Physics 445: Problem Set 1

Carlos Wagner, Spring 2006

Due Friday 14 April, 2:30 p.m.

1. Consider a complex scalar field, whose action is invariant under a global U(1) symmetry group.

a) Find the expression of the Noether current associated with the global U(1) symmetry in terms of the complex scalar field.
b) Express this current in terms of the radial and phase fluctuations of the scalar field.

Assume that the U(1) symmetry is spontaneously broken.

$$\phi = \frac{1}{\sqrt{2}} \exp(i\zeta(x)/v)\ (v + \rho(x)) \tag{1}$$

and show that ζ becomes a properly normalized Nambu-Goldstone mode.
c) Demonstrate that the Nambu-Goldstone boson interaction terms may be written as

$$\mathcal{L}_{\rm int} \simeq \frac{1}{v} \zeta\ \partial_\mu j^\mu \tag{2}$$

where j^μ is the Noether current (In general, j^μ is the current whose charge

$$Q = \int d^3x j_0 \tag{3}$$

generates the symmetry which is spontaneously broken). The coupling is inversely proportional to the scale of symmetry breaking v.

Extra credit: Try to do the same for unitary, non-abelian symmetry groups.

2. Consider a U(1) $\times$ U(1) gauge theory with coupling constants g, g'. A complex scalar field Φ carries charge (1,-1) under the gauge group and $| < \Phi > | = v$. Compute the spectrum of gauge bosons in this theory.

Do the same for a gauge symmetry group $SU(2)$ and a scalar field in the adjoint representation of the group. How does the resulting spectrum compare with the Standard Model one ?