Physics 323. Problem set I. Thursday, March 31, 2005

DUE THURSDAY, APRIL 7, 2005

Problem 1.
The doors of a garage are arranged in such a way that its front door opens at the same time as the rear door closes in the reference frame K, where the garage is at rest. The garage has a length L_{G}, as measured by an observer in the reference frame K.

An incoming car, moving towards the garage at a speed v in the reference frame K, has a length L_C, as measured by an observer in the reference frame K' where the car is at rest. Assuming that the car enters the garage through the rear door, which is open, and exits through the front door, which is closed before the car enters the garage, and assuming that the front door opens at the precise moment in which the car is about to hit it:
a. What is the length of the car, as measured by an observer in the reference frame K ?
b. What is the minimal speed v necessary for the car to be able to go through the garage without colliding against any of the two doors ?
c. Describe the same process, as described by an observer in the reference frame K', in which the car is at rest.

Problem 2
Imagine a frame K' moving at speed $\vec{v} = v\hat{x}$ with respect to the inertial frame K, and a system K'' moving at speed $\vec{v'} = v'\hat{x}$ with respect to K'.

a. If a particle moves with velocity $\vec{u} = u_x\hat{x}$ in the system K, derive the expression of its velocity in the reference frame K''.
b. Interpret the result as a function of the relative velocity of K'' with respect to K.
c. Imagine that the relative velocities of the frames would be interchanged (that is, K' moves with velocity $\vec{v'}$ with respect to K and K'' moves with velocity $\vec{v}$ with respect to K'). Do the operations of change of frame commute

in this case,

$$S(\vec{v})\ S(\vec{v'}) = S(\vec{v'})\ S(\vec{v})\ ? \tag{1}$$

Explain this result.

d. In general, if the velocities are not alligned, would you expect the operations of change of frame to commute,

$$S(\vec{v_1})\ S(\vec{v_2}) = S(\vec{v_2})\ S(\vec{v_1})? \tag{2}$$

Problem 3.

Determine the relativistic uniformly accelerated motion, i.e. the rectilinear motion for which the acceleration in the proper reference frame remains constant at each instant of time. In particular, find the equation of motion, $x = x(t)$, in an inertial frame in which the particle is at rest at time $t = 0$. Find also the proper time variation of the uniformly accelerated particle.

Hint: Consider that the particle has a velocity $v_x = v$ at a time t, and evaluate the cuadrivector

$$w^\mu = \frac{d^2 x^\mu}{c^2 d\tau^2} \tag{3}$$

at that particular time. Derive the condition for which $w^\mu w_\mu$ is a constant, $-w^2/c^4$, (and therefore equal the square of the acceleration in the frame in which the particle is at rest $w^\mu = (0, w/c^2, 00)$), and show that it leads to the relation

$$\frac{d}{dt}\frac{v}{\sqrt{1 - v^2/c^2}} = w. \tag{4}$$

Use the fact that $d\tau = \sqrt{1 - v^2/c^2}dt$

Problem 4.

Use the solution to problem 3 to solve problem 11.6 in Jackson's book, third edition, page 569.