QCD challenges at the Large Hadron Collider

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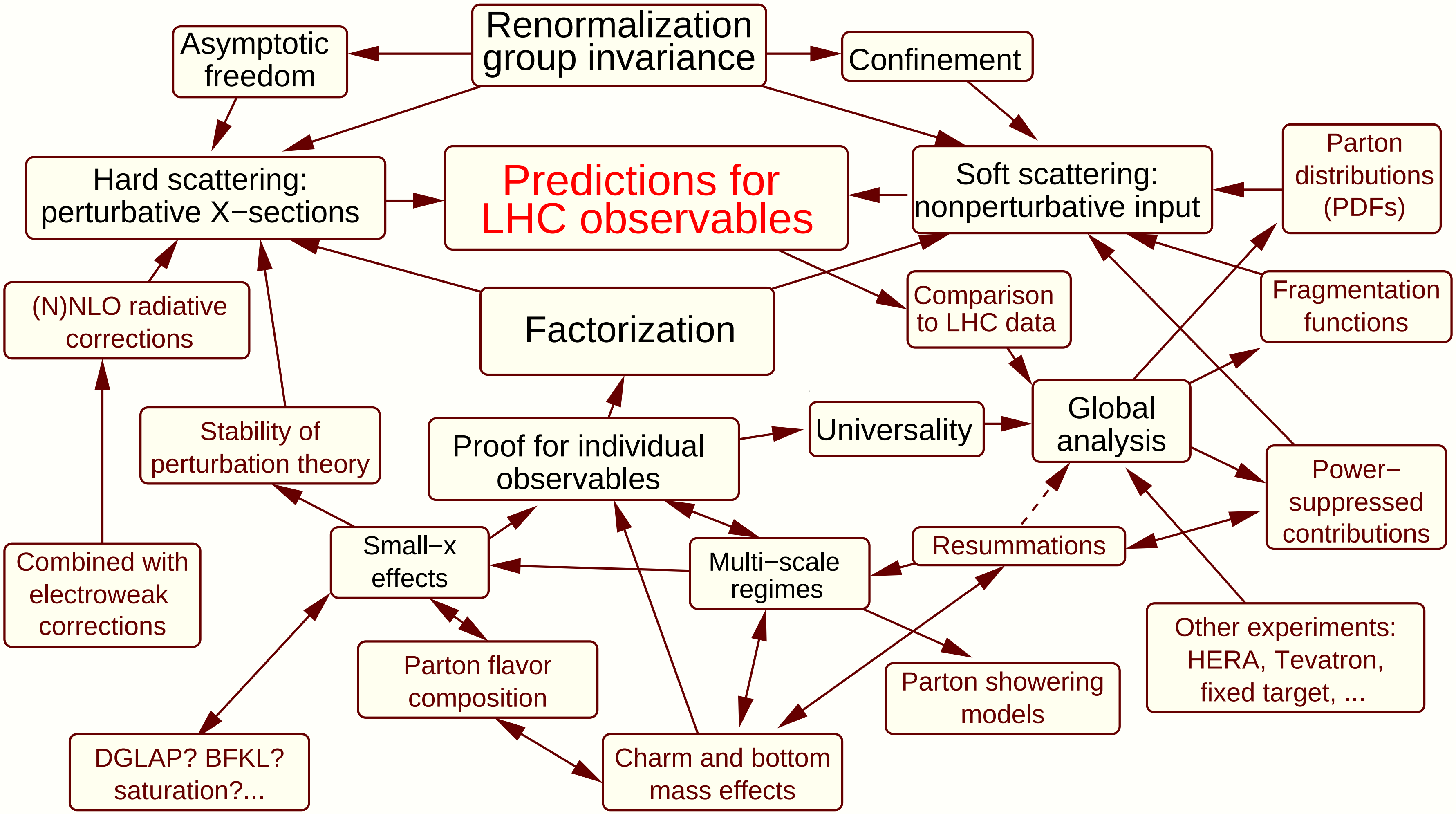
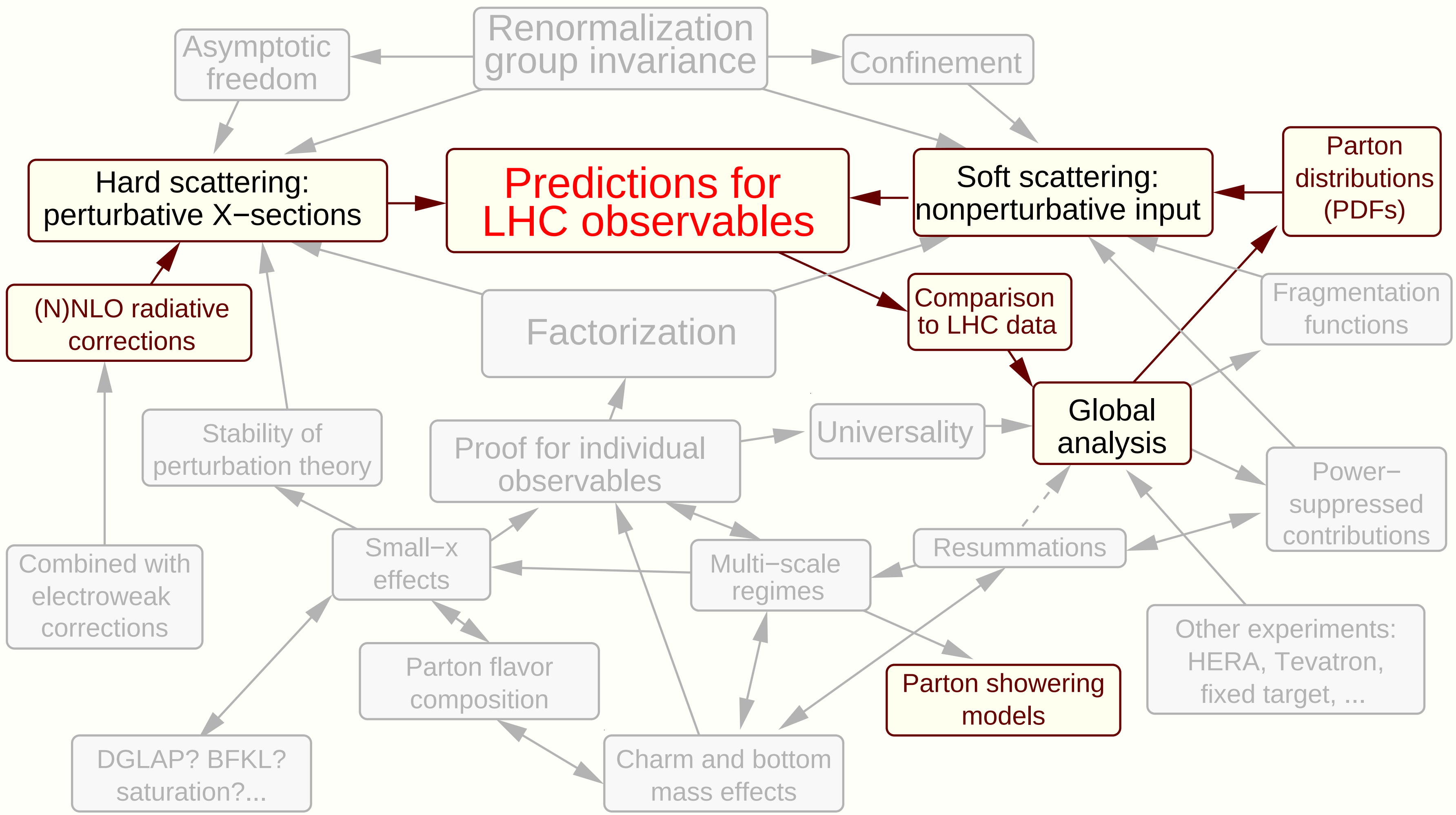
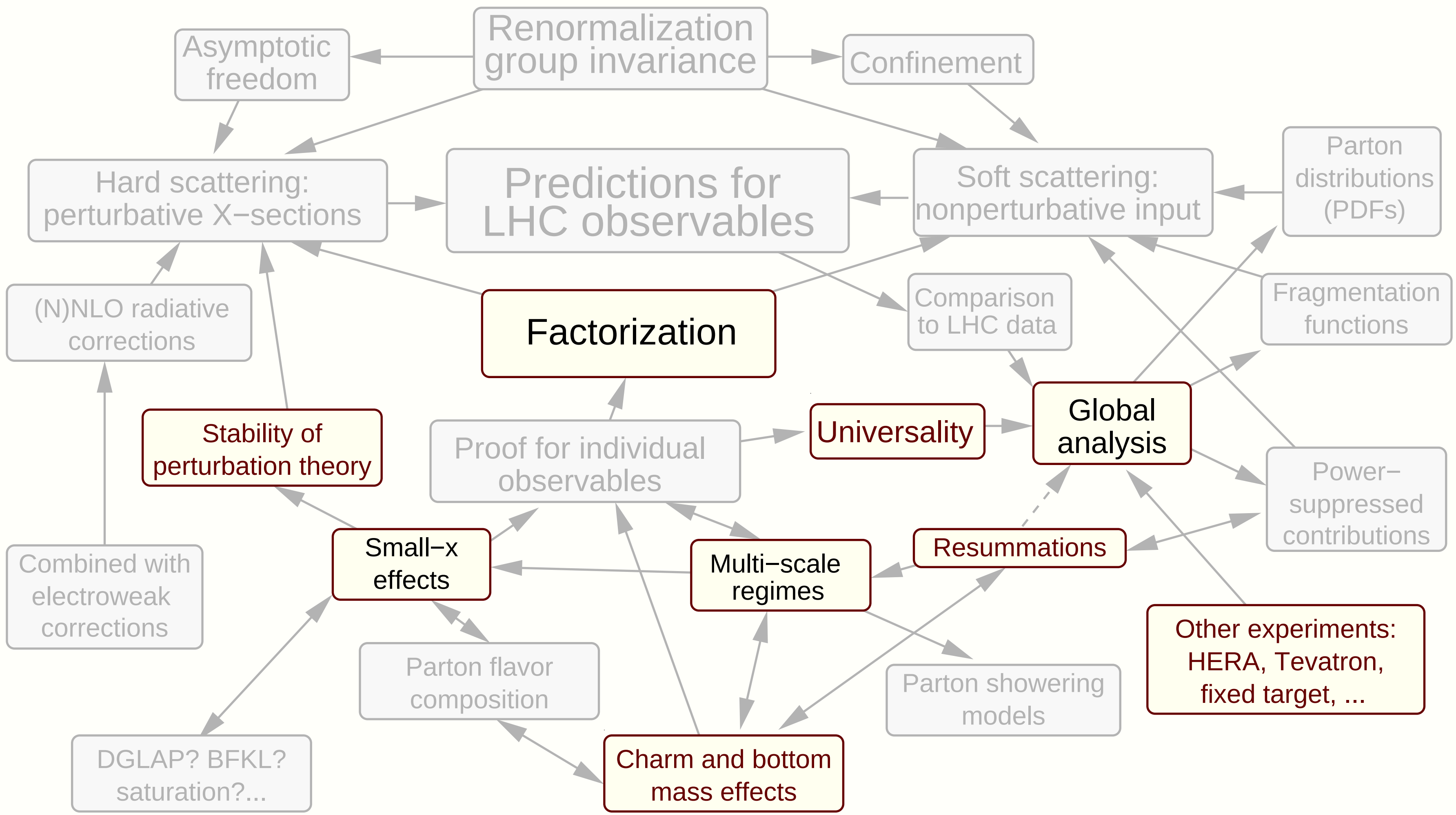
Strong interactions at LHC



(N)NLO perturbative calculations (F. Petriello's talk)



Factorization issues (this talk)



Main point

❒ scattering at the LHC $\neq$ "rescaled" scattering at the Tevatron

- dominance of sea parton scattering
- small typical momentum fractions x in several key searches (Higgs, lighter superpartners, ...)
- intensive QCD backgrounds
- complicated selection of new physics candidates
  $\Rightarrow$ reliance on differential distributions

At the LHC, many cross sections must be predicted at a few percent accuracy in an untested kinematical regime

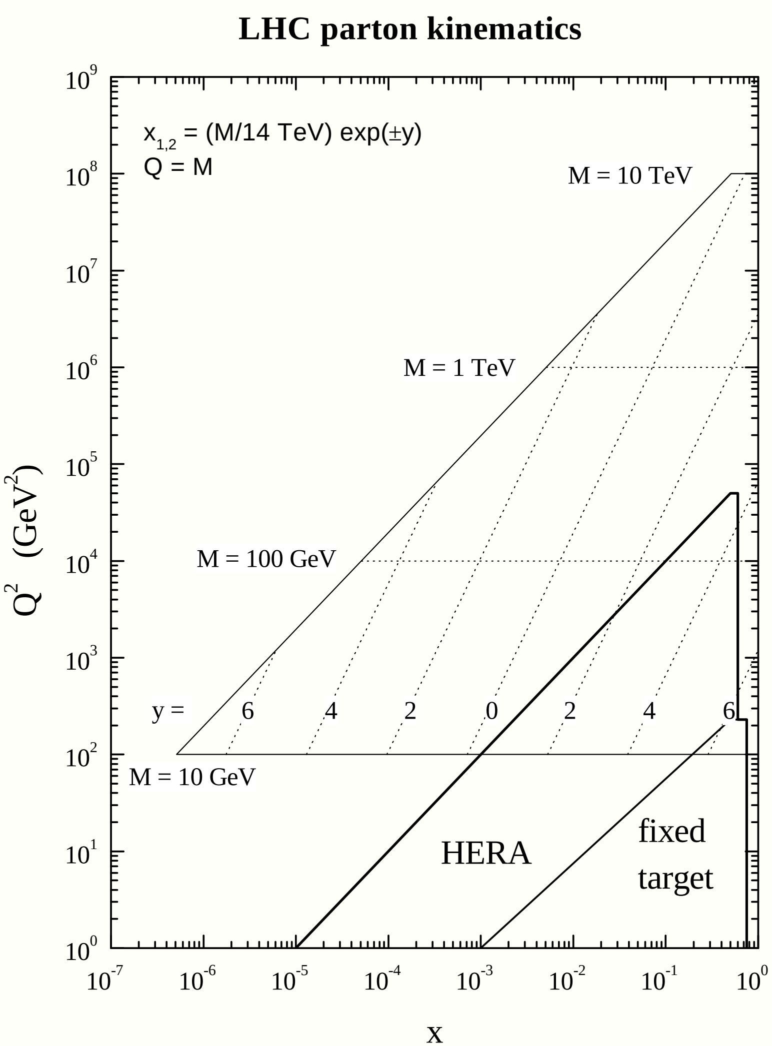
Some issues to be pursued are

- ❒ precision exploration of factorization (PDFs) and perturbative stability
- ❒ development of factorization for many-scale observables (resummation)
- ❒ understanding of new effects at small x, and in gluonic and heavy-flavor channels

I will refer to Drell-Yan-like processes (production of $\gamma^*, W, Z, H, \gamma\gamma$, ...) to find examples of relevant effects

Studies of QCD processes at small x, small Q, or forward rapidities at Tevatron and HERA are crucial for reducing uncertainties at the LHC

LHC [$\sqrt{s} = 14$ TeV] vs. Tevatron [$\sqrt{s} \approx 1.96$ GeV]: basic differences



- In units of $\sqrt{s}$:
  - Kinematics of a heavy particle with $M \sim 7M_Z \sim 700$ GeV is similar to the kinematics of W and Z bosons at the Tevatron
  - $M \sim M_Z \sim 100$ GeV at the LHC is kinematically similar to $M \sim 14$ GeV at the Tevatron (messy region!)
- Hadronic dynamics does not scale!
  - logarithmic scaling violations [$\alpha_s(M)$]
  - different particle contents (W, Z, t, H at ~ 100 GeV)
  - pp vs. $p\bar{p}$ scattering: reduced qq cross sections; enhanced qg and gg cross sections

QCD components at LHC: precision factorization for single-scale observables

Assumptions:

$$\begin{aligned} Q^2 &\gg 1\ \text{GeV}^2 \\ \{X\} \equiv \left\{\frac{p_i \cdot p_j}{Q^2}\right\} &\sim 1 \text{ for } i \neq j \\ \Leftrightarrow \{\ln(X)\} &\sim 0 \\ p_i^2 \equiv m_i^2 &\sim Q^2 \ (\text{ or } \ll Q^2) \end{aligned}$$

Examples:

- DIS structure functions $F_i(x, Q^2)$
- total cross sections for heavy particle production σ_{tot} ($Q^2 \sim M_h^2$)
- $d\sigma/dq_T$ at large q_T ($q_T^2 \sim Q^2$)

Inclusive structure functions for lepton-nucleon deep inelastic scattering

$$F(x,Q) = \sum_{a=u,\bar{u},\dots,g} \int_x^1 d\xi H_a\left(\frac{x}{\xi}, \frac{Q}{\mu}\right) f_{a/p}(\xi,\mu) + \mathcal{O}\left(\frac{1}{Q}\right)$$

☞ "Hard parts" $H_a(x/\xi, Q/\mu)$ can be calculated perturbatively

☞ μ is a factorization scale; $\mu \sim Q$ to avoid large logs $\ln^m(Q/\mu)$ in $H_a(x/\xi, Q/\mu)$

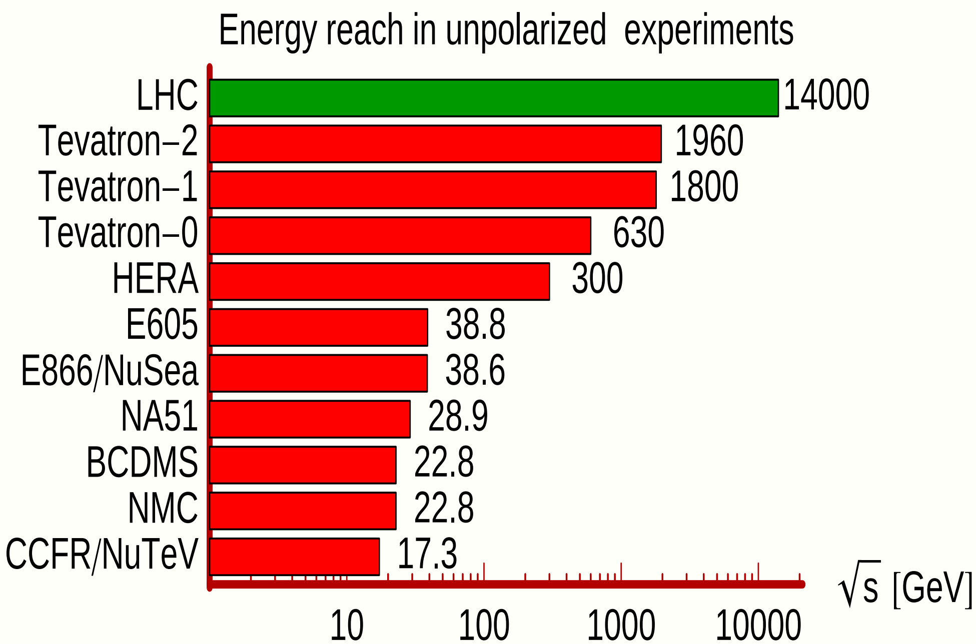
☞ $f_{a/p}(\xi,\mu)$ are *universal* parton distribution functions (PDF's) satisfying the known evolution equations (DGLAP):

$$\mu \frac{df_{a/p}(x,\mu)}{d\mu} = \sum_b \int_x^1 \frac{dy}{y} P_{a/b}\left(\frac{x}{y}\right) f_{b/p}(y,\mu).$$

☞ Parameterizations of $f_{a/p}(\xi,\mu)$ at $\mu = \mu_0 \sim 1$ GeV are found by fitting to the data (DIS, vector boson and jet production,...)

Global analyses of hadronic data *(CTEQ, MRST)*

Minimization of 20 PDF parameters + experiment normalizations + control parameters for correlated systematic errors (about 100 parameters in total)

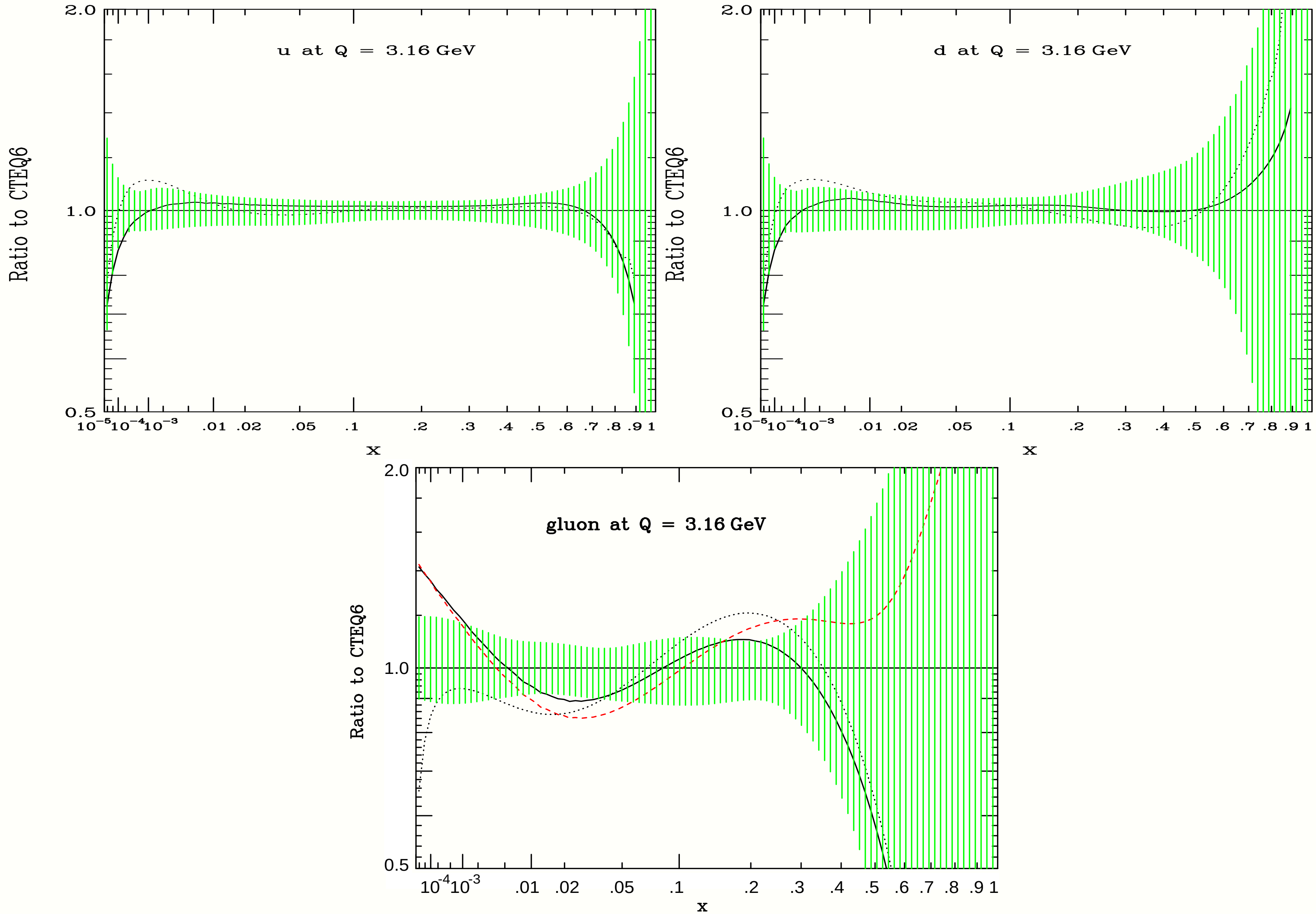


11 (12) experiments at $\langle Q^2\rangle > 4\,(2)$ GeV^2, $\langle x\rangle > 6\cdot 10^{-5}$

- deep-inelastic scattering in a large range of Q, for a variety of targets (p, d, Fe) and lepton beams (e, μ, ν)
- fixed-target Drell-Yan pair production
- W-boson production
- Tevatron jet production

- NLO (CTEQ), N$^{3/4}$NLO (MRST)
- Large number of experiments needed for flavor separation
- $\bar{u}(x) \neq \bar{d}(x) \neq \bar{s}(x)$;
  $s(x) \neq \bar{s}(x)$? *(CTEQ, 2003)*; $u^p(x) \neq d^n(x)$? *(MRST, 2003)*

Uncertainties in $f_{a/p}(x,Q)$ at $Q^2 = 10$ GeV^2 (CTEQ6)



Global vs. DIS-based fits

- ❒ Global fits *(CTEQ; MRST)*
  - ◆ Data from DIS, lepton pair and inclusive jet production
    - ✔ Separation of most parton flavors
    - ✗ Non-negligible tensions between different experiments
    - ✗ Acceptable PDF sets are selected by applying an approximate criterion
      - ✧ CTEQ “tolerance”: $\delta\chi^2 = 100 \Leftrightarrow 90\%$ c.f.
      - ✧ MRST “tolerance”: $\delta\chi^2 = 50$
- ❒ DIS-based fits *(Alekhin; H1, ZEUS; Giele et al.)*
  - ◆ DIS experiments only
    - ✔ PDF uncertainties can be introduced according to the exact statistical definition ($\delta\chi^2 = 1 \Leftrightarrow 65\%$ c.f.)
    - ✗ Insufficient information about sea parton flavors ($\bar{d} - \bar{u}$, g, ...)
    - ✗ disagree with a part of pN scattering data

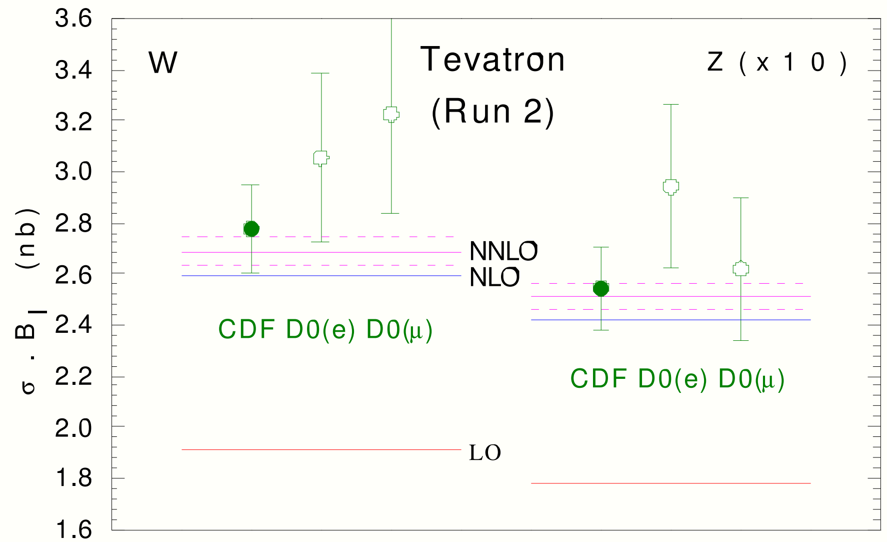
Total W and Z cross sections

- Monitors of the beam and parton luminosity at future colliders
  (Dittmar, Pauss, Zurcher; Khoze, Martin, Orava, Ryskin; Giele, Keller)

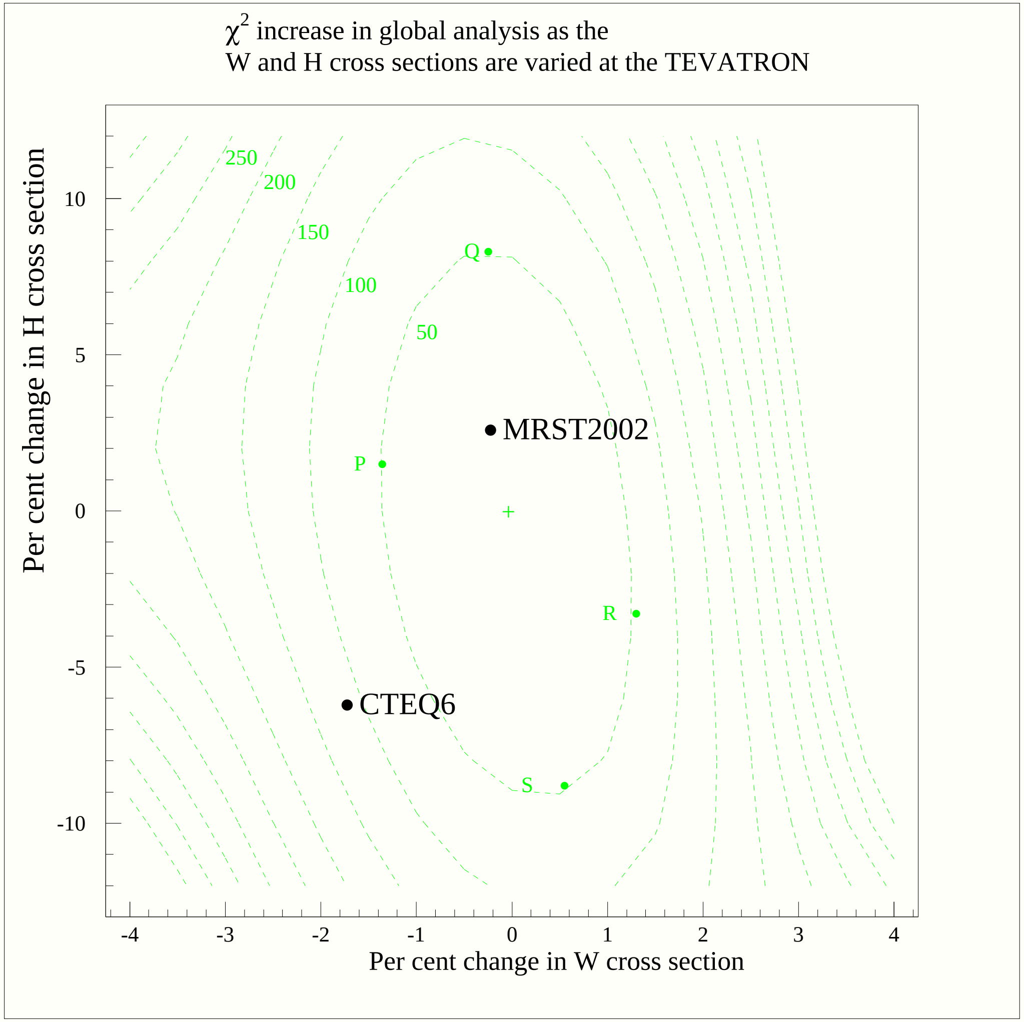
Total cross sections: NNLO QCD corrections

$$\sigma_{tot}(p\bar{p} \to V) = \sum_{partons} \int dx_1 dx_2 f_{a/p}(x_1) f_{b/\bar{p}}(x_2) \hat{\sigma}_{tot}(ab \to V)$$

- NNLO hard cross section $\hat{\sigma}_{tot}(ab \to V)$
  (Hamberg, van Neerven, Matsuura, 1991; Harlander and Kilgore, 2002)
- Partial NNLO results for parton distributions $f_{a/p}(x)$
- Scale dependence of order 1%
- NNLO K-factor is about 1.04 at the Tevatron and 0.98 at the LHC
  (MRST'03)



Sources of differences between CTEQ & MRST

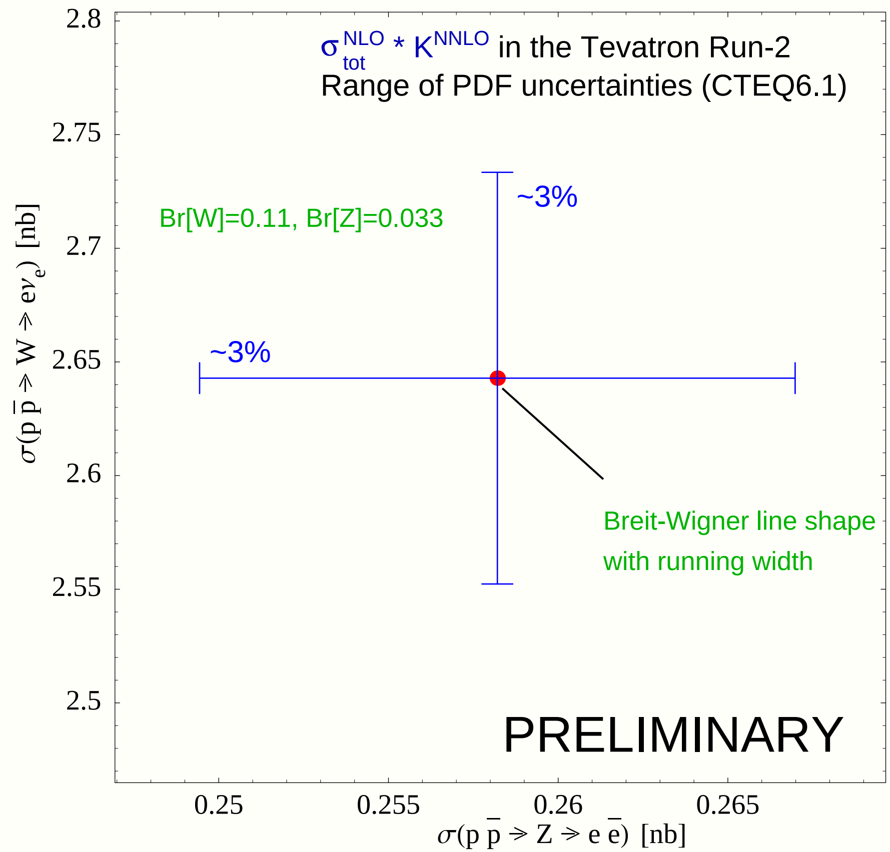


- Selection & weighting of data in the fit
- Parametric form of PDFs at $\mu = \mu_0$
- Definition of α_s at (N)NLO
- Assumptions about sea flavor symmetries
- Treatment of heavy flavors
- Electroweak corrections
- acceptance

(Frixione, Mangano, 2004)

Electroweak effects in σ_{tot}

- tree-level approximation insufficient!
  ☞ EW corrections, updated EW parameters

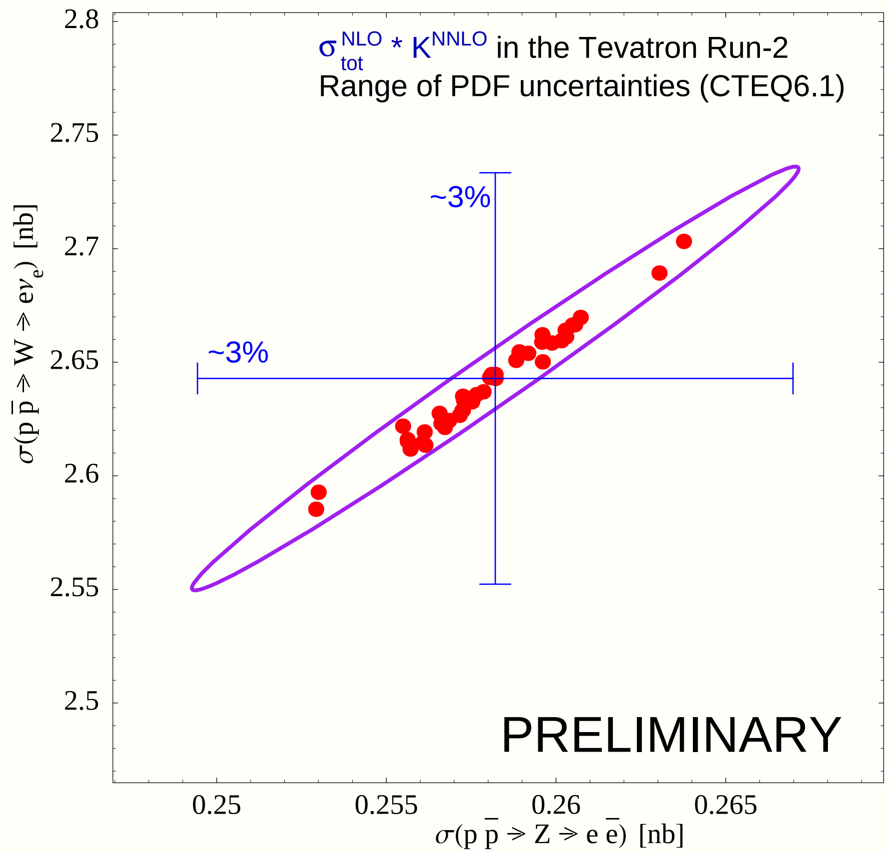


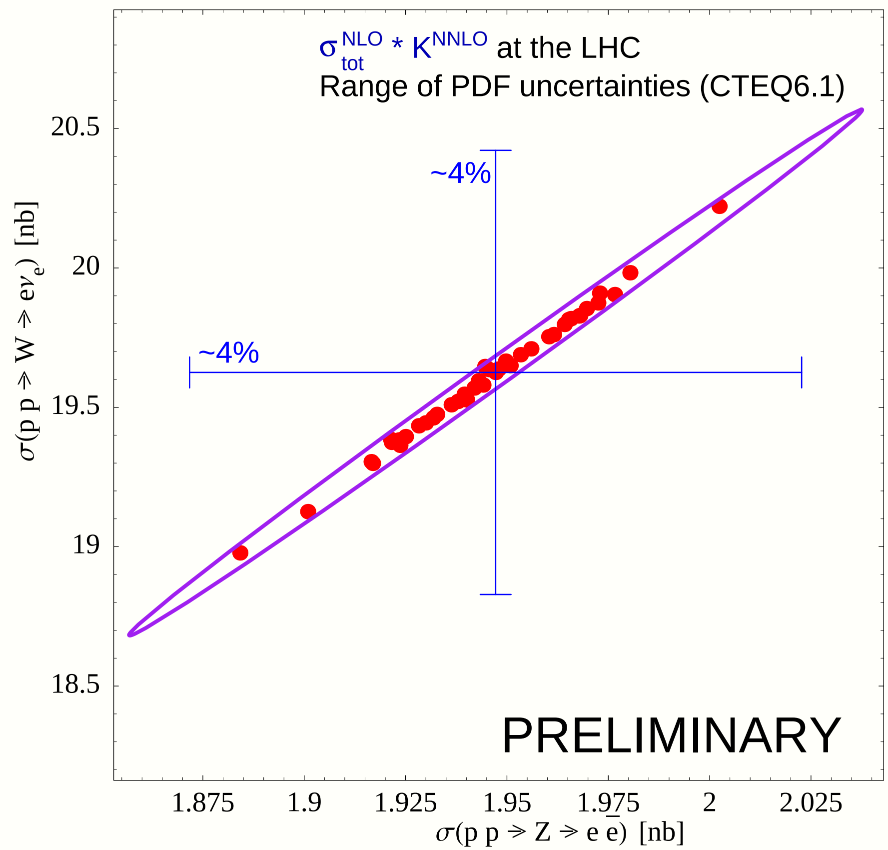
$\sigma_{tot}(W)$ and $\sigma_{tot}(Z)$: standardization of theory predictions (in progress)

Collider/ program	Cross section (pb)	CTEQ6M	MRST 2002(NLO)
Tevatron ($\sqrt{s} = 1.96$TeV) wttot	$\sigma(W \to \ell\nu)$ (SigmaTot1)	2526	2548
	$\sigma(W)$ at Q=80.423 GeV	23773	23988
	$\sigma(W) \cdot 0.1068$	2539	2562
	$\sigma(W) \cdot 0.1084$	2577	2601
ResBos	$\sigma(W \to \ell\nu)$	2588 ± 6	2606 ± 6
MRST'02 paper	$\sigma(W)\cdot 0.1068$		2600 (1.4% above WTTOT)
LHC ($\sqrt{s} = 14$TeV) wttot	$\sigma(W^+ \to \ell\nu)$ (SigmaTot1)	11525	11444
	$\sigma(W^- \to \ell\nu)$ (SigmaTot1)	8497	8500
	$\sigma(W \to \ell\nu)$ (SigmaTot1)	20022	19944
	$\sigma(W)$ at Q=80.423 GeV	188549	187885
	$\sigma(W) \cdot 0.1068$	20137	20066
	$\sigma(W) \cdot 0.1084$	20439	20367
ResBos	$\sigma(W^+ \to \ell\nu)$	11899 ± 43	11891 ± 43
	$\sigma(W^- \to \ell\nu)$	8717 ± 29	8799 ± 29
	$\sigma(W \to \ell\nu)$	20616 ± 52	20690 ± 52
MRST'02 paper	$\sigma(W)\cdot 0.1068$		20400 (1.6% above WTTOT)

Cancellation of PDF uncertainties in $\sigma_{tot}(Z)/\sigma_{tot}(W)$

(Huston, P. N., Pumplin, Stump, Tung, Yuan, 2004)





- In spite of different quark flavors, a measurement of $\sigma(Z)$ will constrain $\sigma(W)$ (and possibly other quark-dominated cross sections)!

QCD components at the LHC:

multi-scale factorization (resummation)

$$Q^2 \gg 1\ \mathrm{GeV}^2$$

$$X_{ij} \equiv \frac{p_i \cdot p_j}{Q^2} \gg 1 \quad (\ll 1) \text{ for some } i,\ j$$

$$\Leftrightarrow \left|\ln(X_{ij})\right| \gg 0$$

The perturbation series have to be reorganized to evaluate the sum

$$\sum_{m=0}^{\infty} \alpha_s^m \sum_n c_{mn} \ln^n \left(X_{ij}\right)$$

Examples of resummation

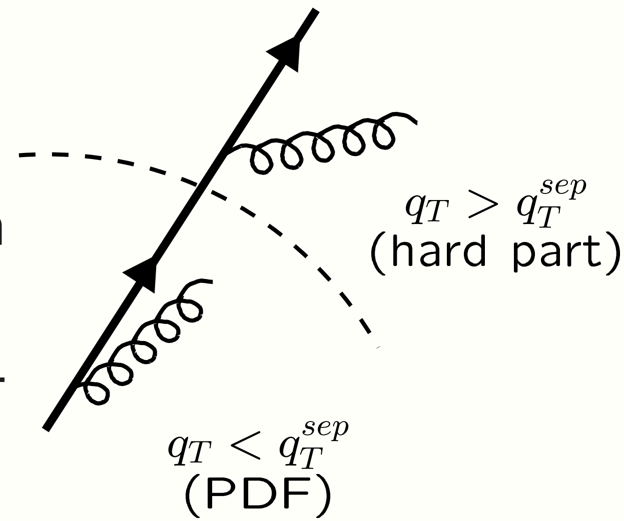
- Mass logarithms $\ln(\mu/m_i)$ *(Dokshitzer, Gribov, Lipatov, Altarelli, Parisi)*
- $\ln(1/x)$ *(Balitsky, Fadin, Kuraev, Lipatov)*
- $\ln(q_T/Q)$ *(Dokshitzer, Dyakonov, Troyan; Parisi, Petronzio; Altarelli, Ellis, Greco, Martinelli; Collins, Soper, Sterman;...)*
- $\ln(1-x)$ *(Sterman; Catani and Trentadue; ...)*
- $\ln(1-x)$ and $\ln(q_T/Q)$ *(Li; Kulesza, Sterman, Vogelsang; ...)*
- $\ln(q_T/Q)$ and $\ln(\mu/M_Q)$ for heavy quarks *(P. N., Kidonakis, Olness, Yuan)*
- Inter-jet energy flow *(Dasgupta, Salam; ...)*
- ...

Resummation for transverse momentum (q_T) distributions

Conventional factorization

✓ is OK for inclusive cross sections

✓ leads to discontinuities and large logarithms in differential cross sections $\frac{1}{q_T^2}\alpha_S^n \ln^m \frac{q_T^2}{Q^2}$, $m = 0, ..., 2n-1$ in q_T distributions as $q_T \to 0$



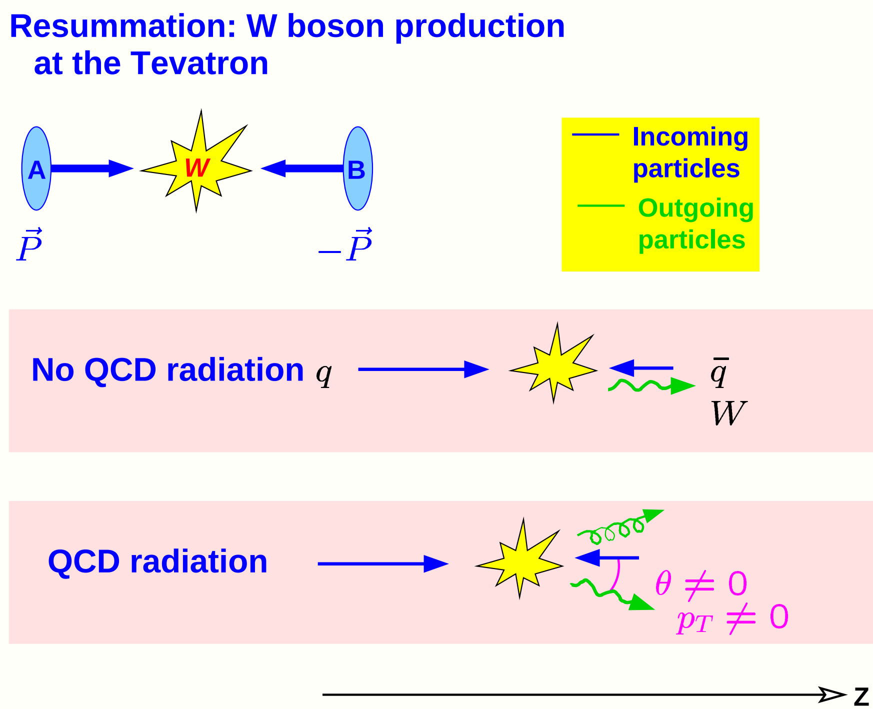
Solution: summation of logarithms through all orders of α_S *(Dokshitzer, Dyakonov, Troyan; Parisi, Petronzio, Altarelli, Ellis, Greco, Martinelli; ..)*

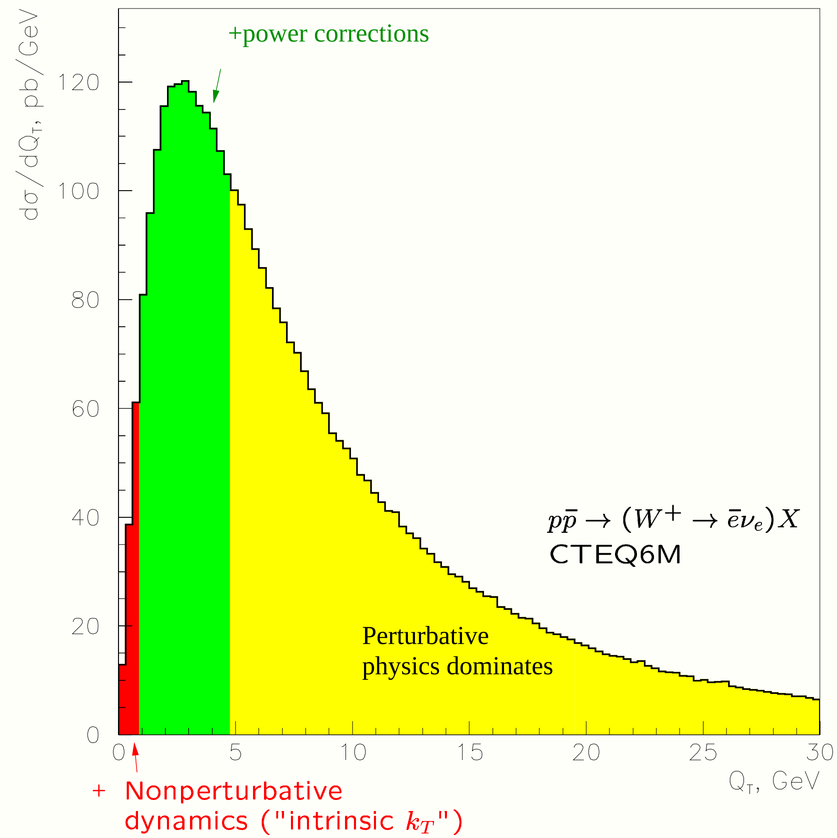
Its validity is proved by a factorization theorem for k_T-dependent PDF's *(Collins, Soper, 1981; Collins, Soper, Sterman, 1985; Collins, Metz, 2004)*

✓ prediction from first principles of QCD

✓ applicable to a wide range of processes at pp, ep, and ee colliders

q_T resummation for vector boson production at the Tevatron





Needed to precisely measure the W-boson mass ($\delta M_W/M_W < 7 \cdot 10^{-4}$)

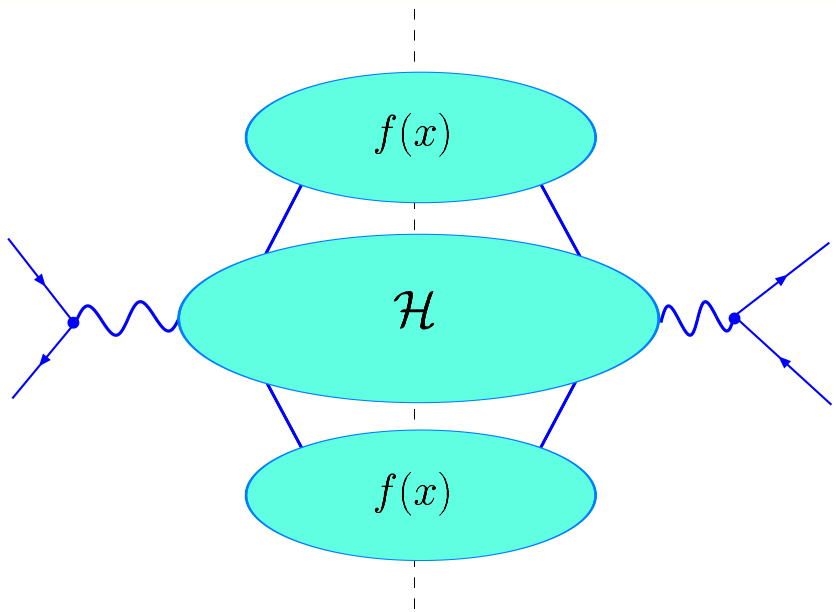
Monte-Carlo integrator ResBos *(Balazs, P. N., Yuan)*: resummed NLO with elements of NNLO in Collins-Soper-Sterman (CSS) method

Resummation describes all q_T range in one unified framework

QCD factorization in hard and soft regions

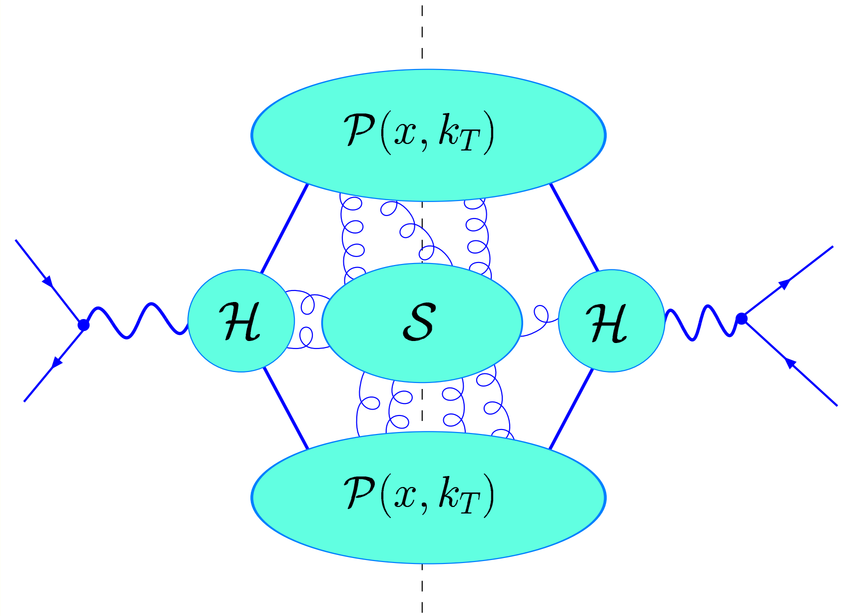
Finite-order (FO) factorization

$$\Lambda^2_{QCD} \ll q_T^2 \sim Q^2$$

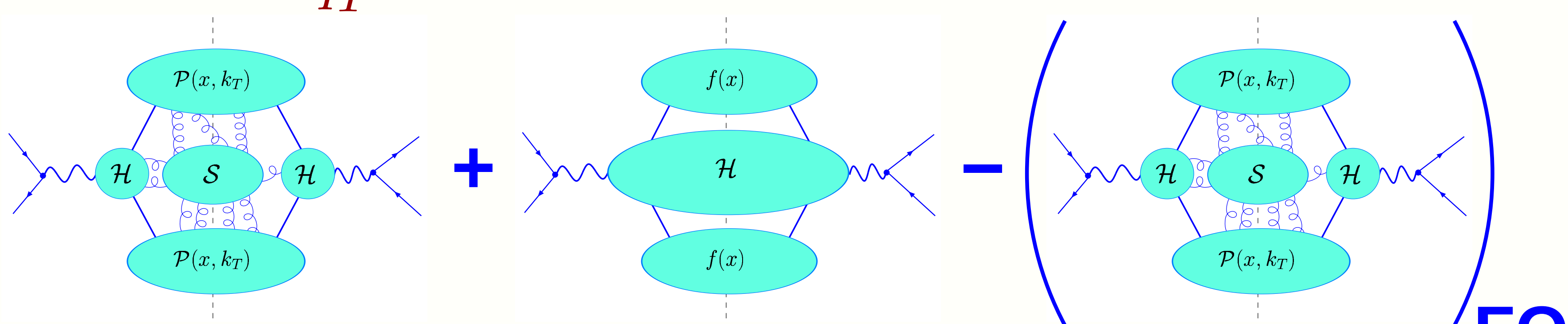


Small-q_T factorization

$$\Lambda^2_{QCD} \ll q_T^2 \ll Q^2$$



Solution for all q_T:



Factorization at $q_T \ll Q$

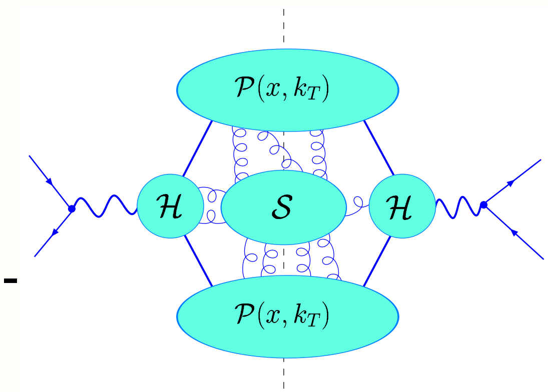
Realized in space of the impact parameter b (conjugate to q_T)

$$\left.\frac{d\sigma_{AB\to VX}}{dQ^2 dy dq_T^2}\right|_{q_T^2\ll Q^2} = \sum_{a,b=g,\overset{(-)}{u},\overset{(-)}{d},\ldots} \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{q}_T\cdot\vec{b}} \widetilde{W}_{ab}(b,Q,x_A,x_B)$$

$$\widetilde{W}_{ab}(b,Q,x_A,x_B) = |\mathcal{H}_{ab}|^2\, e^{-\mathcal{S}(b,Q)} \overline{\mathcal{P}}_a(x_A,b)\overline{\mathcal{P}}_b(x_B,b)$$

$\mathcal{H}_{ab}$ is the hard vertex, $\mathcal{S}$ is the soft (Sudakov) factor, $\overline{\mathcal{P}}_a(x,b)$ is the unintegrated PDF

$$\overline{\mathcal{P}}_a(x,b) = \int d^{n-2}\vec{k}_T e^{-i\vec{k}_T\cdot\vec{b}} \mathcal{P}_a(x,\vec{k}_T)$$



When $b \ll 1$ GeV^{-1}, $S(b,Q)$ and $\overline{\mathcal{P}}_a(x,b)$ are calculable in perturbative QCD;

$\overline{P}_a(x,b)$ factorizes as

$$\overline{\mathcal{P}}_{a/A}(x,b) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{C}_{ja}(\xi,\mu_F b) f_{j/A}(\frac{x}{\xi},\mu_F) + \mathcal{O}(b^2) \equiv \left(\mathcal{C}_{ja}\otimes f_{a/A}\right) + \mathcal{O}(b^2)$$

(note the power-suppressed terms)

Models for $\widetilde{W}_{ab}$ at large b

To perform the Fourier transform to q_T space, we must know $\widetilde{W}_{ab}(b, Q, x_A, x_B)$ at $0 \leq b \leq \infty$

Several models exist for the large-b behavior *(Collins, Soper, 1982; CSS, 1985; Qiu, Zhang, 2001; Kulesza, Sterman, Vogelsang; Guifanti, Smye)*:

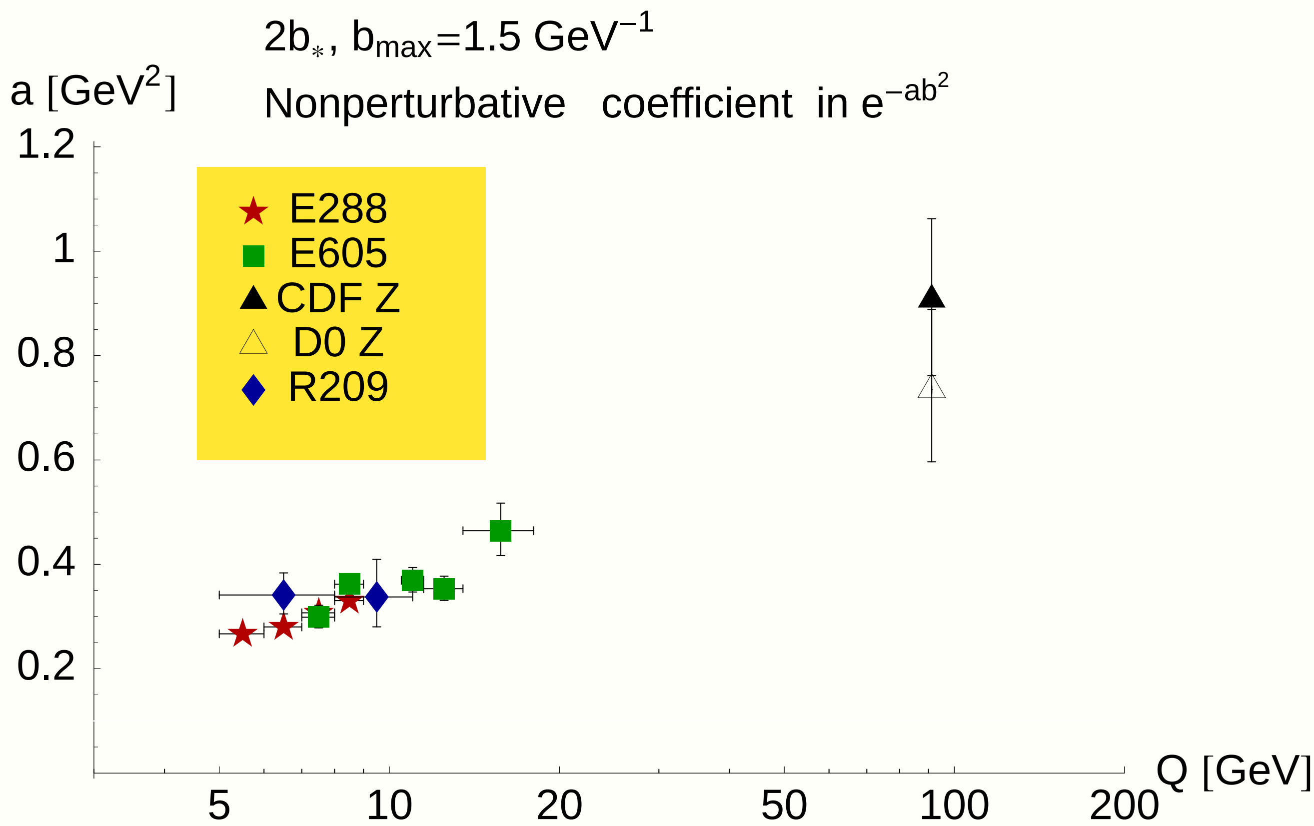
$$\widetilde{W}_{ab}(b, Q, x_A, x_B)\Big|_{all\ b} = \widetilde{W}_{ab}(b_*, Q, x_A, x_B)\Big|_{pert} e^{-S_{NP}(b,Q,b_*,...)}$$

- ❒ $S_{NP}(b, Q)$ is an effective nonperturbative function (universal in Drell-Yan-like and SIDIS processes)

- ❒ Renormalon analysis *(Korchemsky, Sterman; Tafat)*

$$S_{NP}(b, Q) \approx b^2 \{a_1 + a_2 \ln Q + ...\} \oplus \text{small corrections}$$

Energy dependence of the nonperturbative smearing a in Drell-Yan pair and Z boson production *(Konychev, P.N., 2005)*



$$S_{NP}(b,Q) \approx a(Q) b^2,$$

with

$$a \sim \langle k_T^2 \rangle / 2$$

Dependence of best-fit $a(Q)$ on $\ln Q$ is approximately linear, with the slope ($a_2 \approx 0.184 \pm 0.018$ GeV^2) in excellent agreement with renormalon analysis + lattice QCD ($a_2 \approx 0.18^{+0.12}_{-0.09}$ GeV^2)

QCD components at the LHC: factorization at small x

W,Z, and Higgs boson production at the LHC:
typical $x \sim$ a few $10^{-3} \ll 1$

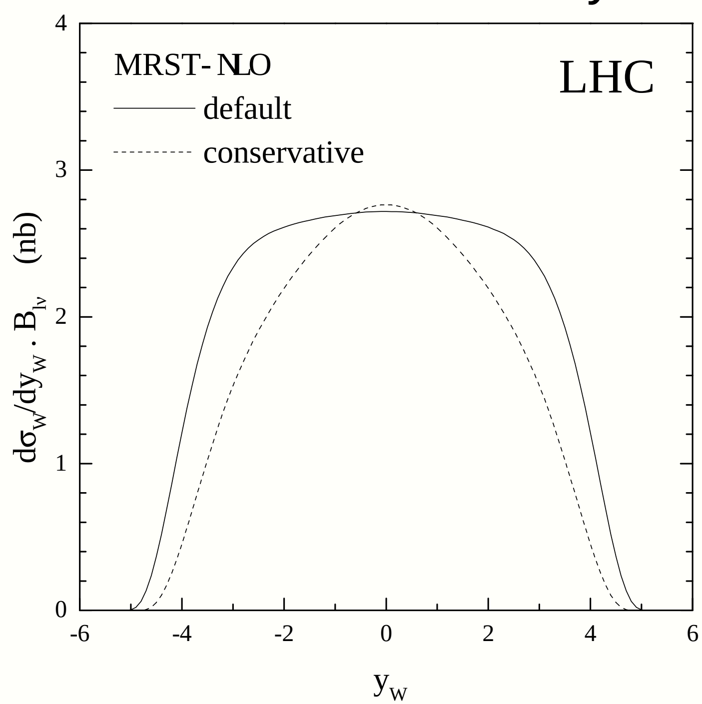
DGLAP factorization (q_T resummation) works well when x is not small

Can small-x effects affect factorization (resummation) at the LHC?

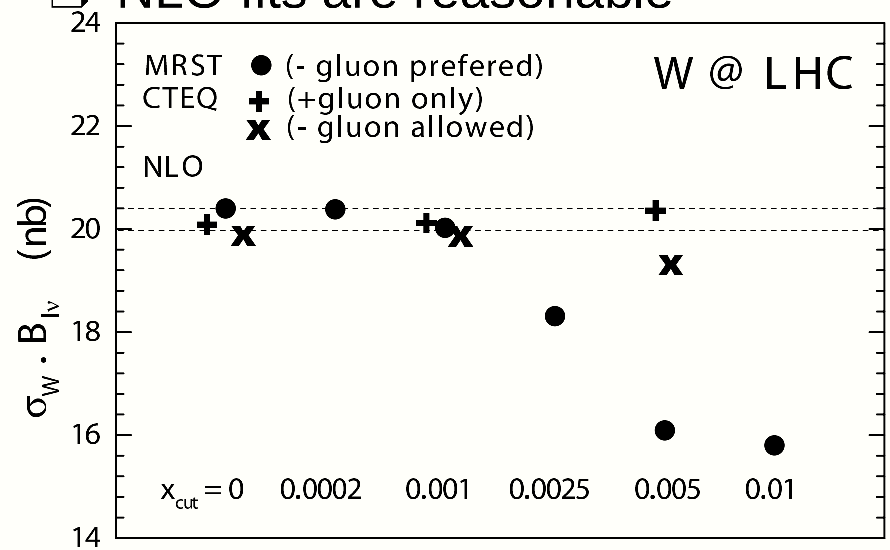
For the same hard scale Q, the magnitude of small-x effects may be negligible in one-scale observables ($\alpha_s(Q) \ll 1$) and important in multi-scale observables ($\alpha_s(q_T) \gtrsim 0.2 - 0.3$)

Tensions in NLO global fits of inclusive (one-scale) data?

- MRST (2002) finds tension between the small-x & Q DIS and high-p_T Tevatron jet data
  - Need for NNLO (resummation of $\ln(1/x)$ terms)?
- The "conservative" (without the small-x DIS data) and "standard" MRST PDFs are very different



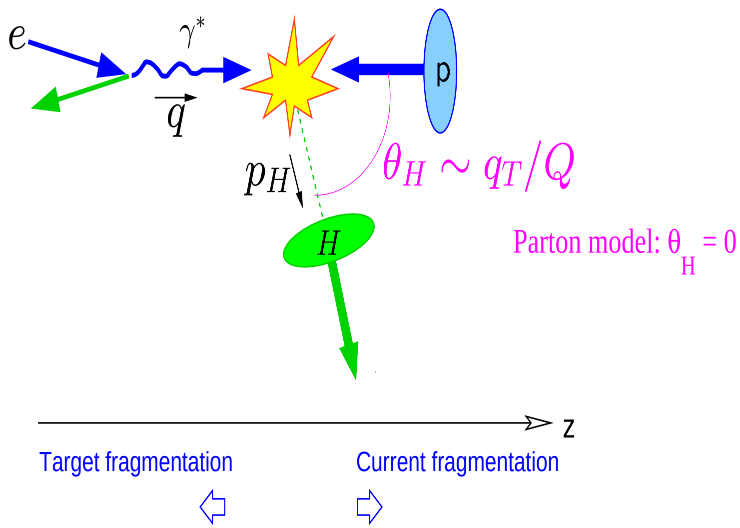
- CTEQ (2005) does not observe such a tension
- Removing the small-x & Q data changes χ^2 by $\delta\chi^2 < 100$ and approximately preserves the form of the PDFs
- NLO fits are reasonable

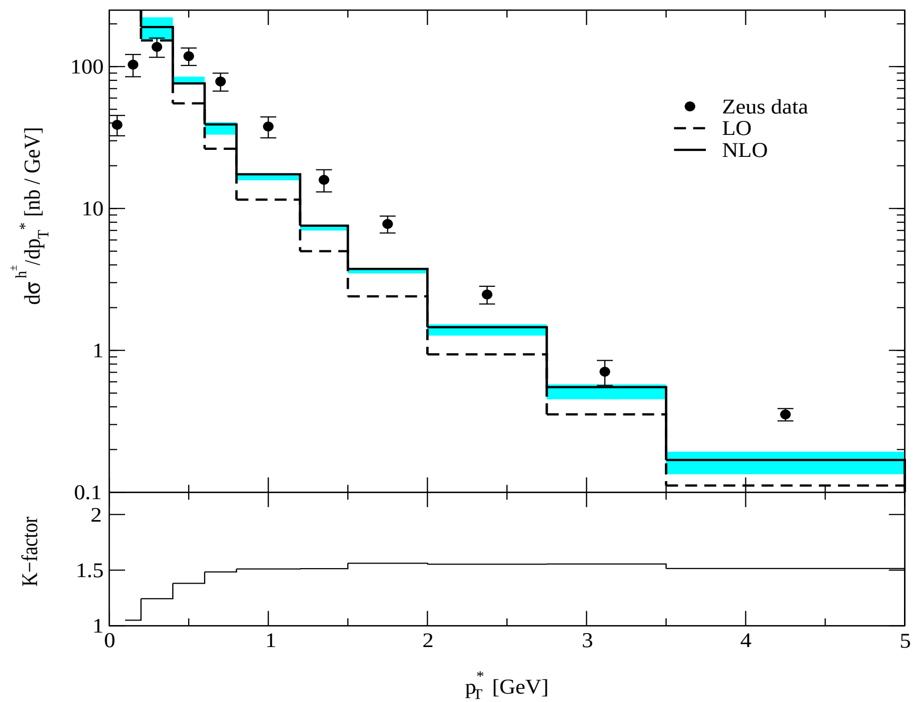


$\mathcal{O}(\alpha_s^2)$ corrections to DIS production of high-p_T hadrons at HERA

(Daleo et al.; Fontannaz et al.; Kniehl et al., 2003-2004)

Semi-inclusive DIS in $\gamma^* p$ c.m. frame





Kniehl, Kramer, Maniatis, 2004

Production of light hadrons: $H = \pi, K, p, ..$
or energy flow

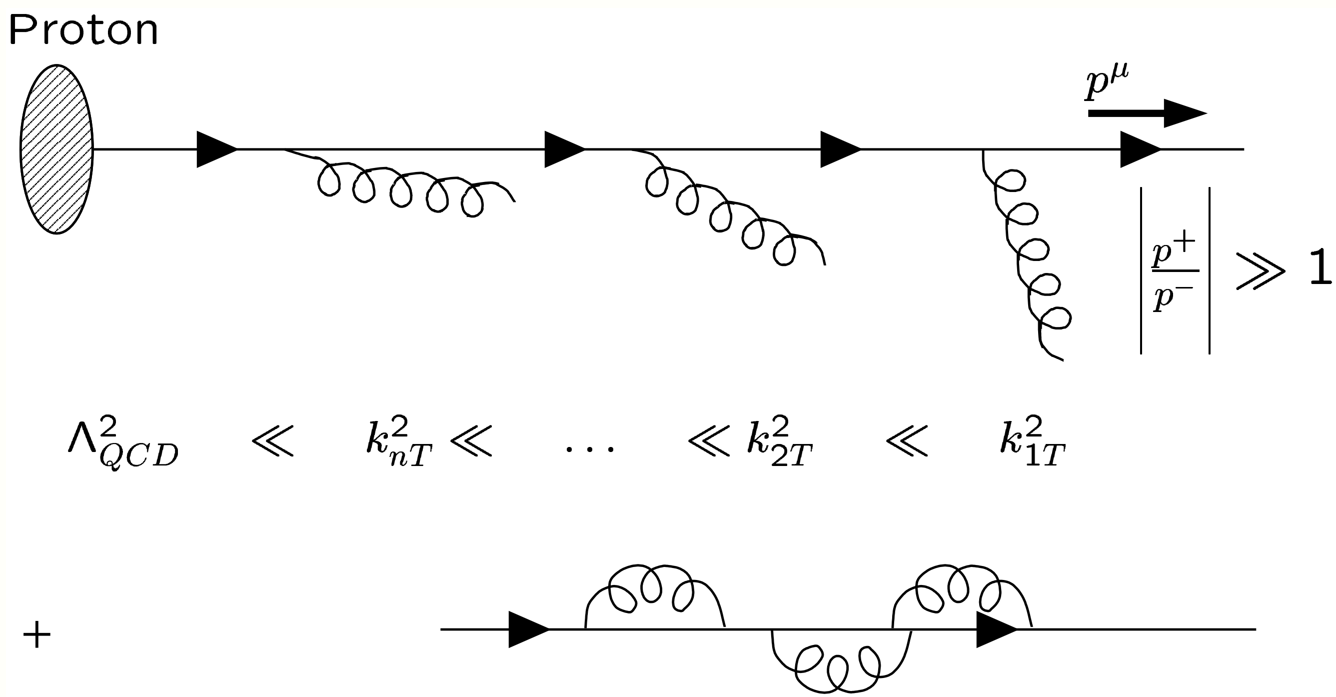
Production of heavy flavors: $H = D, B, ...$

$\mathcal{O}(\alpha_s^2)$ is not sufficient to describe the differential SIDIS data!

Possible small-x effects

- Enhanced gluonic activity at $x \lesssim 10^{-2}$
  - large $\mathcal{O}(\alpha_s^2)$ corrections to hard gluon scattering
    - $gg \to H$
    - $\gamma^* g \to q\bar{q}g$ to dijet and forward jet production at $p_T \sim Q$ in SIDIS at HERA (recently calculated)
    - ...
- Deviations from the DGLAP factorization picture caused by $\alpha_s^m(\mu)\,\ln^n(1/x)$ (to be resummed via BFKL equation)

Physics behind DGLAP factorization (cartoon)



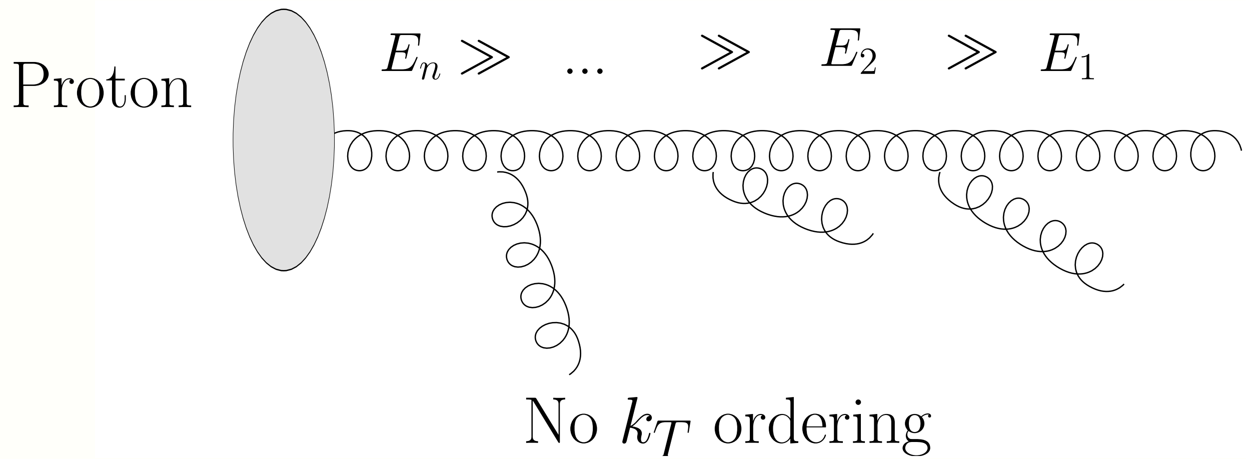
Probed parton is highly boosted throughout all evolution

Soft and collinear radiation

- ❒ dominates parton evolution
- ❒ factorizes from the hard scattering
- ❒ is “collimated” (“k_T-ordered”)

Angular distributions are described in a b-space resummation formalism (Collins, Soper, Sterman, 1985)

The ultimate energy loss scenario (BFKL)

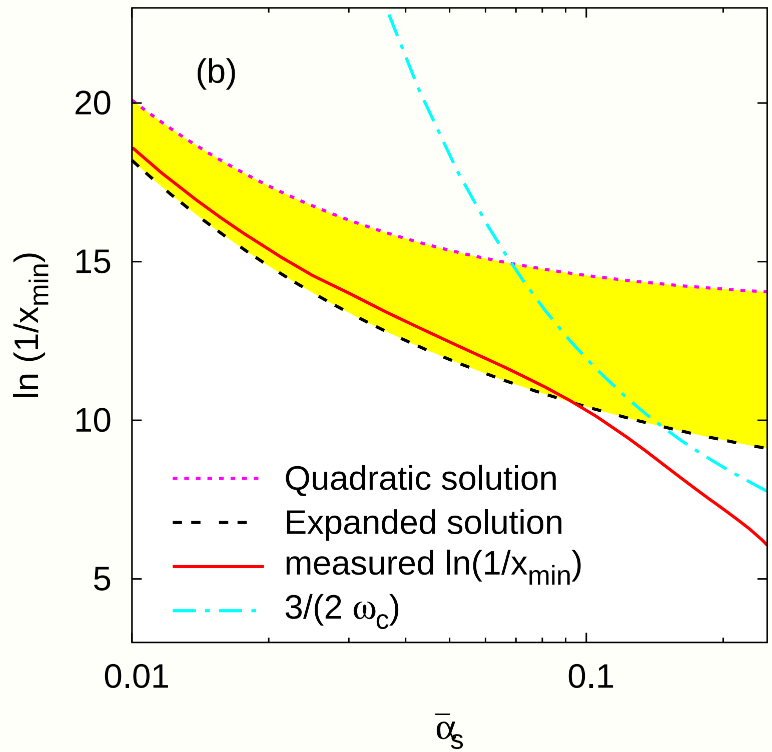


- The probed parton loses practically all energy through radiation
- The radiated partons do not have to be k_T-ordered
- Essential signature: broad angular distributions of the radiated hadrons

As x decreases, k_T-unordered dynamics may turn on faster in q_T distributions than in inclusive cross sections

$$\left[\text{Compare } \alpha_S^n(Q)\ \ln^m(1/x) \text{ and } \alpha_S^n(1/b)\ \ln^m(1/x)\right]$$

Preasymptotic suppression of $P_{gg}(x,\mu)$ at $x < 10^{-4} - 10^{-2}$ and $\mu < 1-2$ GeV *(Ciafaloni, Colferai, Salam, Stasto, 2003)*



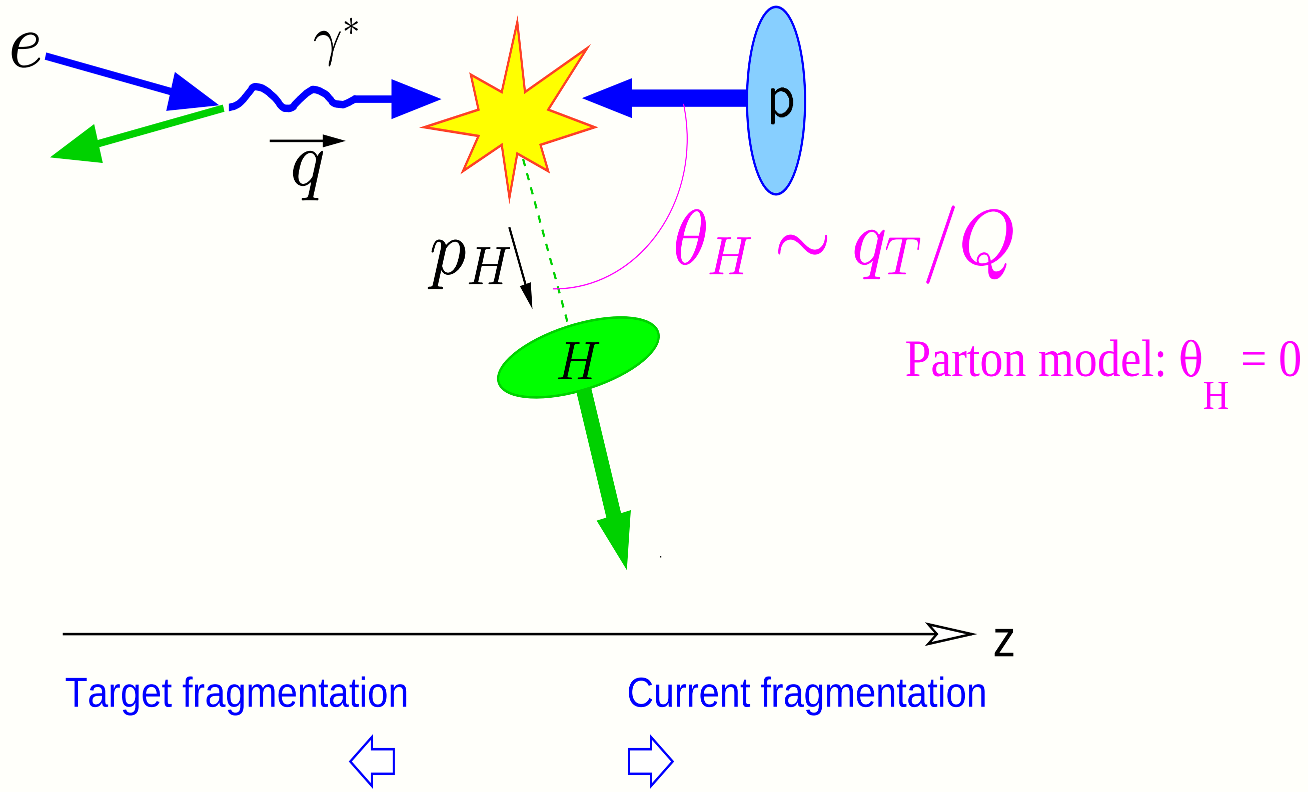
- may suppress $d\sigma/dq_T$ at $q_T < 1-2$ GeV
- together with enhanced gluoproduction at $q_T > 2$ GeV, may lead to hardening ("broadening") of q_T distributions

Resummation for the Tevatron and LHC at small x

(S. Berge, P. N., F. Olness, C.-P. Yuan, hep-ph/0401128; hep-ph/0410375)

$$x < 10^{-2}$$

Semi-inclusive DIS in $\gamma^* p$ c.m. frame

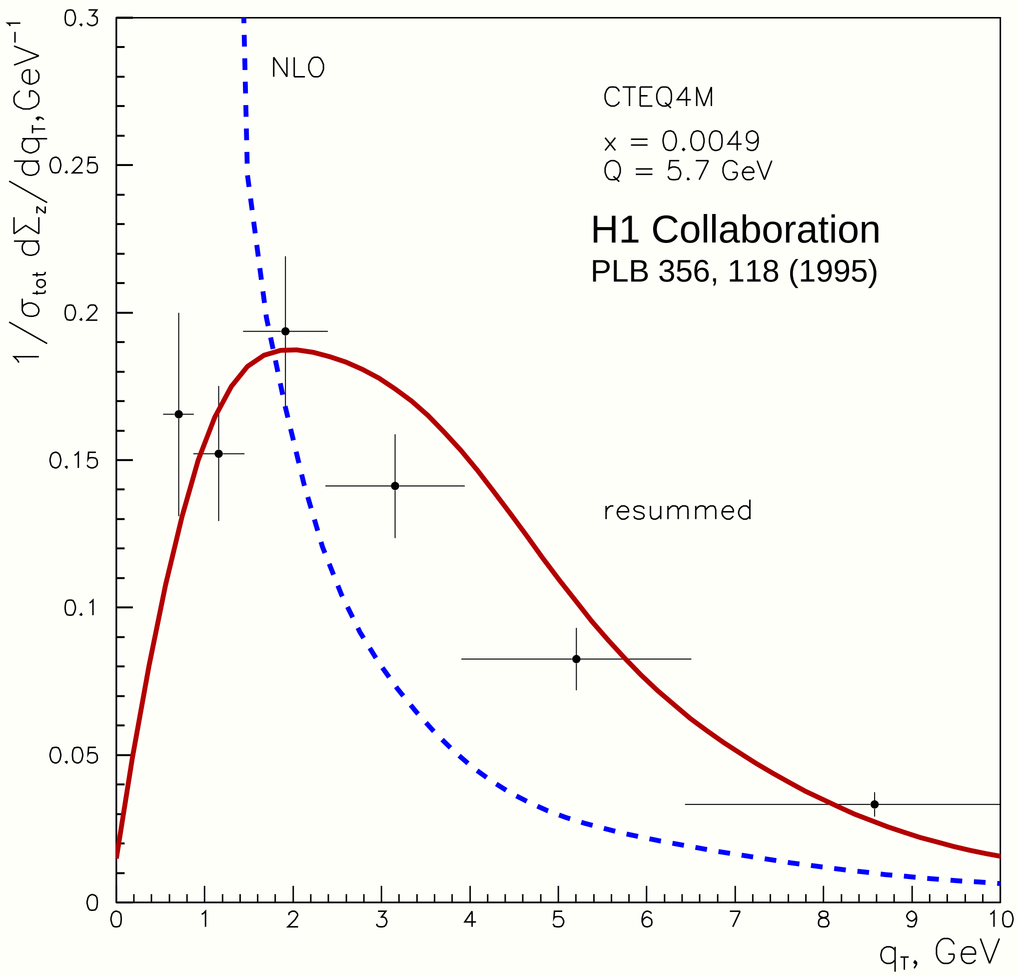


Production of light hadrons: $H = \pi, K, p, ..$
or energy flow

Production of heavy flavors: $H = D, B, ...$

Semi-inclusive hadroproduction at small polar angles

$$\theta_H \sim q_T/Q \longrightarrow 0$$

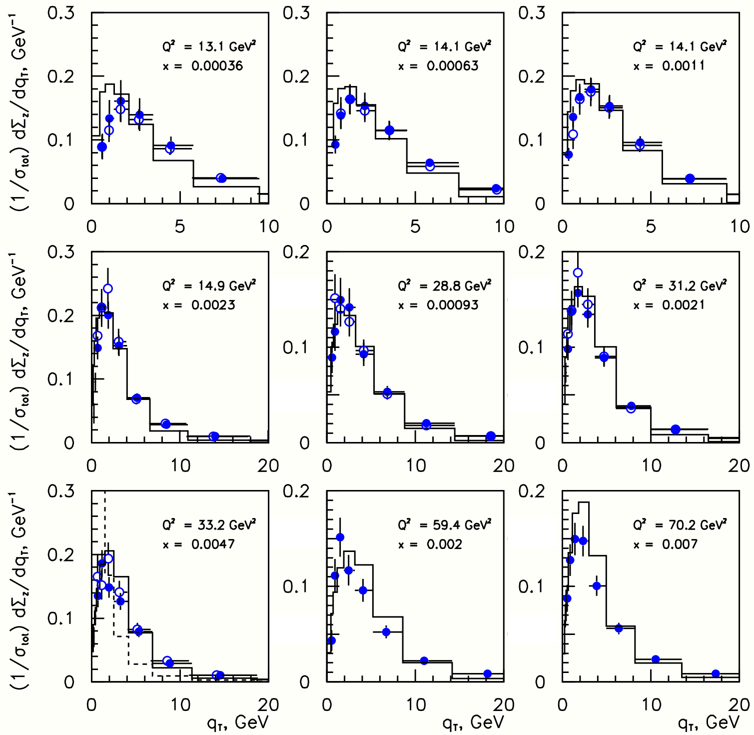


Hadronic energy flow Σ_z at small q_T

- At $\theta_H \to 0$, hadrons recoil against soft or collinear QCD radiation
- Finite-order QCD fails $\Rightarrow$ resummation!
- Instructive tests of soft QCD radiation
  - variety of processes & observables
  - initial- and final-state radiation

q_T dependence of energy flow at small x

Data from *H1 Collaboration*



$13.1 < \langle Q^2 \rangle < 70.2 \quad \text{GeV}^2,$

$8 \times 10^{-5} < \langle x \rangle < 7 \times 10^{-3}$

Resummed E_T-flow: CTEQ5M1 PDFs,

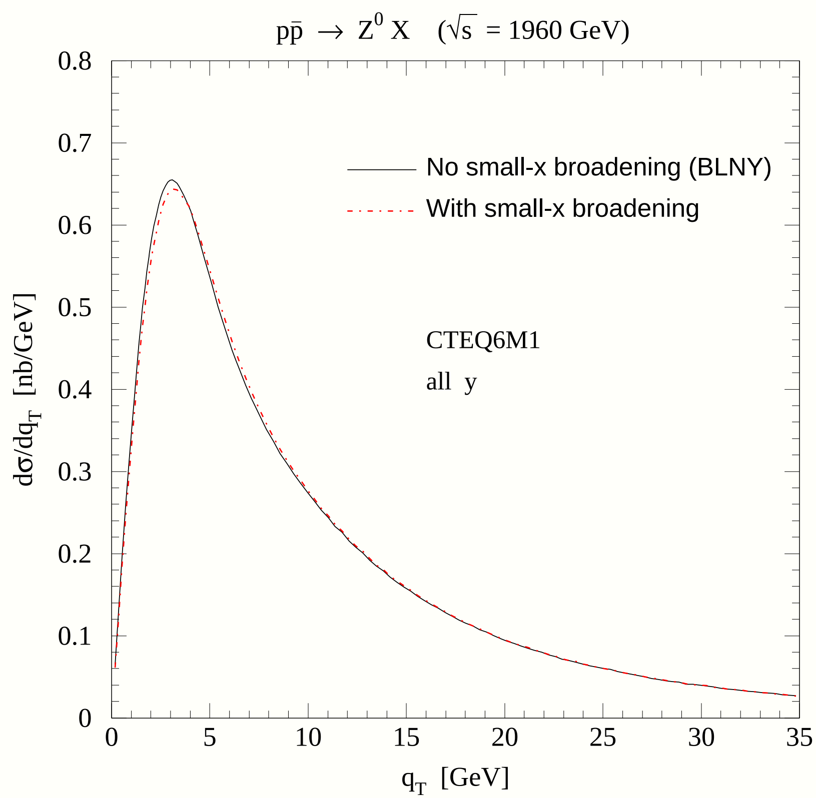
$$S^{NP}_{E_T} = b^2 \left\{ 0.013 \frac{(1-x)^3}{x} + 0.19 \ln \left(\frac{Q}{2 \text{ GeV}} \right) \right\}$$

Possible interpretation:
rapid increase of “intrinsic” k_T when x decreases (first signs of k_T-unordered radiation???)
No mechanism for such increase in the $\mathcal{O}(\alpha_s)$ part of the CSS formula

Can be parametrized as

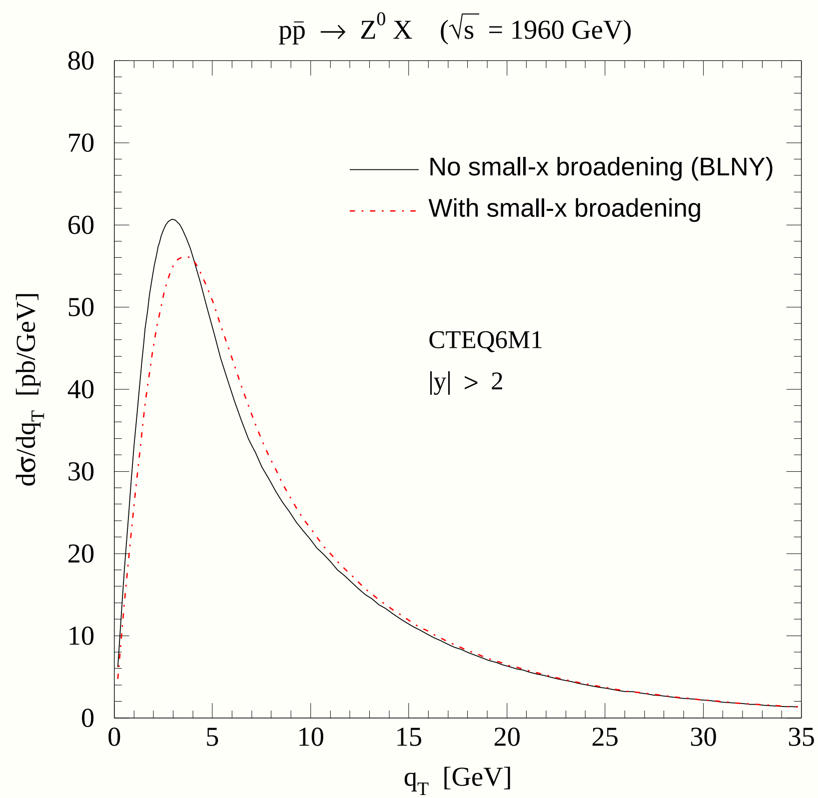
$$\overline{\mathcal{P}}(x,b) = \left(\mathcal{C} \otimes f\right)(x, b_*) e^{-\rho(x) b^2},$$
$$\rho(x) \approx \frac{0.013}{x} \text{ at } x \lesssim 10^{-2}$$

Small-x effects on $p\bar{p} \to Z^0 X$ at the Tevatron



No cuts: no visible effects

(the dominant contribution comes from $x|_{y\approx 0} \approx 0.05$)

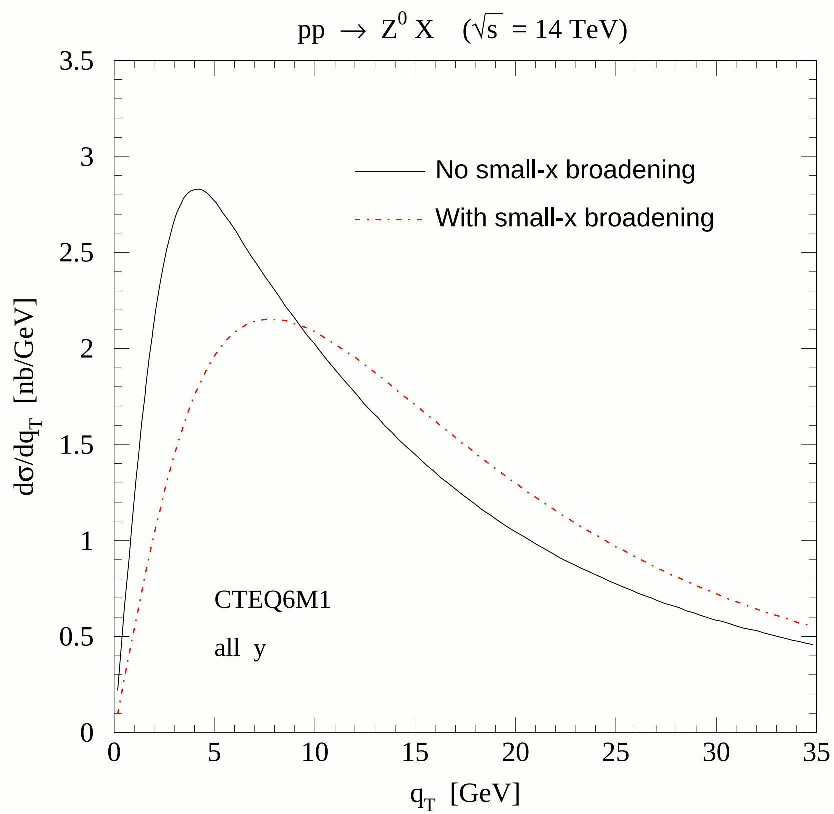


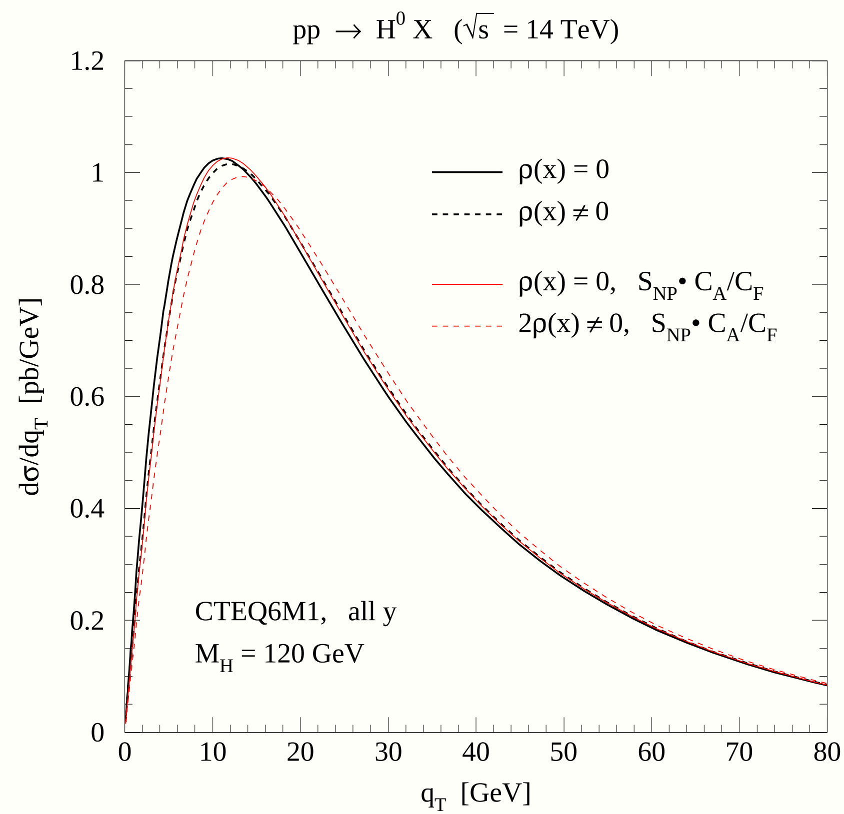
$|y| > 2$

Visible broadening in the forward region

Effect measurable in the Tevatron Run-2; may change the measured M_W by 10-20 MeV

Small-x effects at the LHC: visible even in the central region!





Z^0 production ($x|_{y\approx 0} \approx 0.007$)
Drastic differences

SM Higgs boson production Effects less pronounced due to (a) harder q_T spectrum and (b) narrower rapidity distribution

Broadening increases in magnitude as y grows

Summary

- LHC will be particularly sensitive to sea parton interactions, small-x scattering; percent-level predictions will be needed in this new QCD regime
- Recent developments include
  - precision methods for understanding the PDFs and their uncertainties at NNLO
  - resummations for 2- and 3-scale observables (depending on Q, q_T, x,$M_{c,b}$, ...)
- Studies of QCD processes at small x, small Q, or forward rapidities at Tevatron and HERA are crucial for reducing uncertainties at the LHC
- If broadening of $d\sigma/dq_T$ is observed in forward Z boson (Drell-Yan pair) production in the Tevatron Run-2, it will strongly affect predictions for W and Z production at the LHC

Backup slides

Small-q_T cross section is related to a form factor in the impact parameter (b) space

$$\left.\frac{d\sigma}{dx\,dz\,dQ^2\,dq_T^2}\right|_{q_T\to 0} \propto \int \frac{d^2b}{(2\pi)^2} e^{-i\vec{q}_T\cdot\vec{b}}\,\overline{\mathcal{D}}(z,b)\,e^{-S(b,Q)}\,\overline{\mathcal{P}}(x,b)$$

$S(b,Q)$: soft (Sudakov) factor (same as in DY process)

$\overline{\mathcal{P}}(x,b) \approx (\mathcal{C}\otimes f)\,(x,b)$: b-dependent parton distribution

$\overline{\mathcal{D}}(z,b) \approx (D\otimes\mathcal{C})\,(z,b)$: b-dependent fragmentation function

From angular distributions of the hadronic transverse energy flow,

$$\begin{aligned}\frac{d\langle E_T\rangle}{dx\,dQ^2\,dq_T^2} &\propto \int dz\cdot z\cdot\frac{d\sigma}{dx\,dz\,dQ^2\,dq_T^2}\\ &\propto \text{const}\cdot\int\frac{d^2b}{(2\pi)^2}e^{-i\vec{q}_T\cdot\vec{b}}e^{-S(b,Q)}\,\overline{\mathcal{P}}(x,b),\end{aligned}$$

one can learn about the x-dependence of $\overline{\mathcal{P}}(x,b)$ and make predictions for the Drell-Yan processes

What is q_T in DIS?

$q_T \sim$ polar angle θ_H in the $\gamma^* p$ c.m. frame

$$q_T^2 = \frac{(p_T)_H^2}{z^2} = Q^2 \frac{(1-x)}{x}\left(\frac{\theta_H^2}{4} + ...\right)$$

Discontinuities in $d\sigma/d\theta_H$ are due soft and collinear radiation:

$$\left(\frac{d\sigma}{d\theta_H^2}\right)_{\theta_H \to 0} \propto \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \left\{ a_n \delta(\theta_H) + \frac{1}{\theta_H^2}\sum_{m=0}^{2n-1} v_{nm} \ln^m\left(\frac{\theta_H^2}{4}\right)\right\}$$

The all-order sum is

$$\left.\frac{d\sigma}{dx\,dQ^2\,dz\,d\theta_H^2}\right|_{\theta_H \to 0} = \frac{\sigma_0}{S}\int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{q}_T \cdot \vec{b}}\widetilde{W}(b, Q, x, z) + ...$$