QFT 3: PROBLEM SET 3

This is discussed in P+S section 16.6 – I'll just refer to that.

Key is (16.131) $$C_{r,j} = \frac{C(r)}{(4\pi)^2}\left(\frac{1}{3}d(j) - 4C(j)\right)$$

spin opts in Lorentz rep

coeff of trace over Lorentz generators

(a) For chiral fermions $d(j) = \frac{1}{2}d(j)_{\text{Dirac}}$
– we keep only 2 opts

If you follow through the algebra, the Lorentz generator becomes something like

$$S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]\frac{1-\gamma^5}{2}$$

→ end up giving $C(j) = \frac{1}{2}C(j)_{\text{Dirac}}$

$$\Rightarrow C_{r,j}^{\text{chiral}} = \frac{1}{2}C_{r,j}^{\text{Dirac}}$$

$$= -\frac{4}{3}\frac{C(r)}{4\pi^2}$$

$$\Rightarrow \beta(g)_{\text{chiral}} = -\frac{n_f}{2}\left(-\frac{4}{3}\right)\frac{C(r)}{(4\pi)^2}g^3$$

(b)

For scalars

$$C_{r,j} = \frac{C(\overset{\text{fundamental} = \frac{1}{2}}{N})}{(4\pi)^2}\left(\frac{1}{3}\overset{=1}{d(j)} - 4\,\overset{=0\ (\text{scalar})}{C(j)}\right) = \frac{1}{6(4\pi)^2}$$

$$\Rightarrow \beta_{\text{scalar}} = \frac{g^3}{(4\pi)^2}\frac{n_f}{6}$$

(d) Standard model: gauge bosons + ghosts:

$$\beta(g) = g^3\left[\frac{1}{2}C_{G,1} - C_{G,0} + \frac{n_f}{2}C_{r,1/2} + n_H C_{r,0}\right]$$

$$= \frac{g^3}{(4\pi)^2}\left[\frac{1}{2}N\left(-\frac{20}{3}\right) - \frac{N}{3} + \frac{n_H}{6} + \frac{n_f}{3}\right]$$

$$= -\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}N - \frac{n_f}{3} - \frac{n_H}{6}\right)$$

SUSY → also have Majorana fermions in the adjoint

$$11/3\,N \to 11/3\,N - \frac{2}{3}N = 3N$$

for each Higgs, get chiral fermion.

$$n_H^{eff} = n_H + 2n_H \Rightarrow \frac{n_H}{6} \to \frac{n_H}{2}$$

for each chiral, get a scalar.

$$n_f^{eff} = n_f + \frac{n_f}{2} \Rightarrow \frac{n_f}{3} \to \frac{n_f}{2}$$

QFT: PS3

So for SYM

$$\beta = \frac{g^3}{(4\pi)^2}\left(3N - \frac{N_f}{2} - \frac{N_H}{2}\right)$$

(e)

	$SU(3)_L$	$SU(3)_E$
SM	$-\frac{22}{3} + \frac{1}{6} + \frac{12}{3} = -\frac{19}{6}$	$-\frac{4}{3}\cdot 3 + \frac{12}{3} + 0 = -7$
SUSY	$-3\cdot 2 + \frac{6}{2} + \frac{3}{2} = 1$	$-3\cdot 3 + 6 + 0 = -3$

$\times\ g^3/(4\pi)^2$

SU(2) no longer asymptotically free in SUSY.

(d2) Equate couplings

SM: $$g_3^2(p) = \frac{g_3^2}{1 + \frac{14 g_3}{(4\pi)^2}\log(p/M_3)} \qquad g_2^2(p) = \frac{g_2^2}{1 + \frac{19 g_2^2}{3(4\pi)^2}\log(p/M_2)}$$

Know $g_3^2(M_Z) = 4\pi \cdot 0.118$

$g_2^2(M_B) = 4\pi \cdot 0.0336$

$\Rightarrow \quad p = 1.2\cdot 10^{17}$ GeV.

QFT3: PS3

For SUSY, do the same thing and get $p = 3 \cdot 10^{16}$ GeV