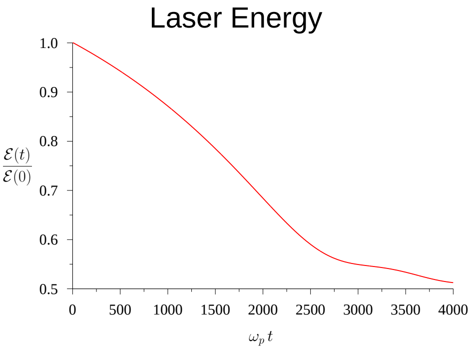

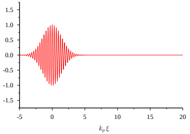

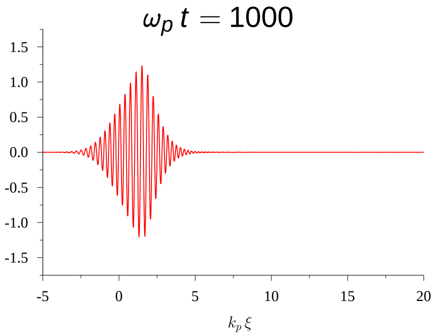

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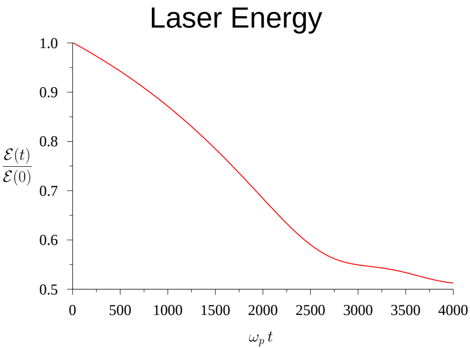

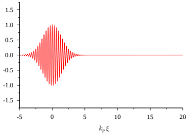

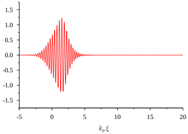

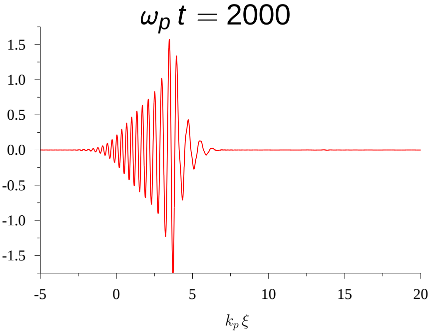

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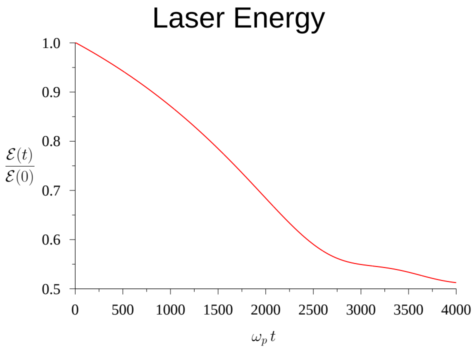

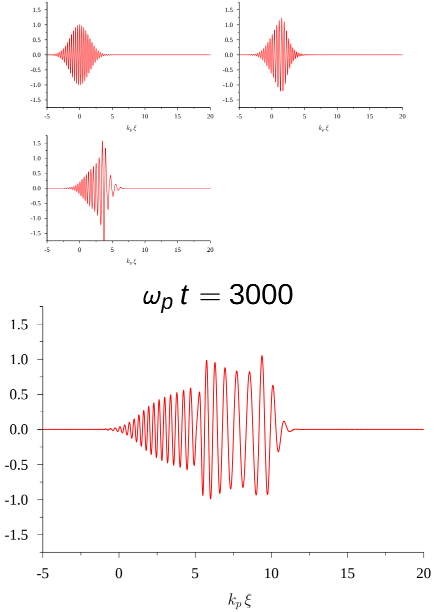

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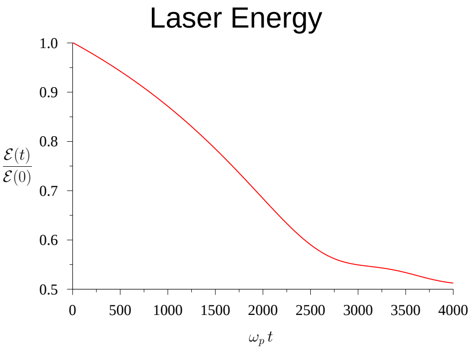

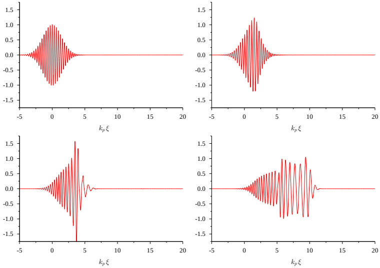

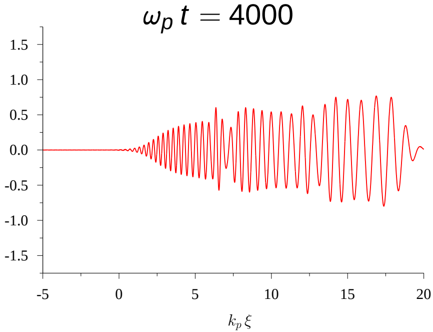

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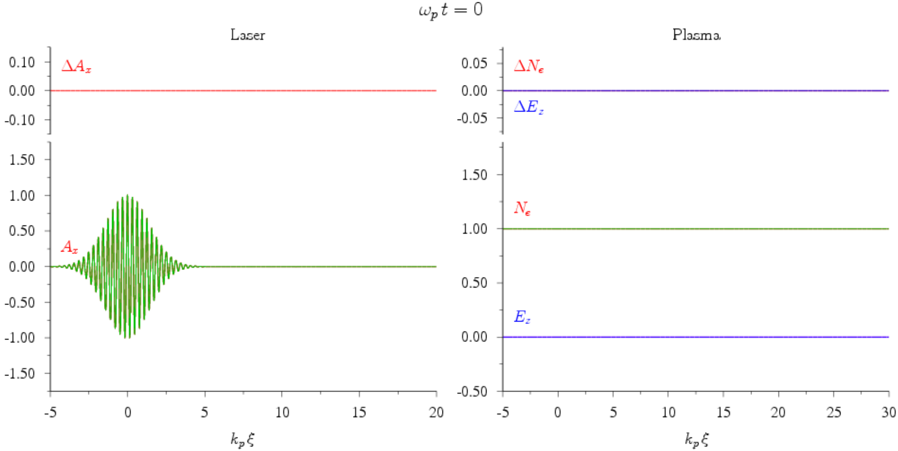

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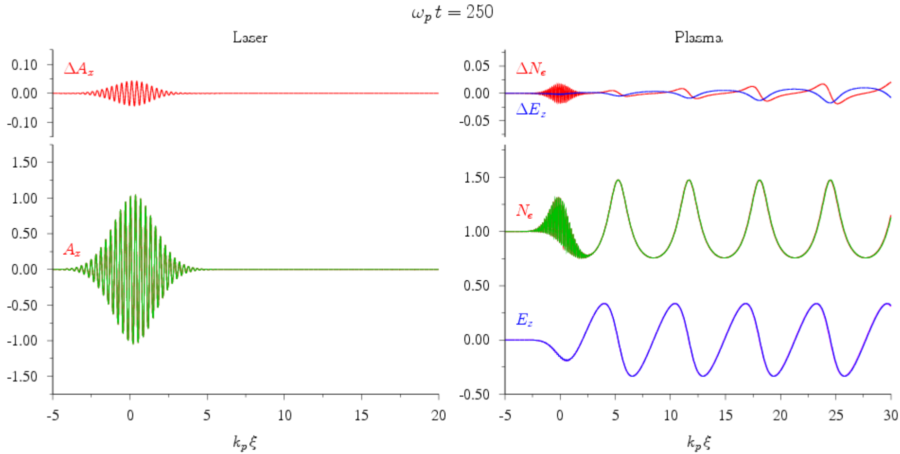

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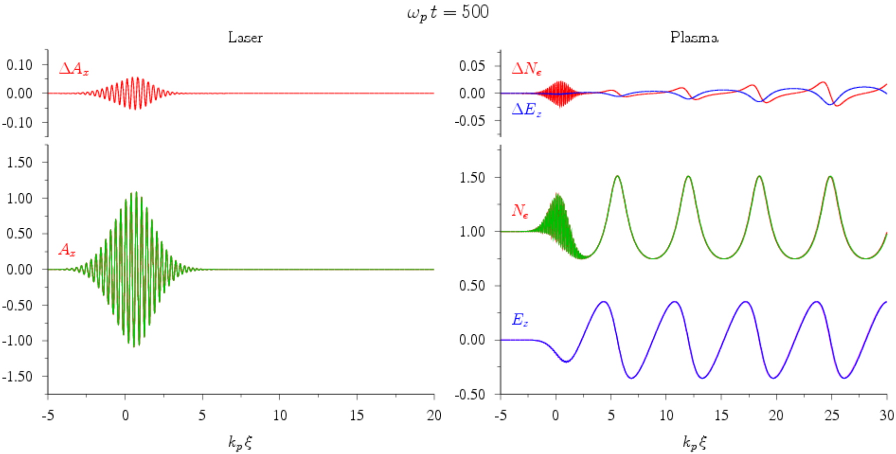

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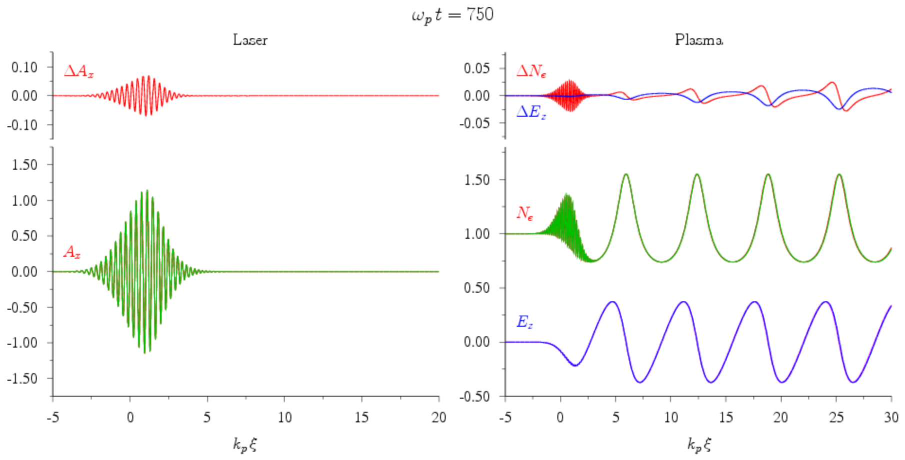

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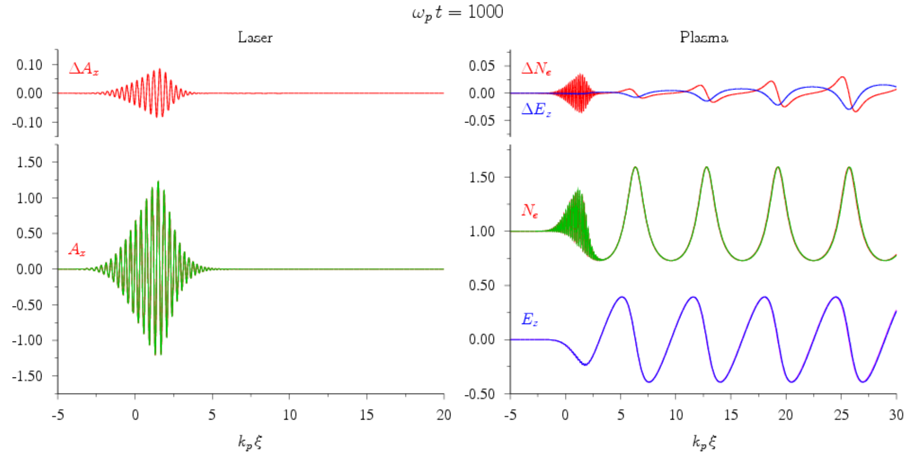

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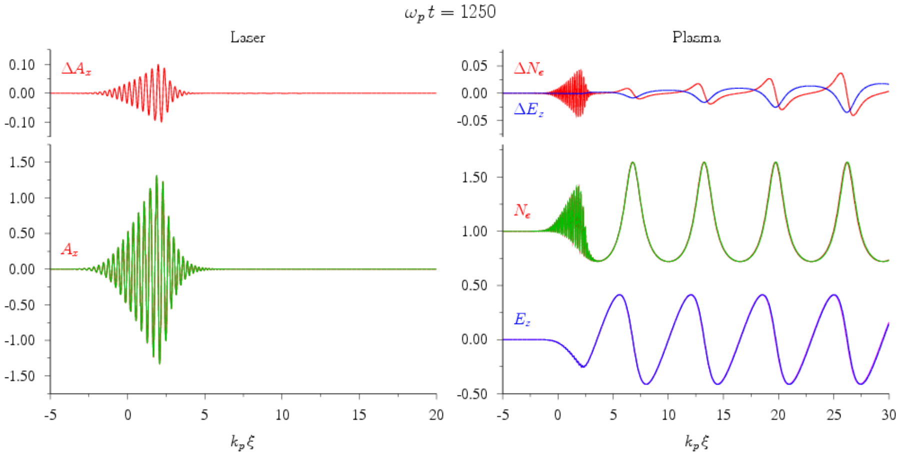

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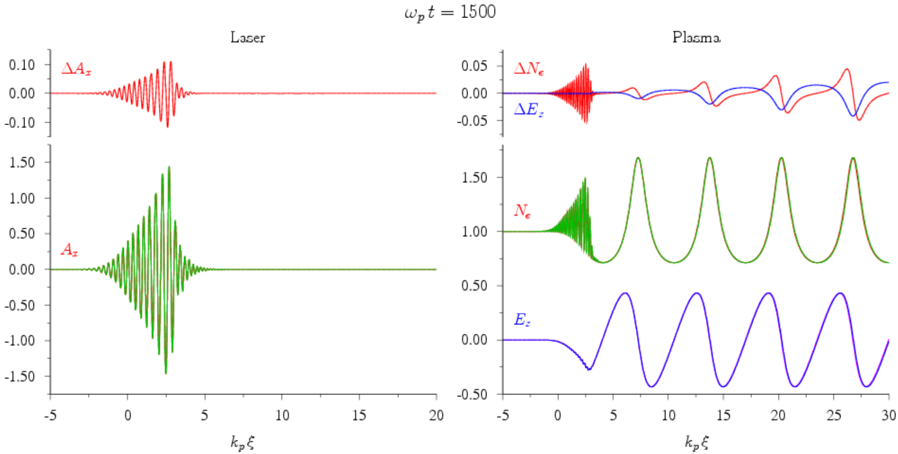

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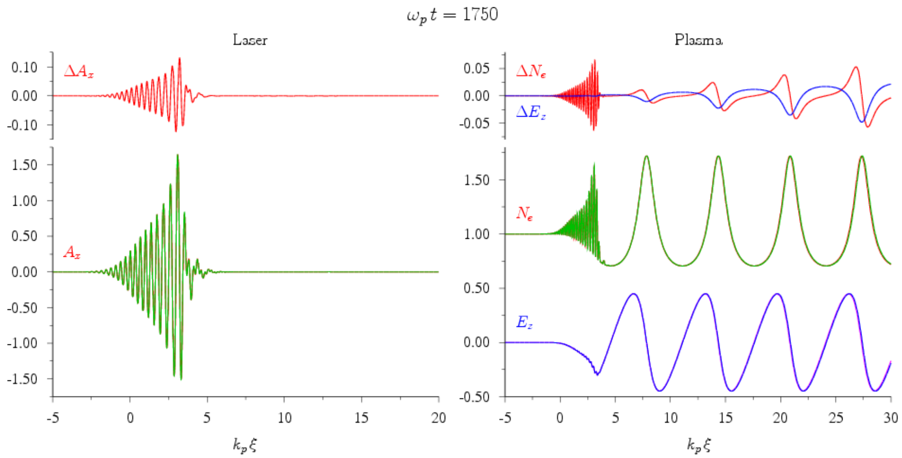

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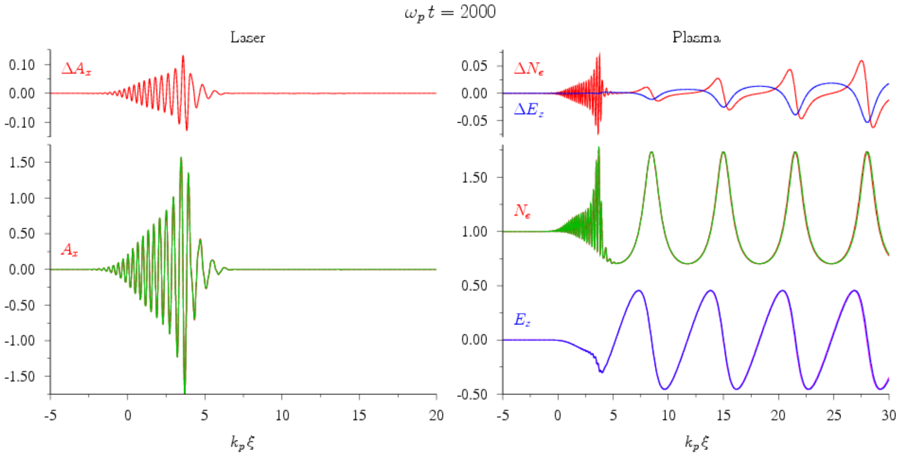

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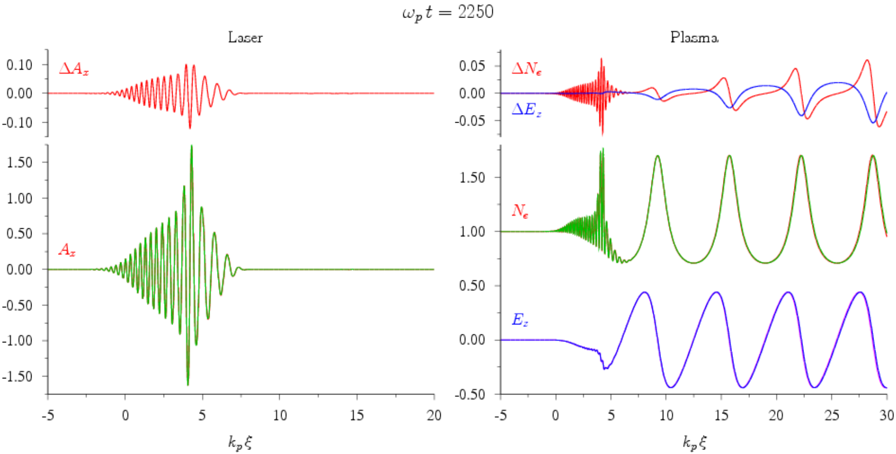

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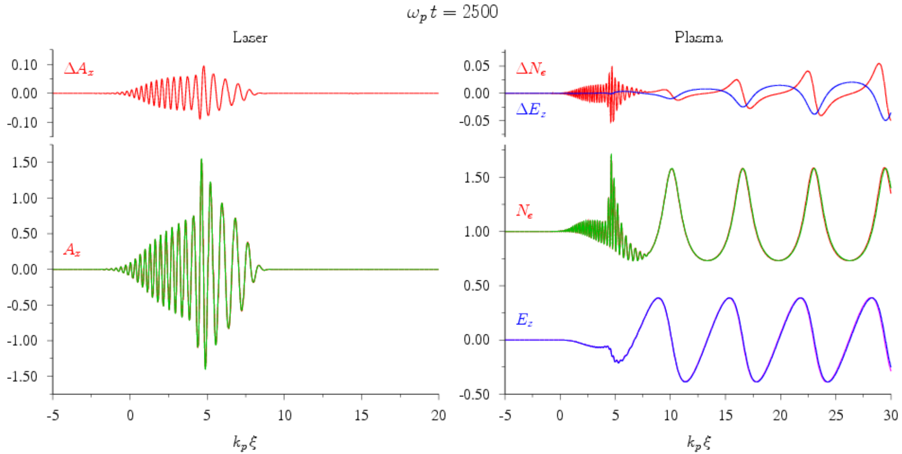

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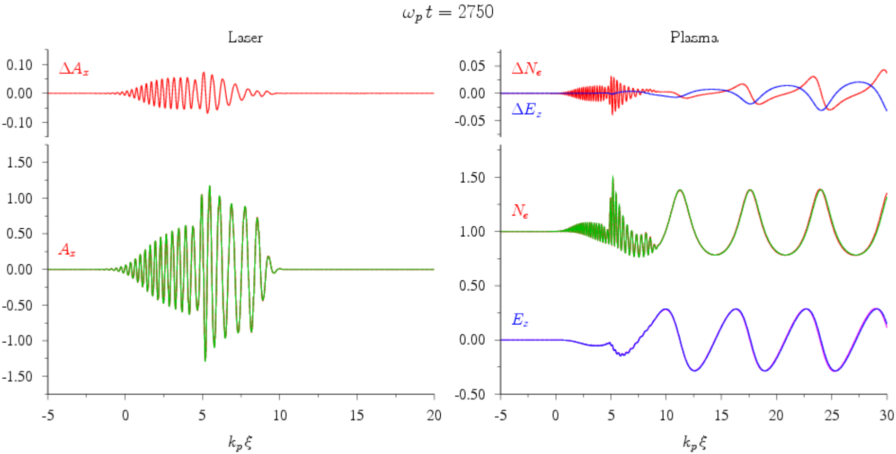

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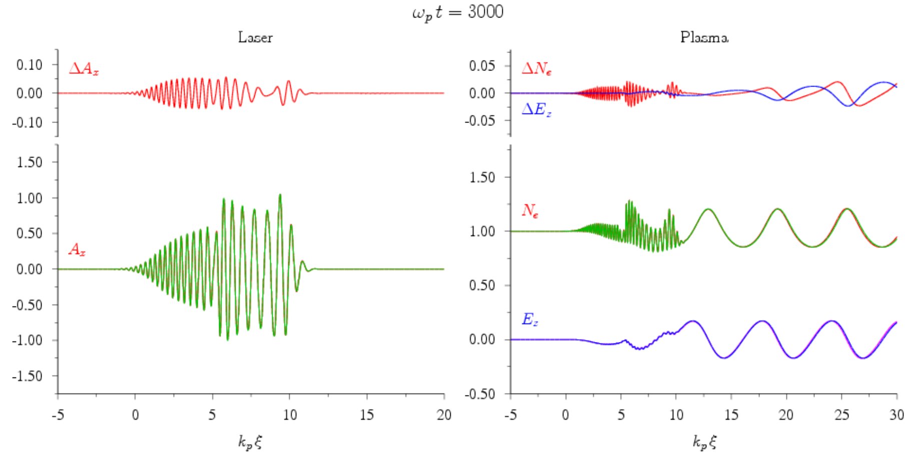

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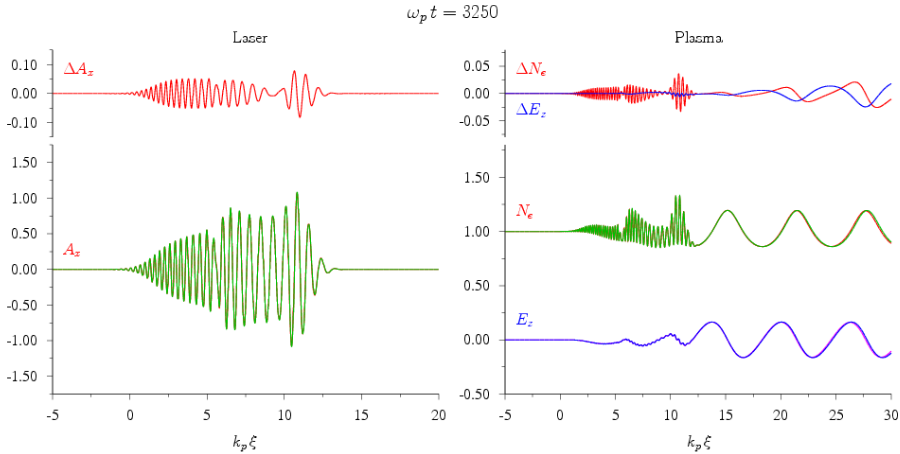

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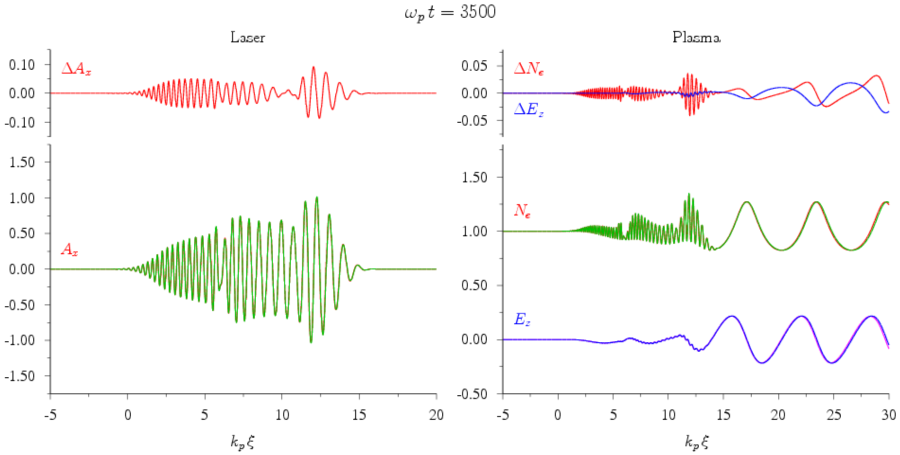

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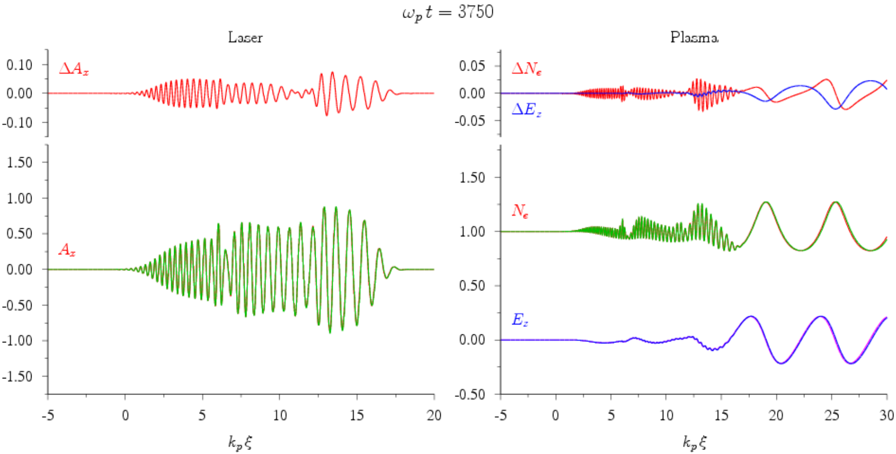

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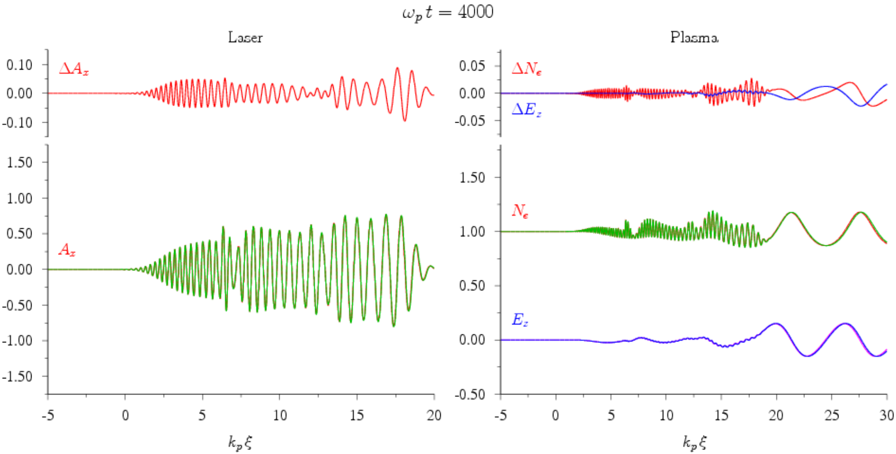

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- See the conference proceedings for the details.

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- For the unaveraged equations, wave action is only an adiabatic invariant.

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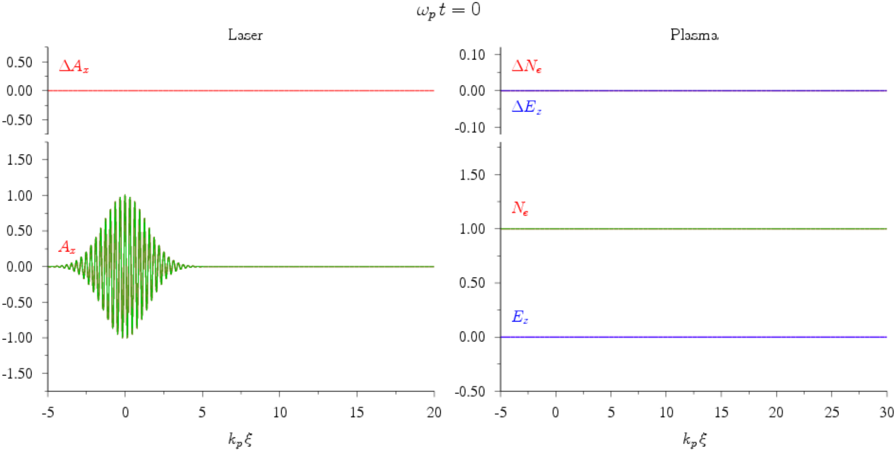

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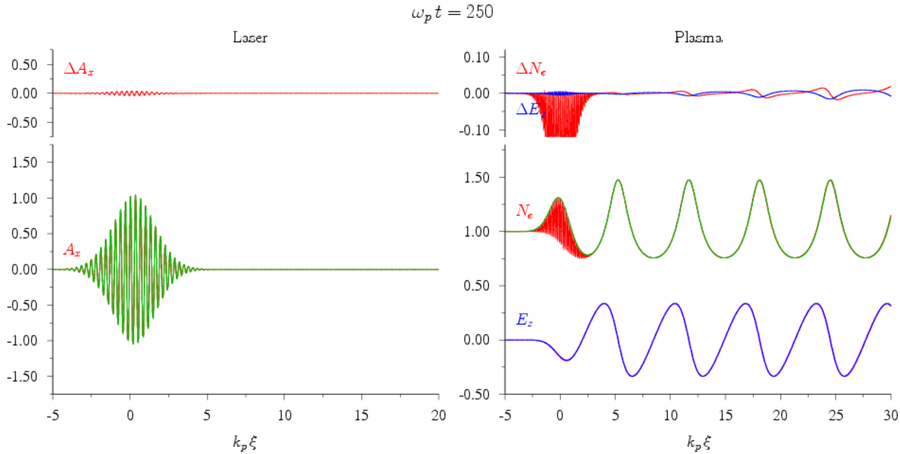

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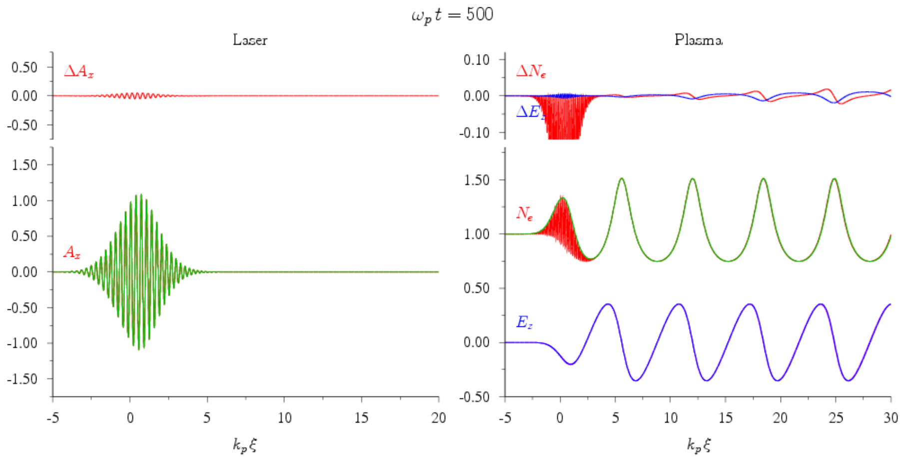

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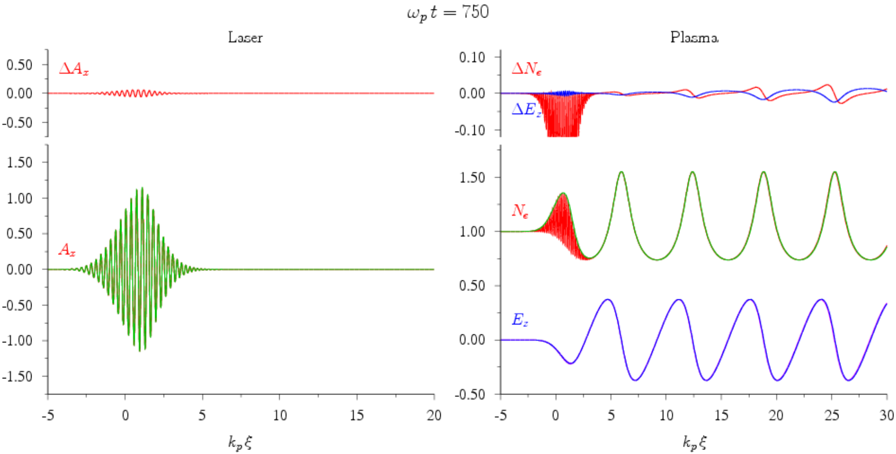

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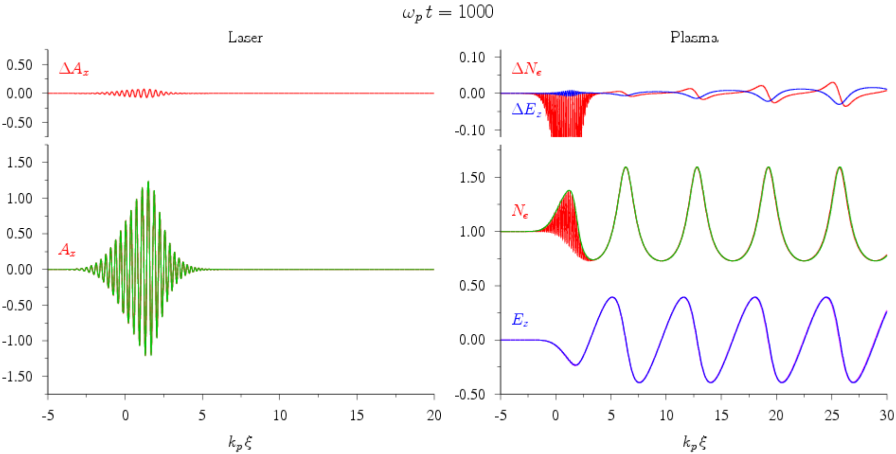

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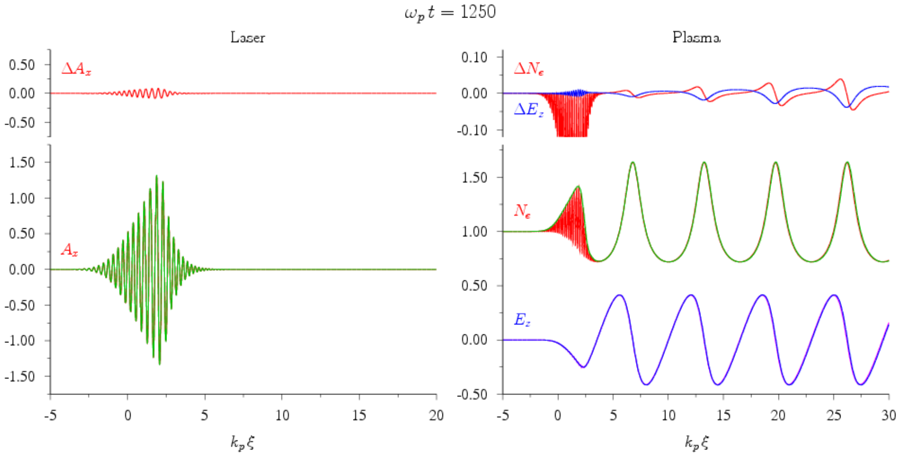

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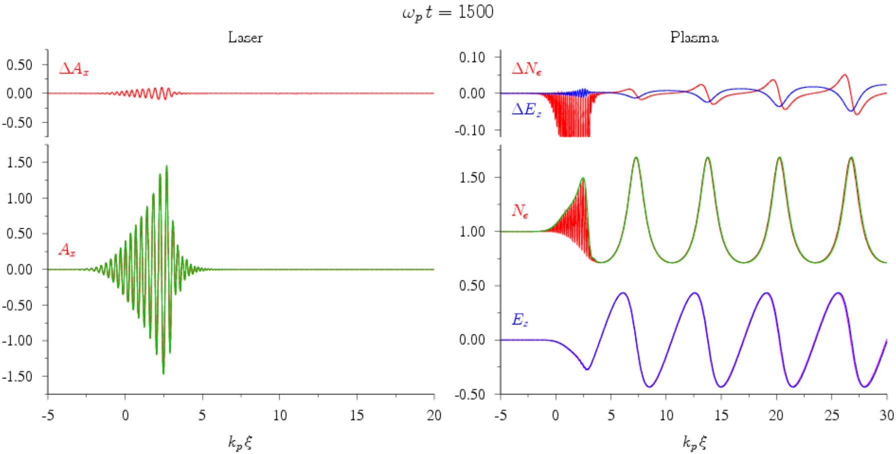

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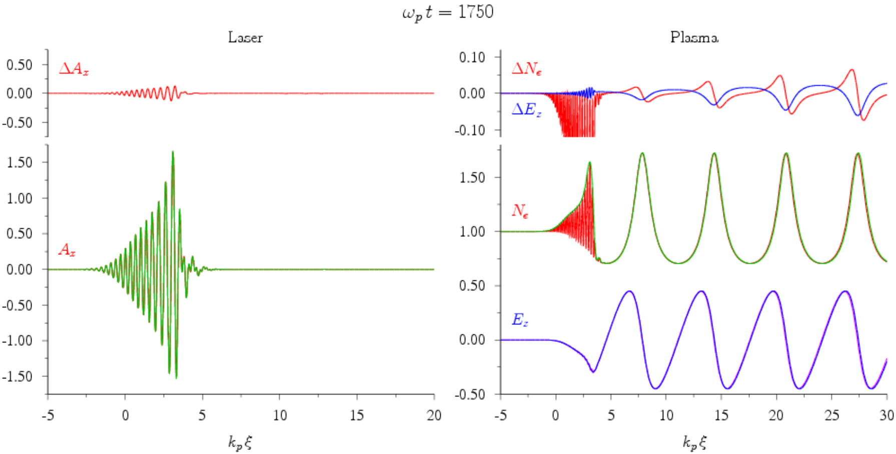

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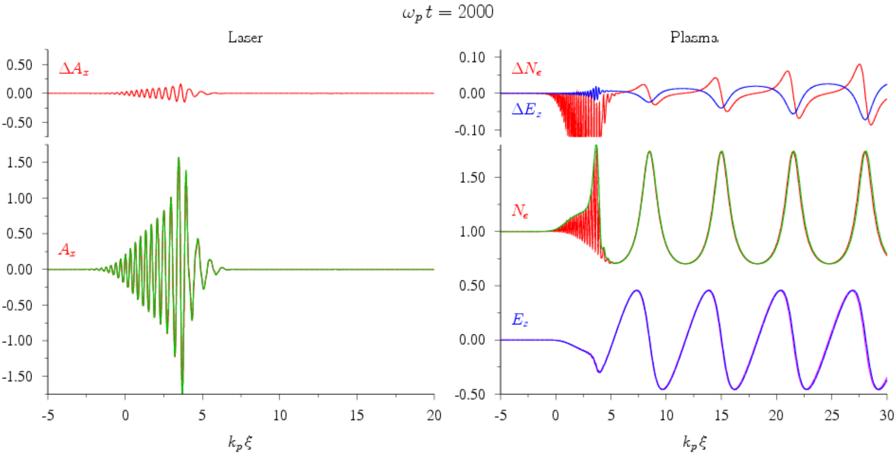

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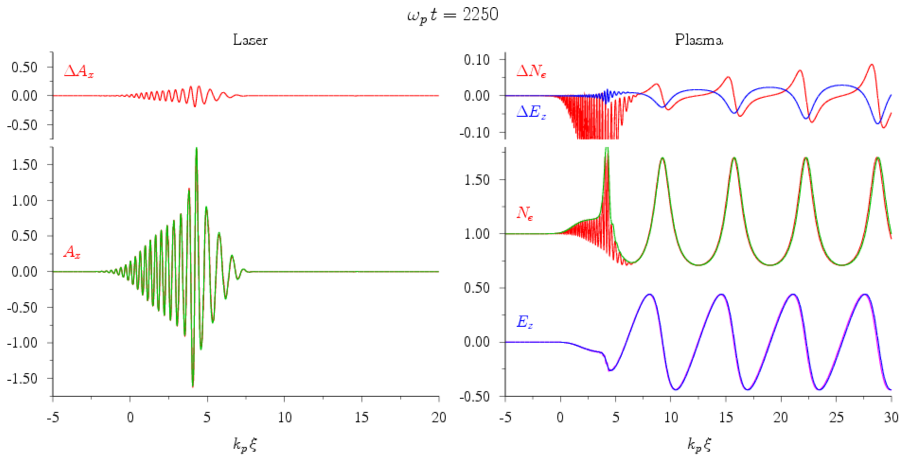

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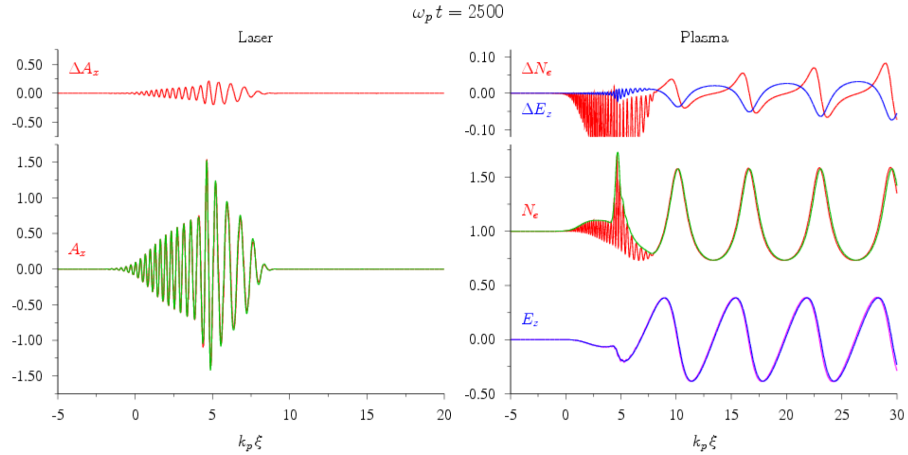

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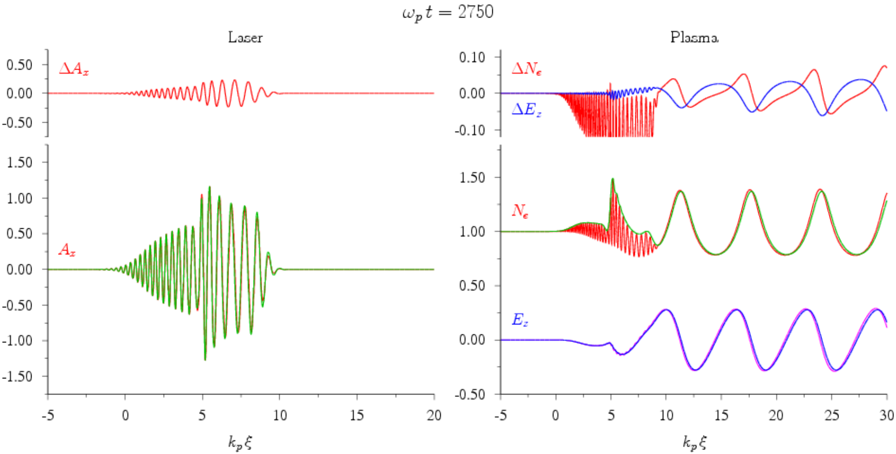

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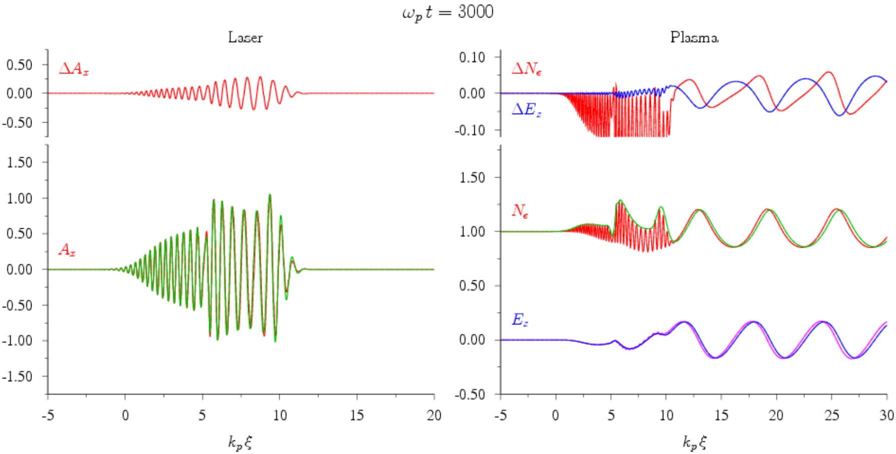

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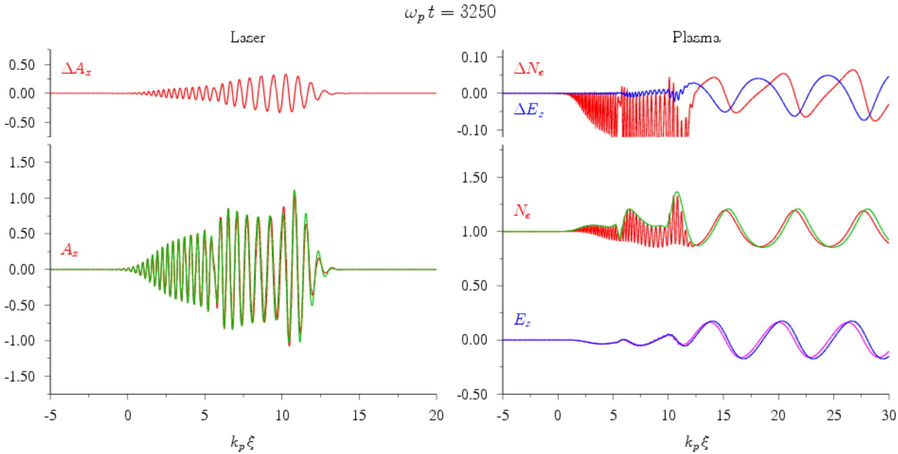

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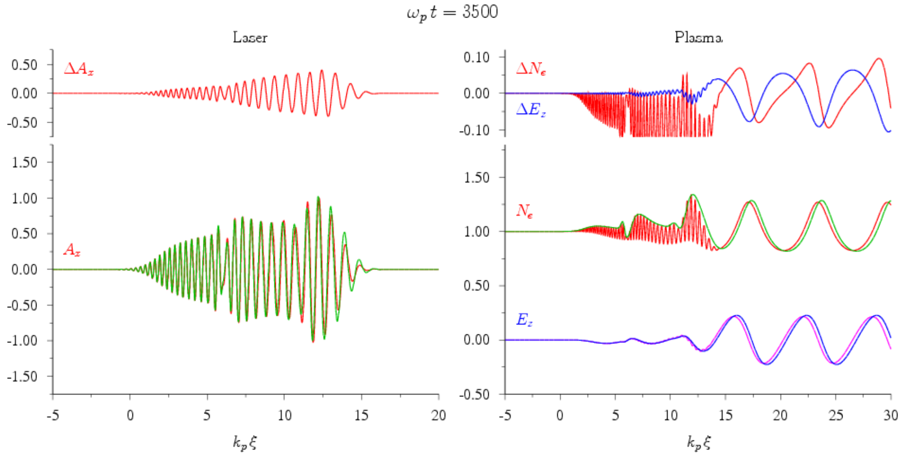

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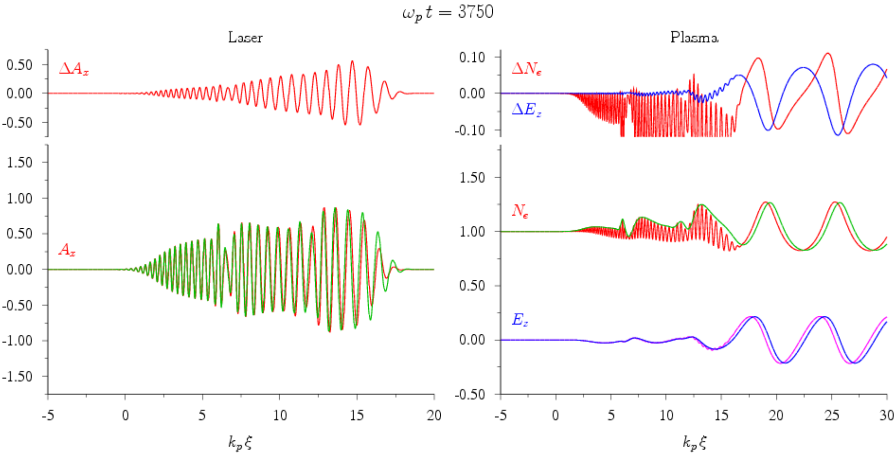

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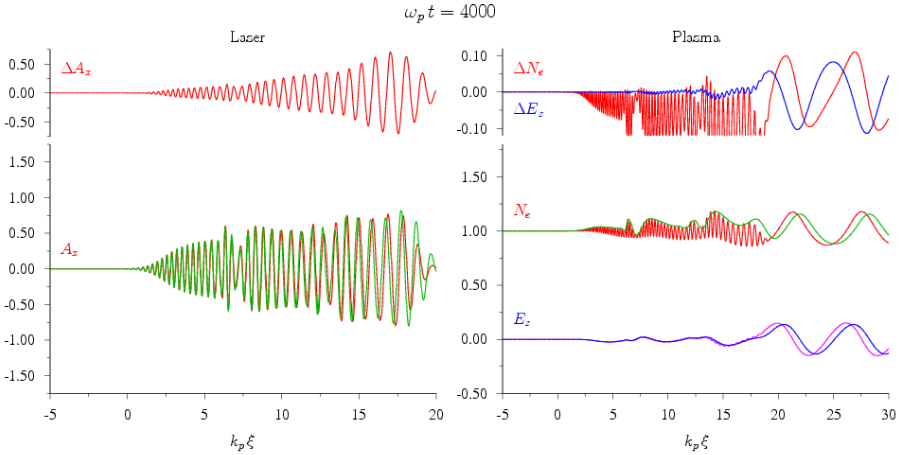

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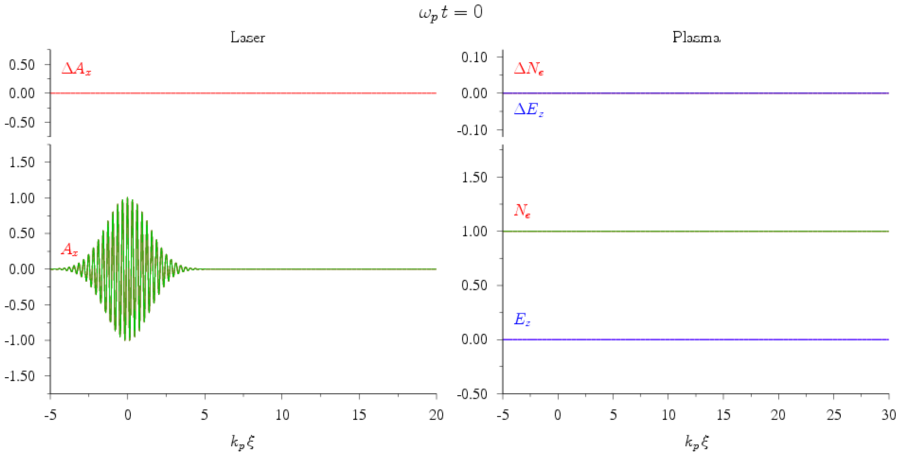

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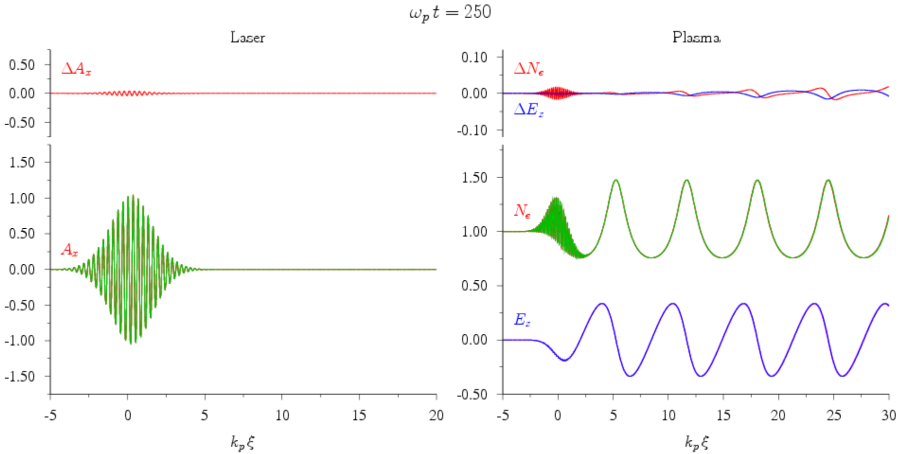

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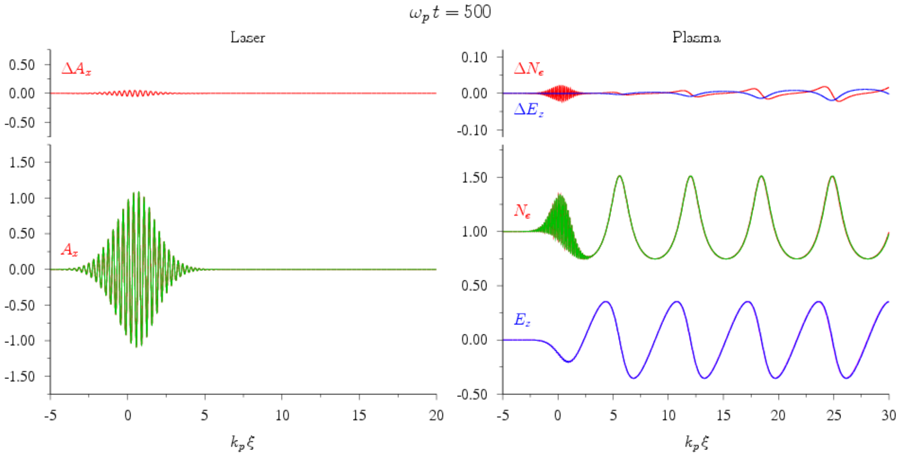

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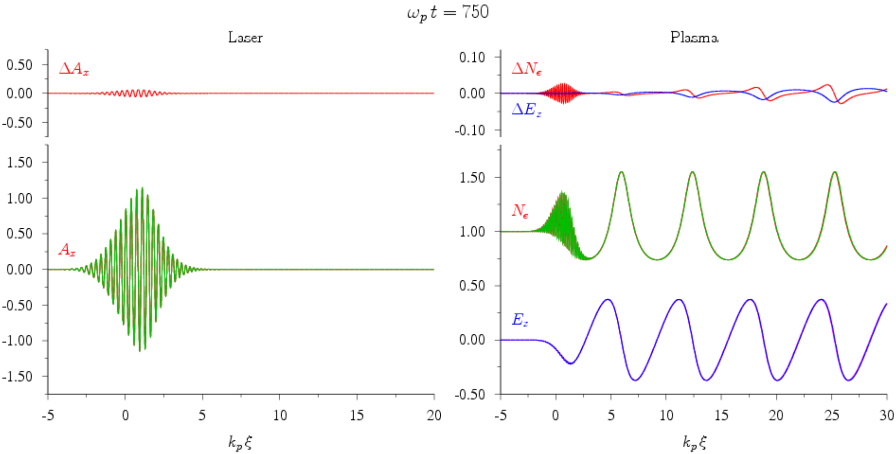

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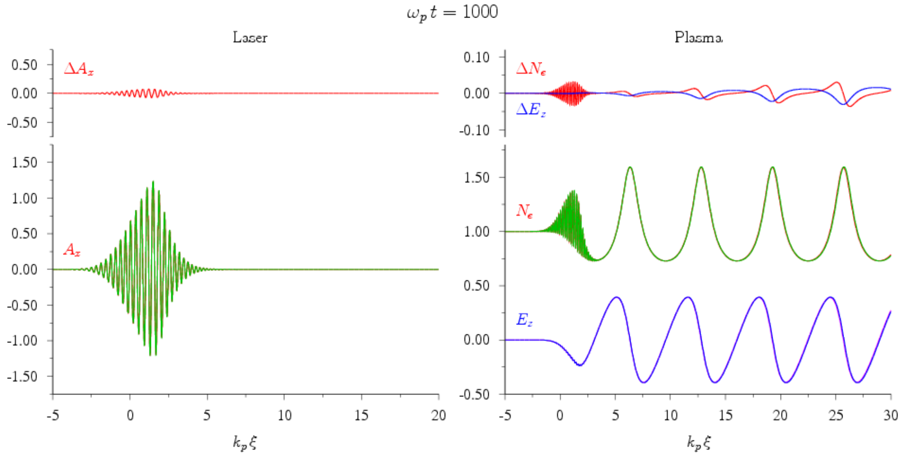

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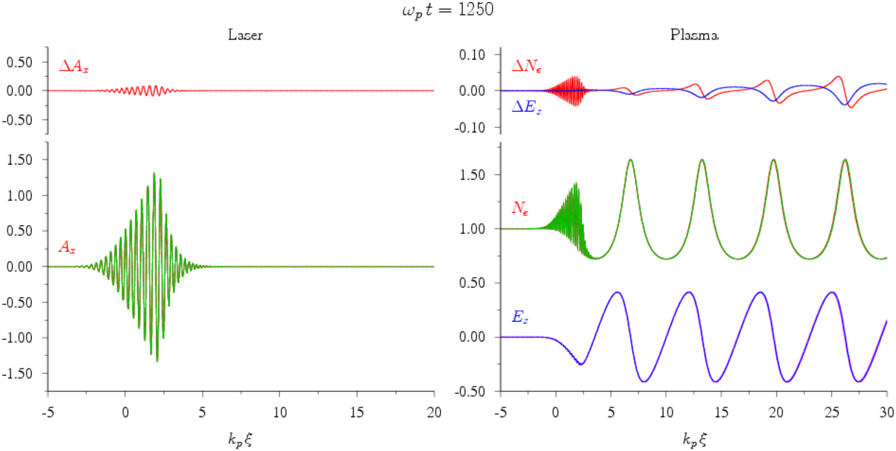

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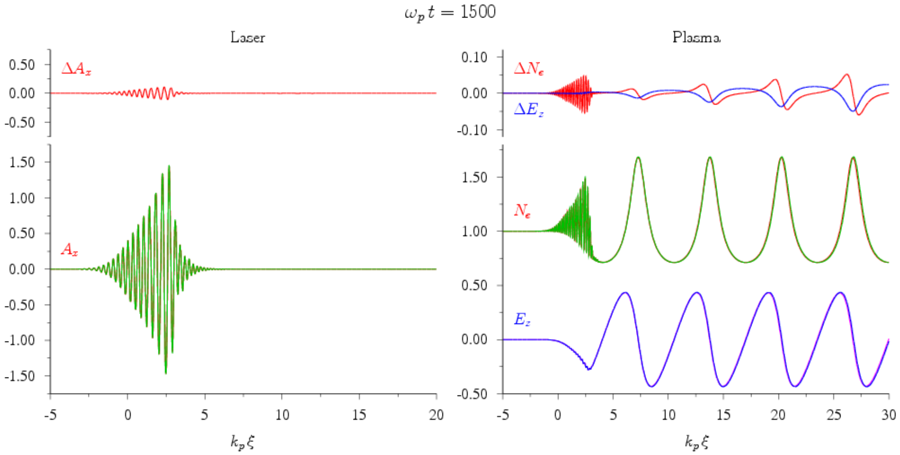

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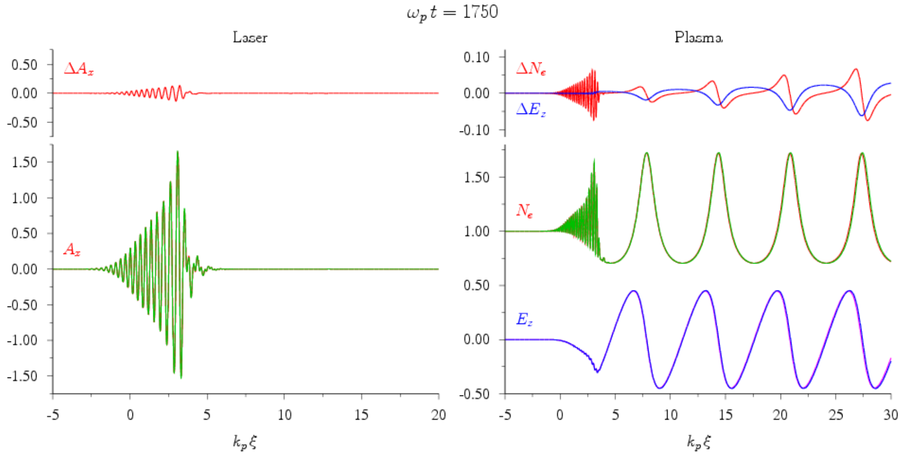

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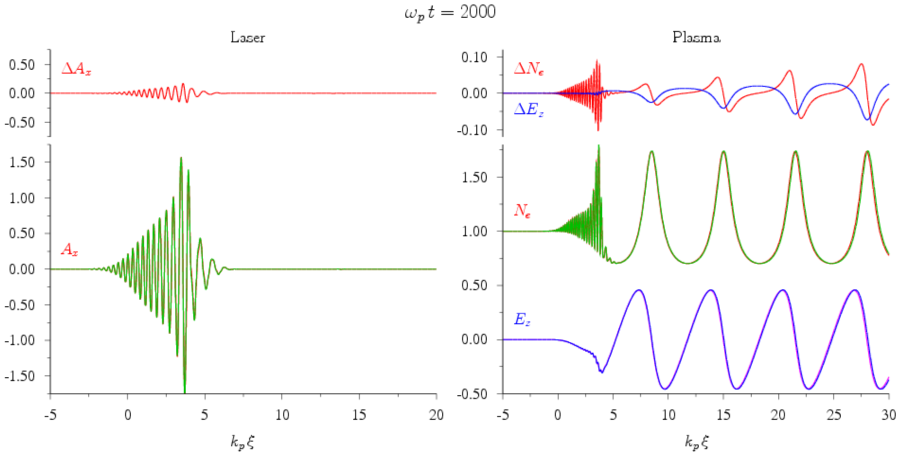

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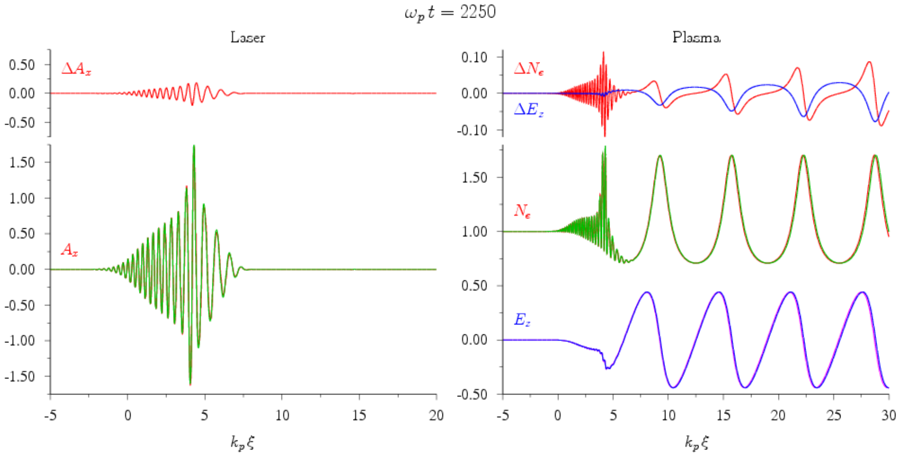

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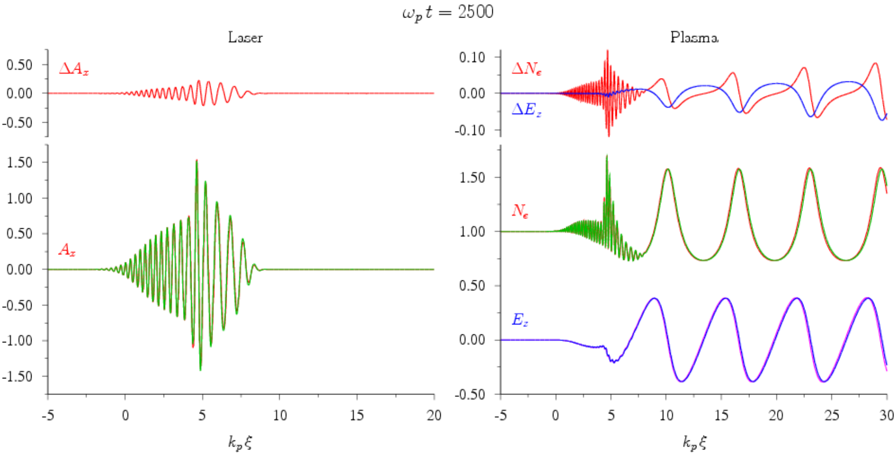

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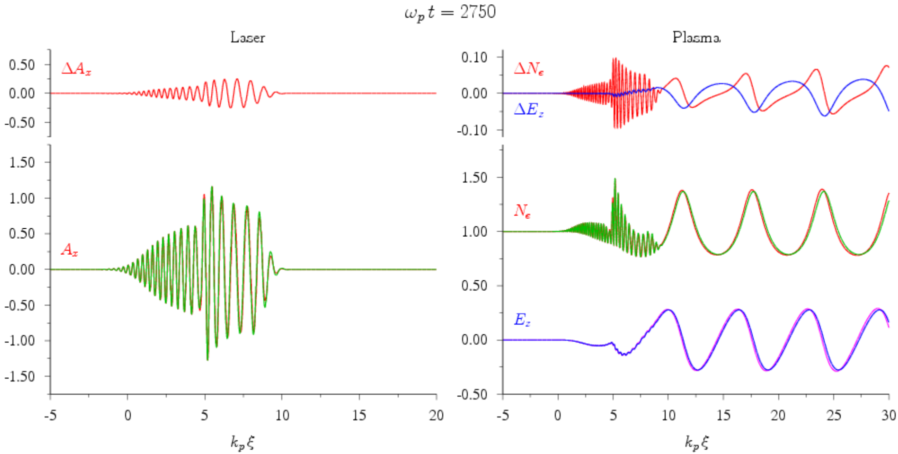

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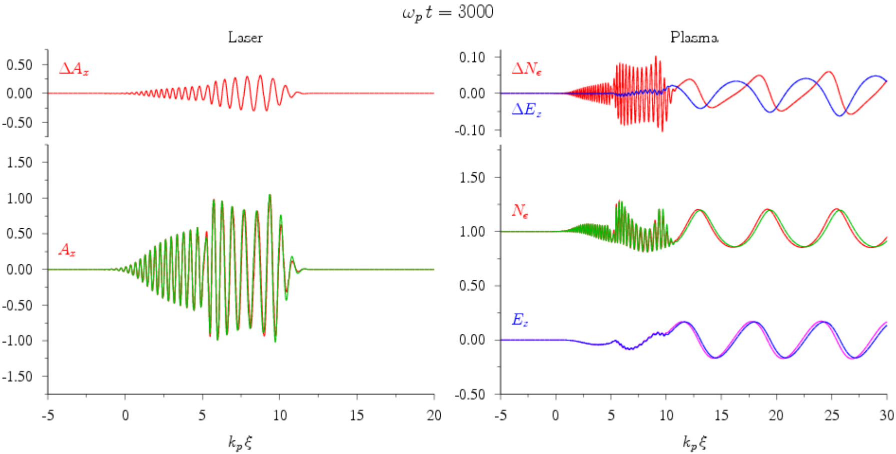

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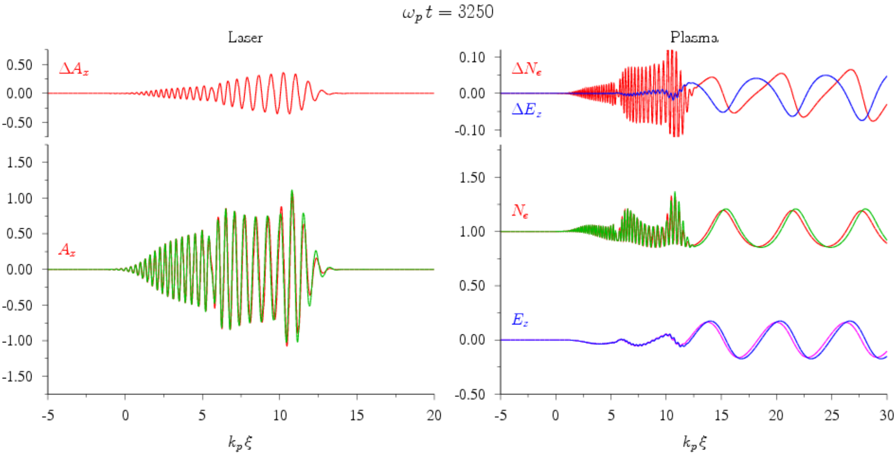

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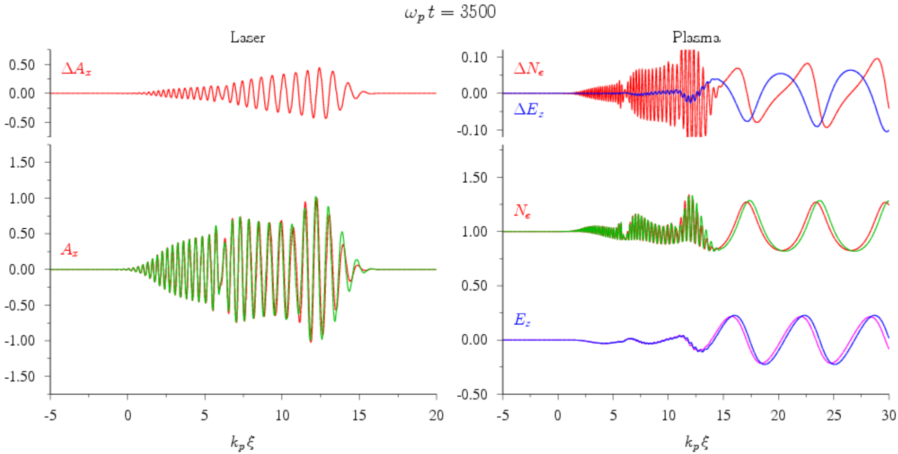

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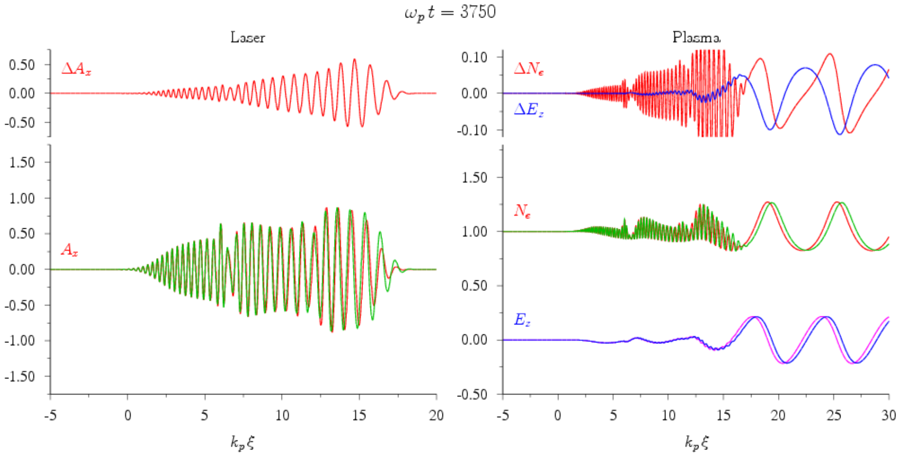

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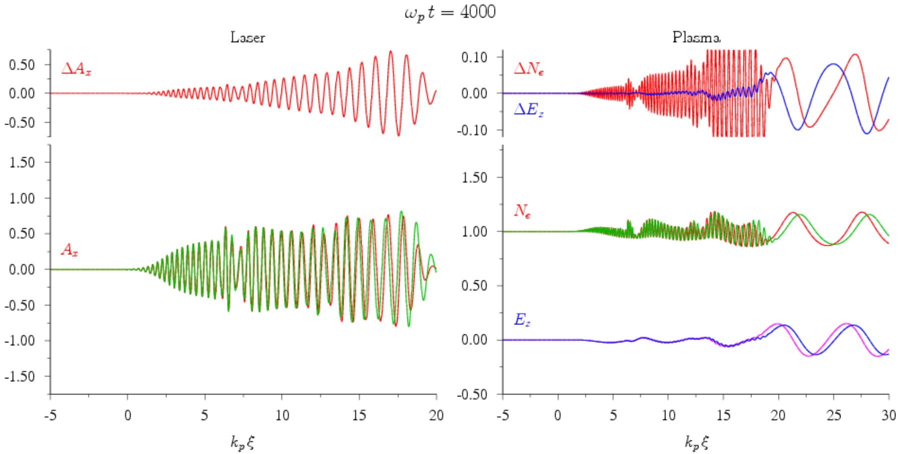

Averaged *vs.* Un-Averaged

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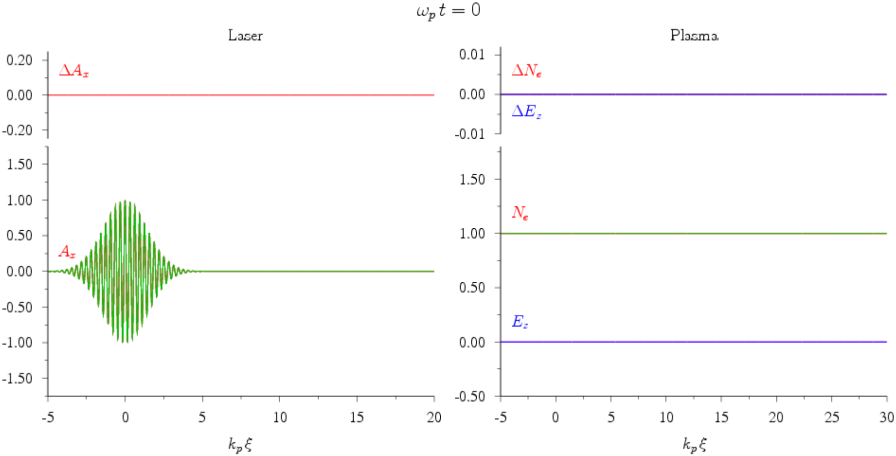
Quasi-Static Reduced Wave Operator, Averaged

Comparing $\lambda_0/\Delta\xi = 100$ to $\lambda_0/\Delta\xi = 25$.



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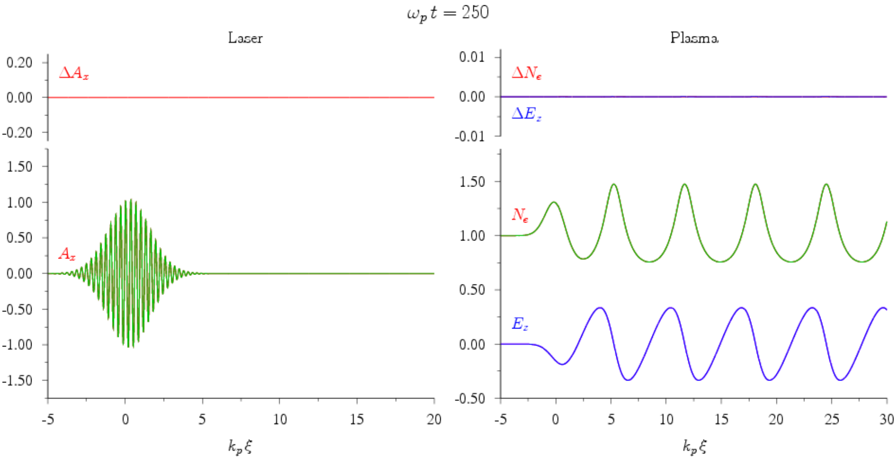

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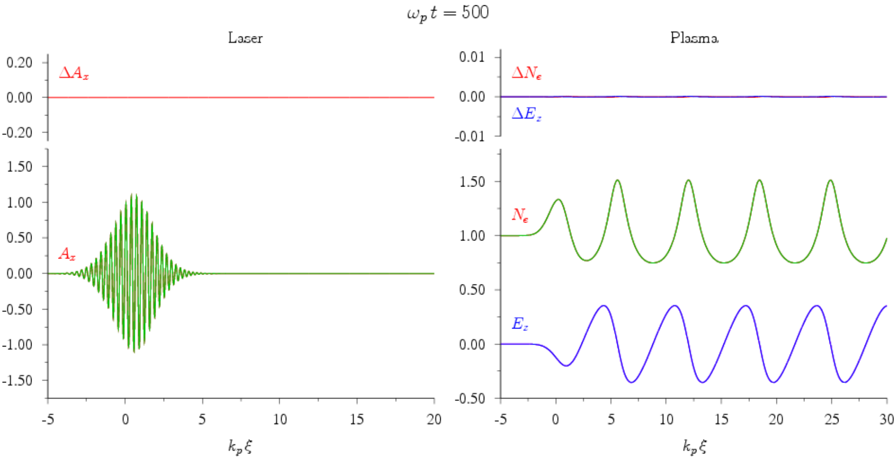

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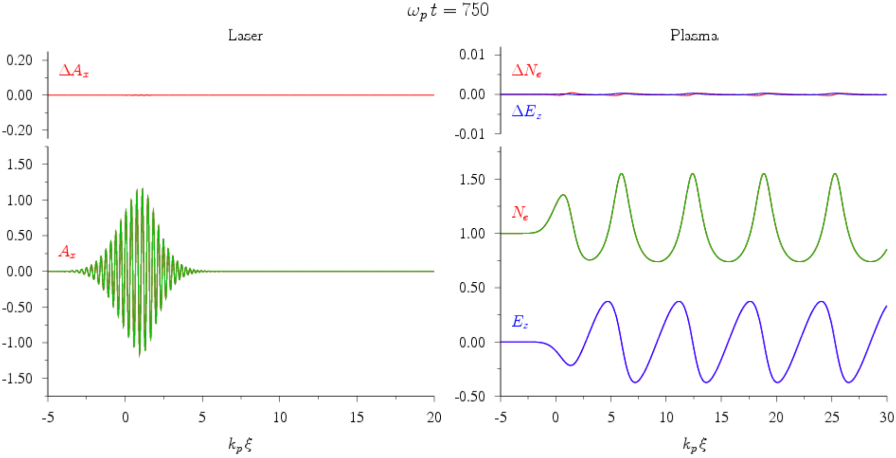

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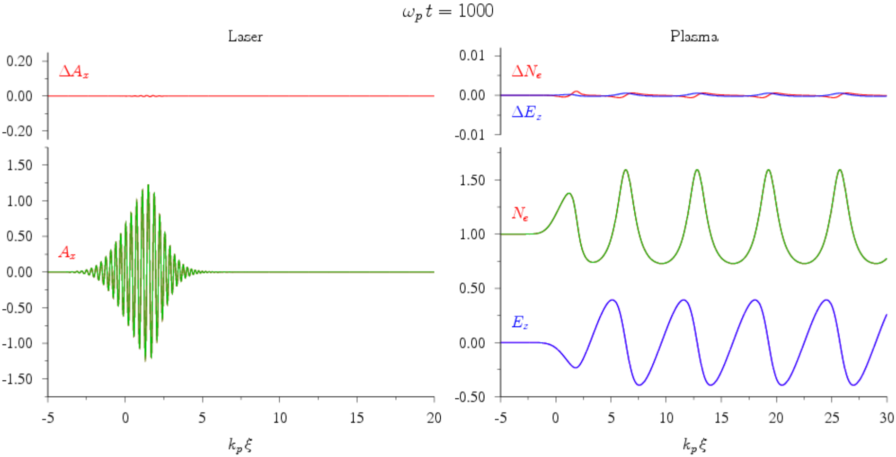

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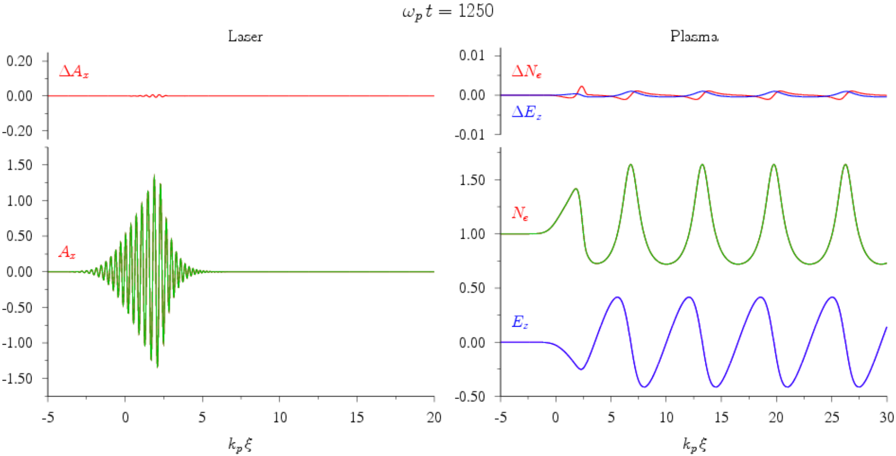

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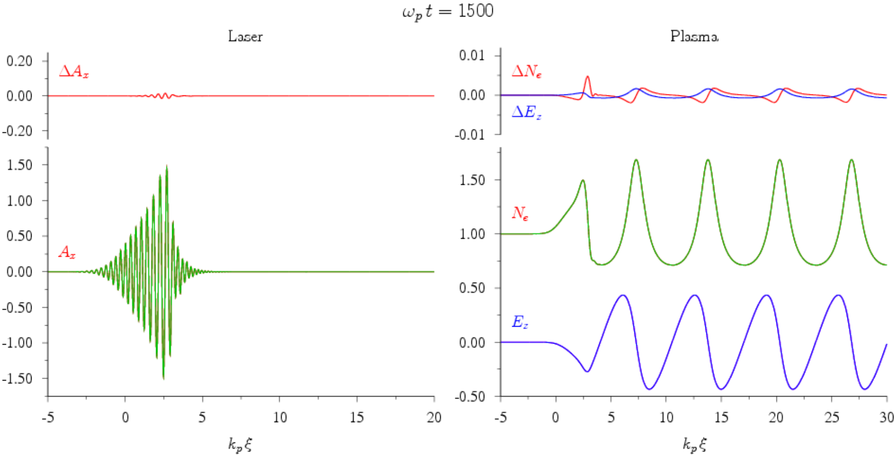

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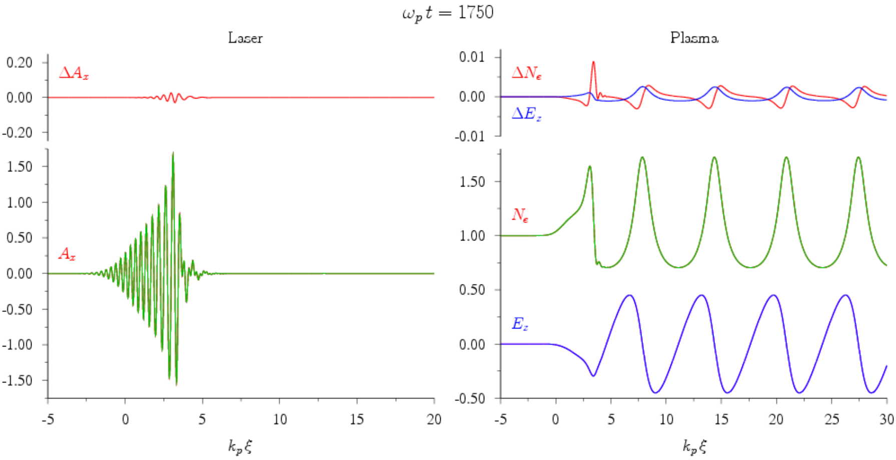

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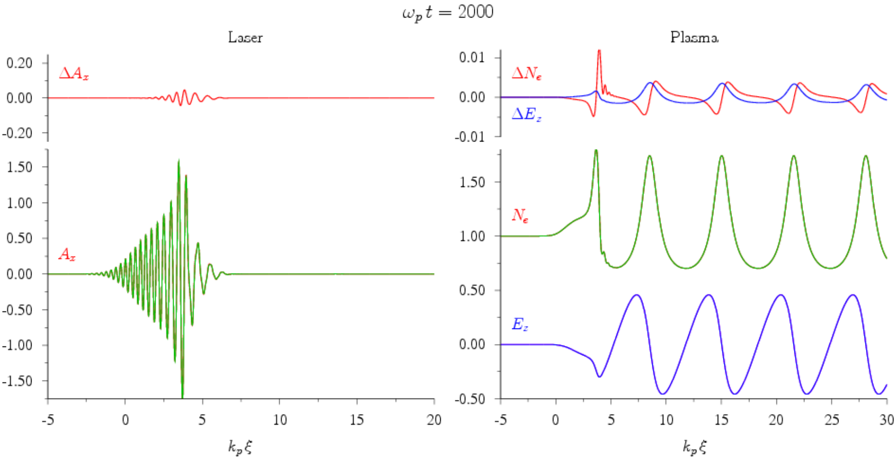

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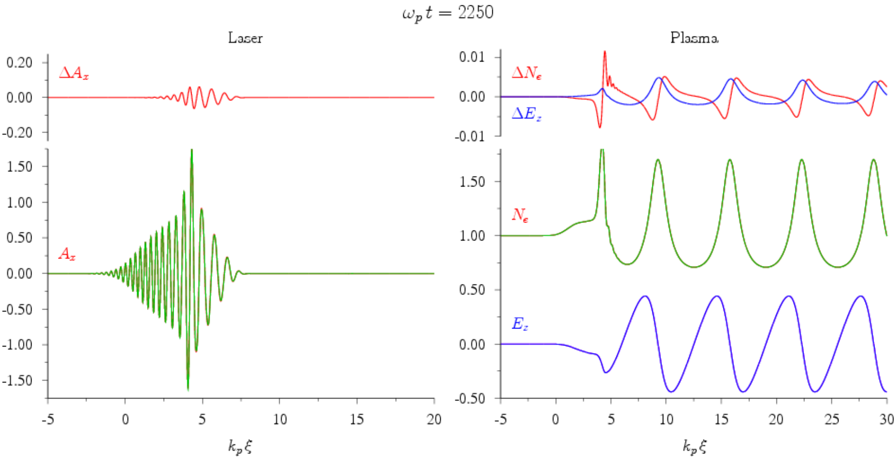

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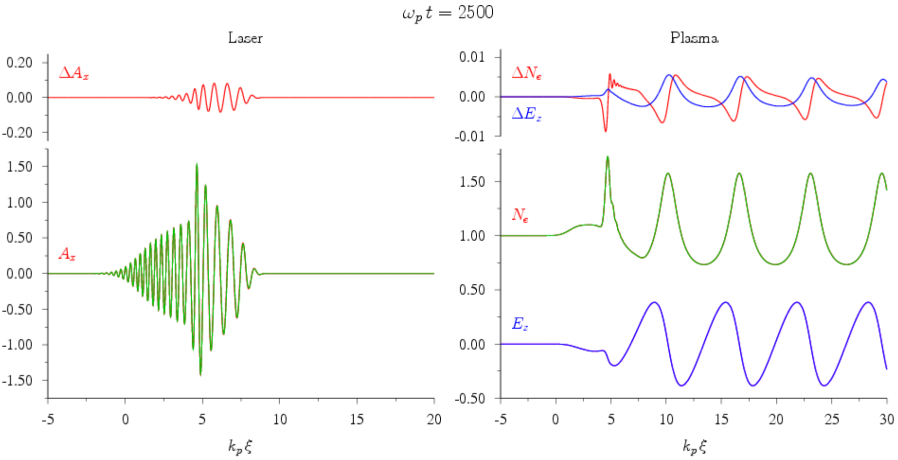

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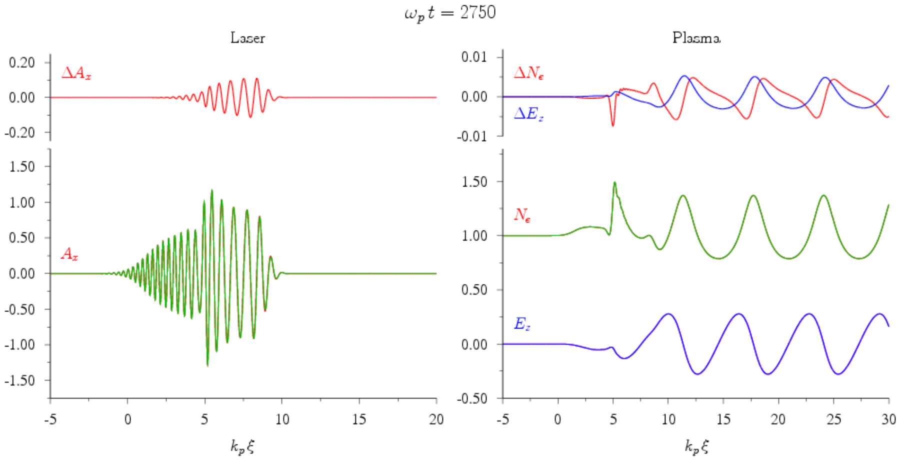

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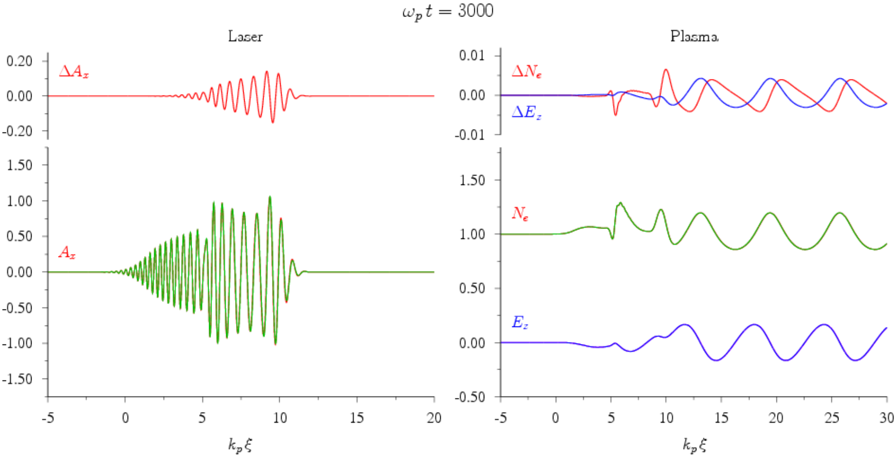

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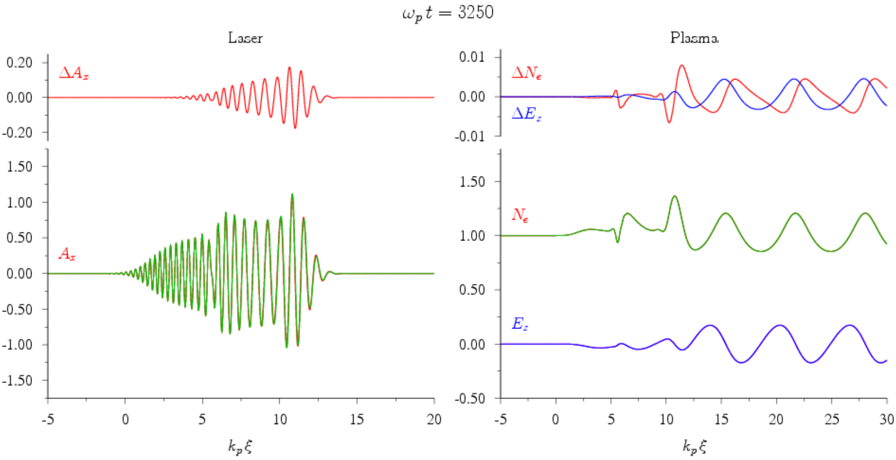

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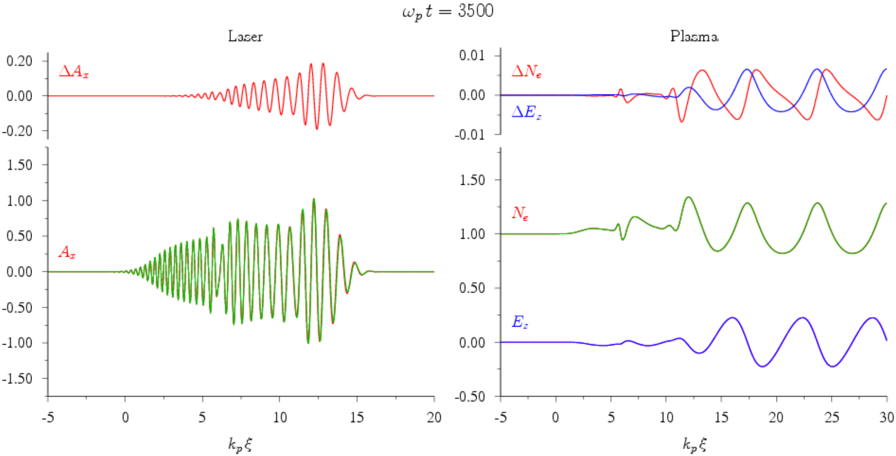

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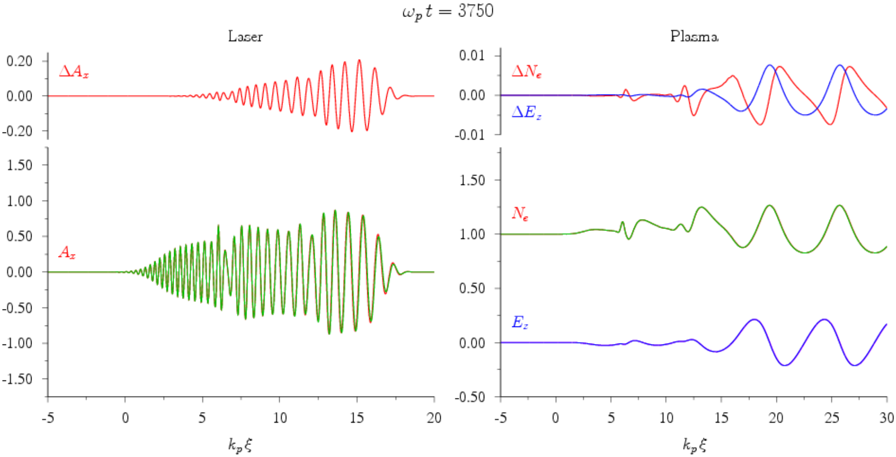

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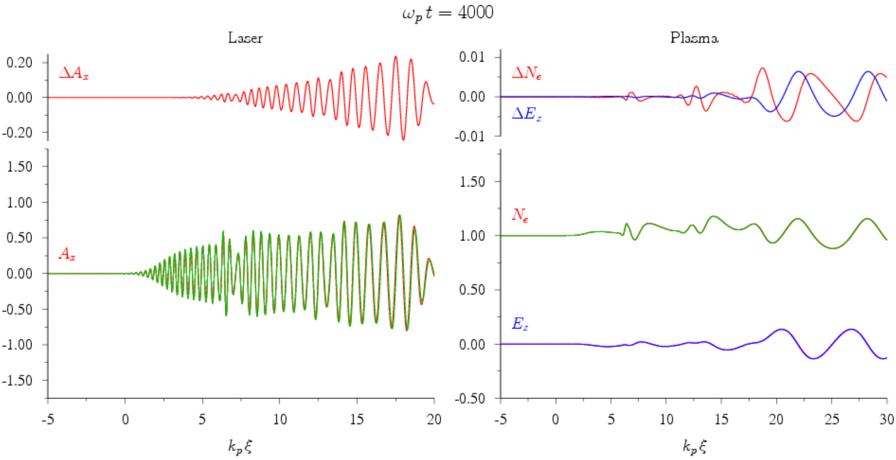

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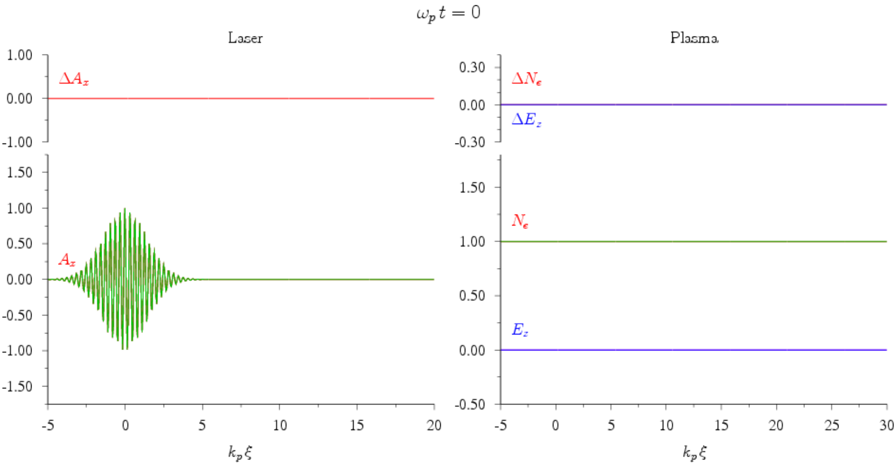

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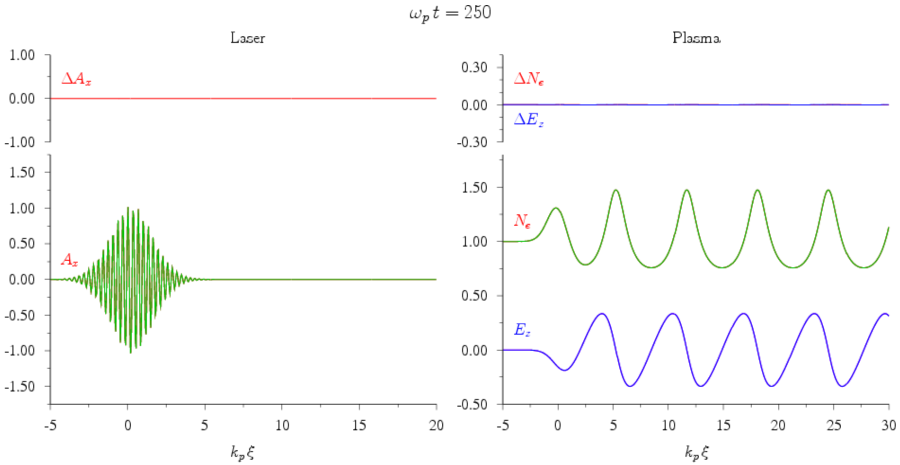

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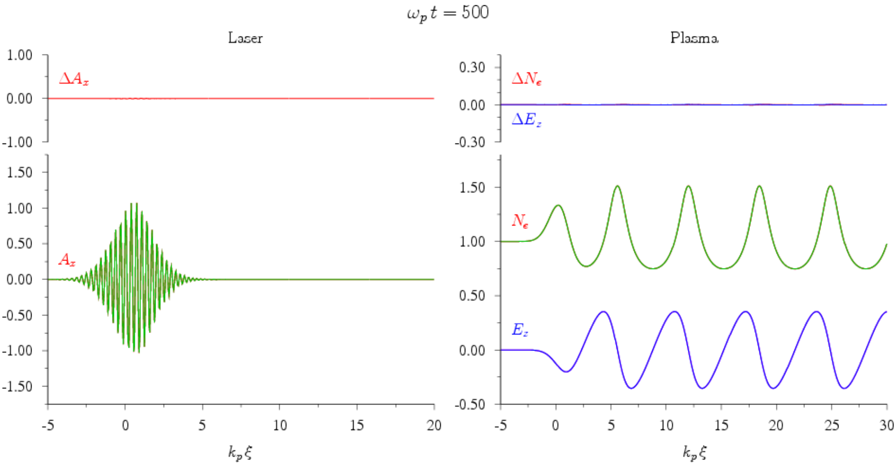

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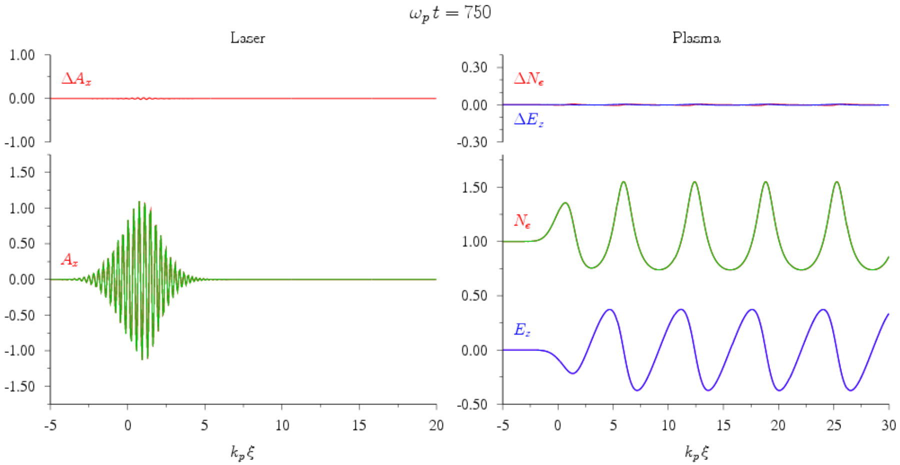

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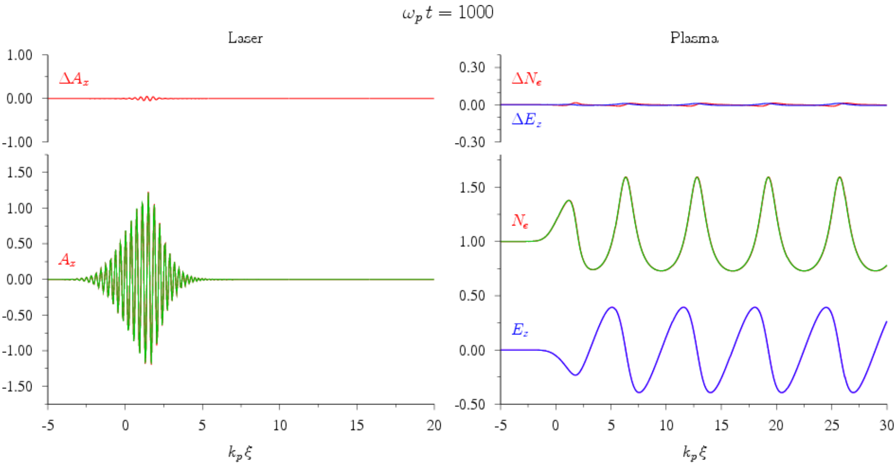

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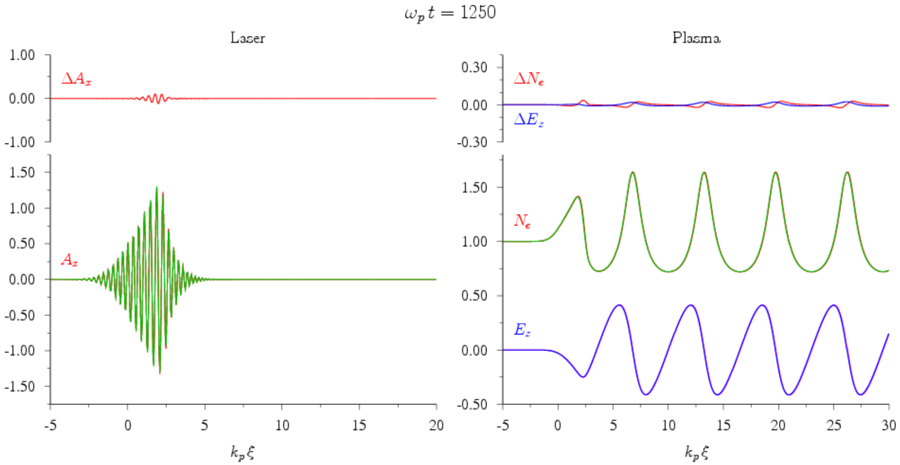

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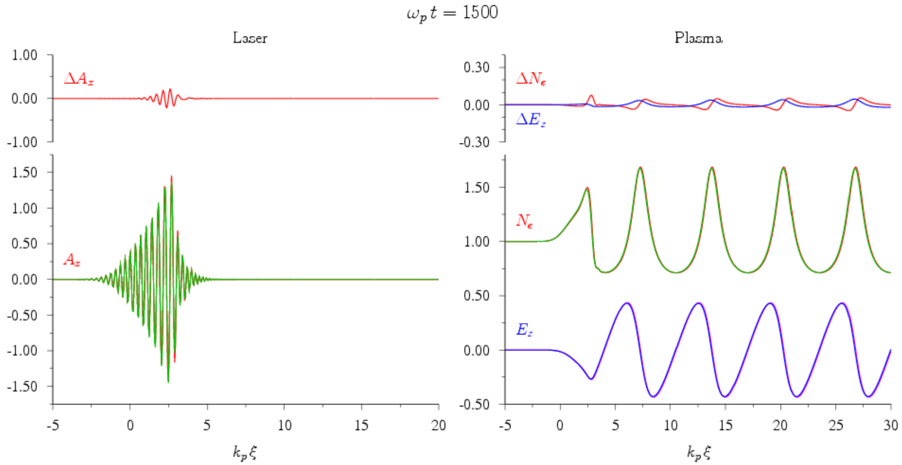

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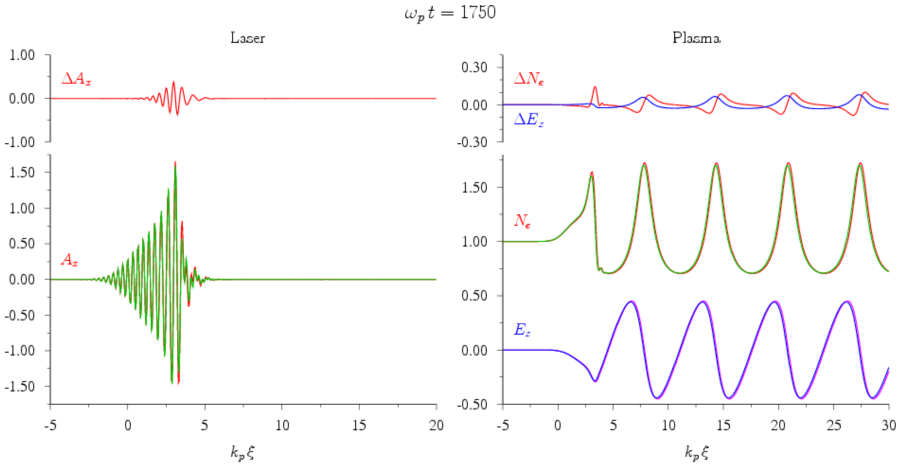

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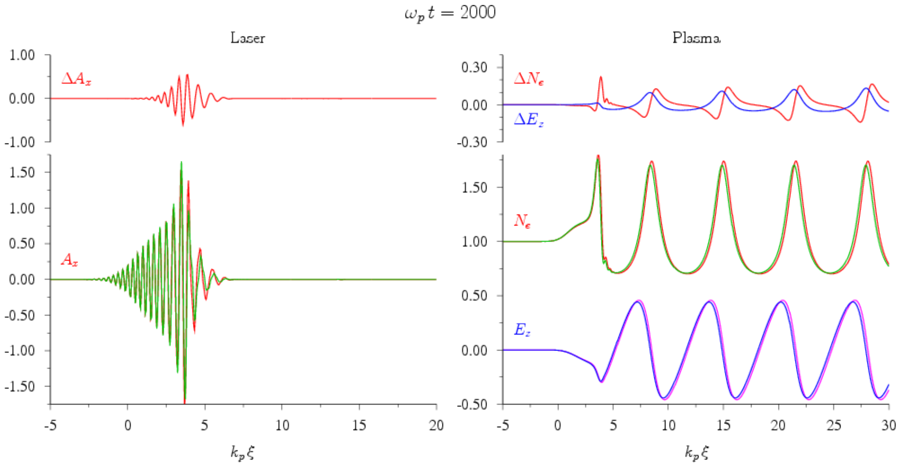

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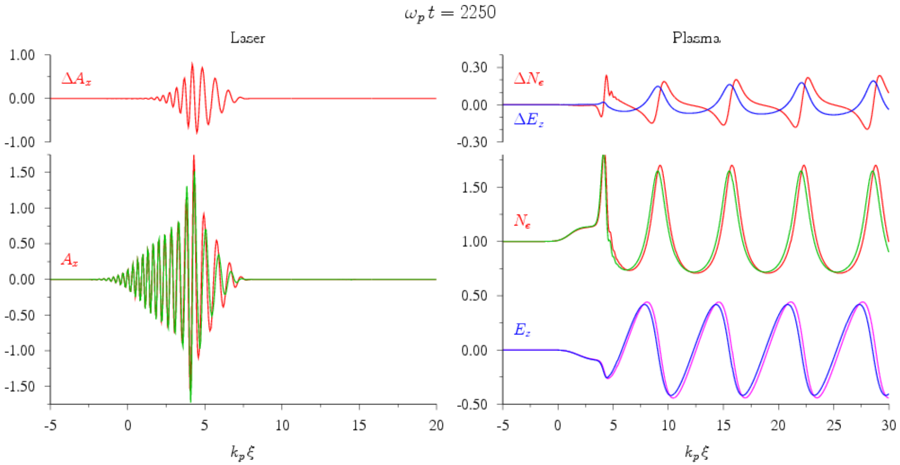

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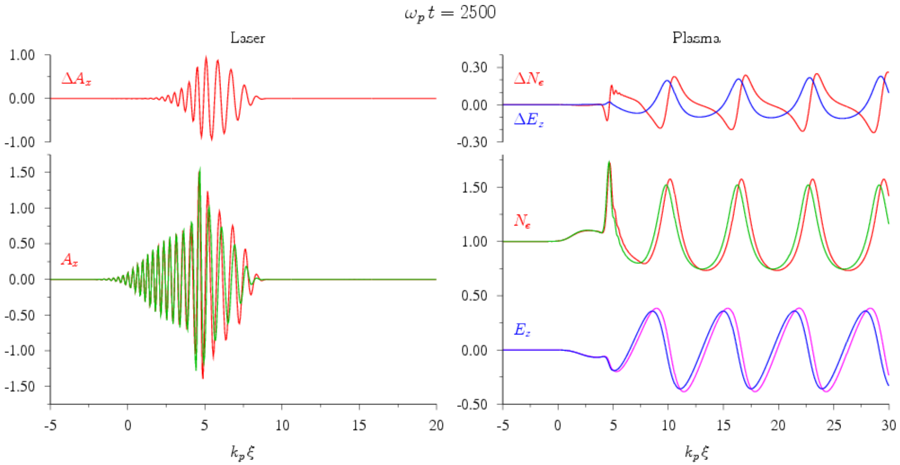

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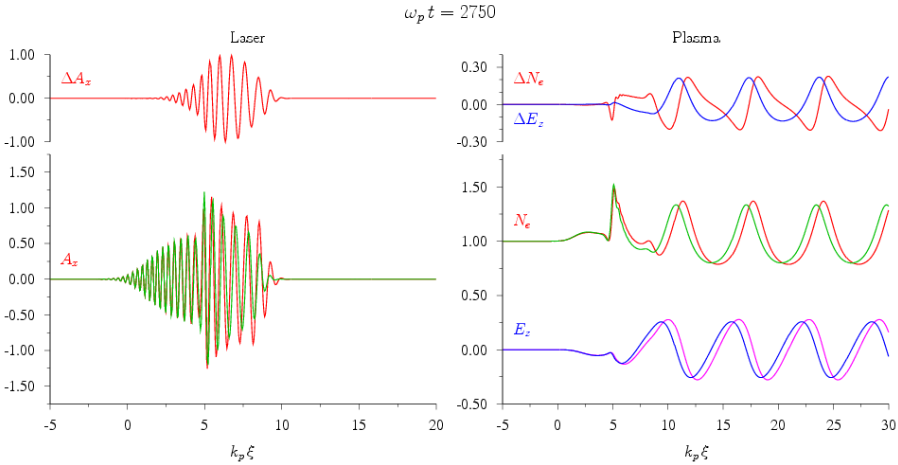

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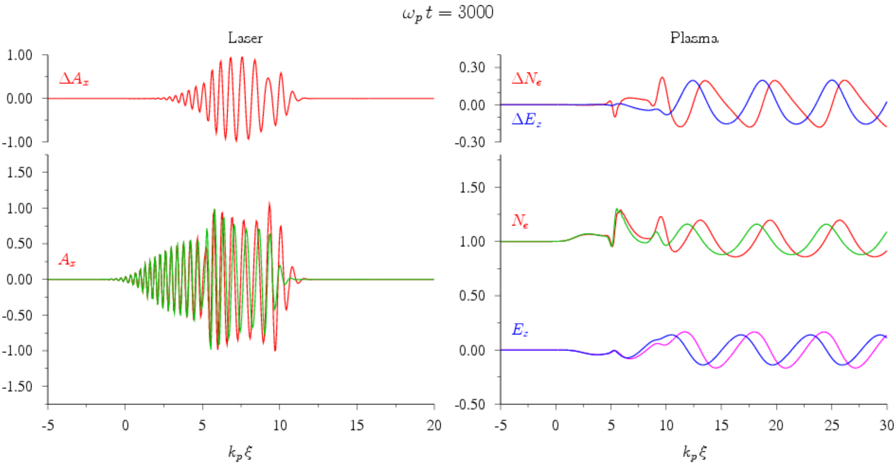

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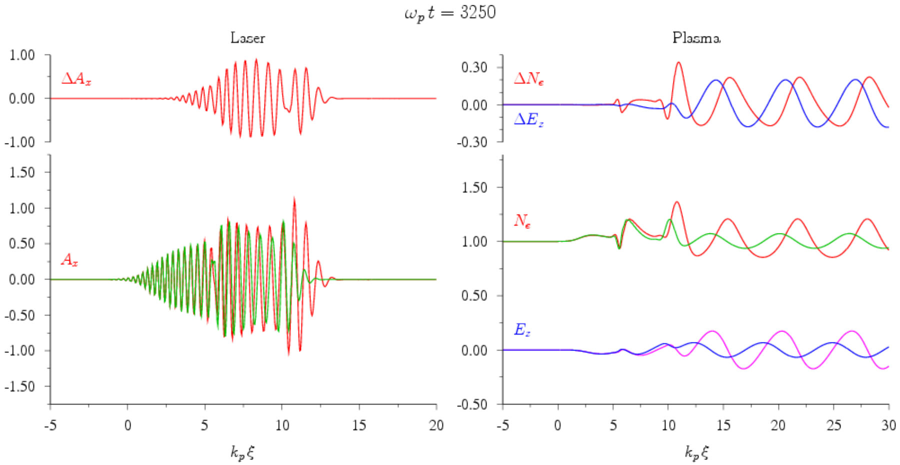

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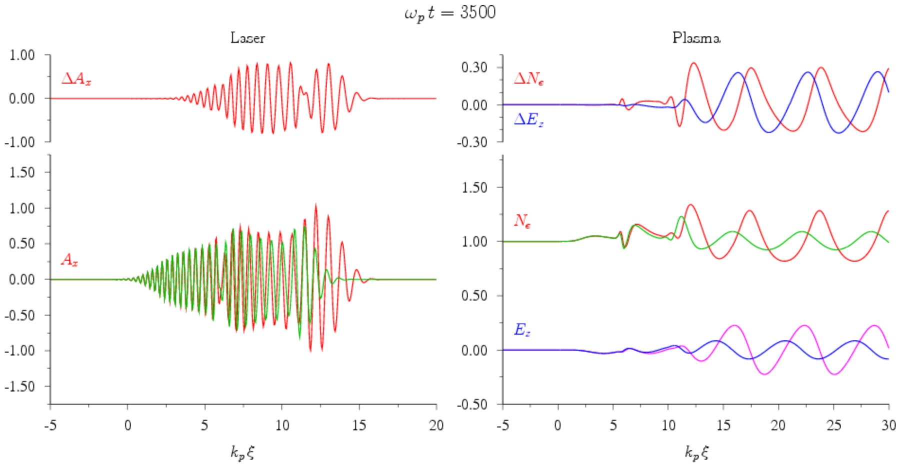

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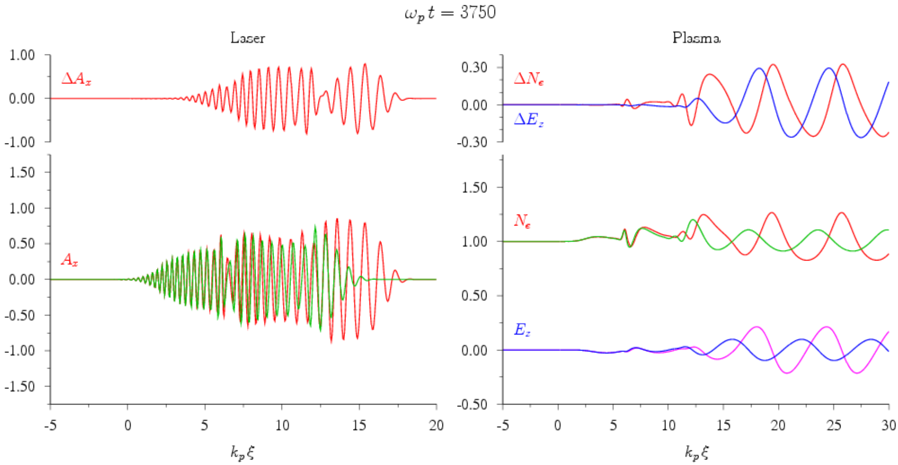

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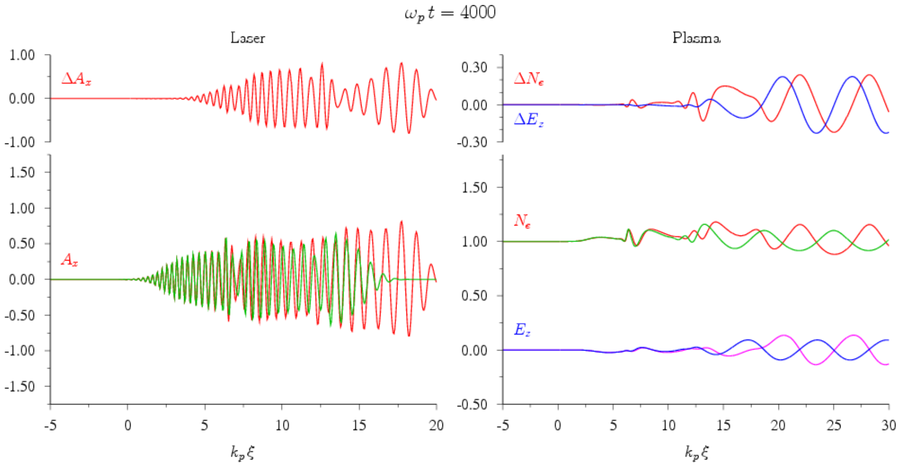

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