Physics 371: Problem Set 2

Carlos E.M. Wagner, Spring 2004

Due Thursday, April 29, 1:30 p.m.

1. Consider the best-fit universe, with density parameters $\Omega_{\rm R0} = 10^{-4}$, $\Omega_{\rm M0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$. Assume that the Universe is flat. Make a plot of the three Ω_i's as a function of the scale factor R, on a log scale, from $R = 10^{-35}$ to $R = 10^{35}$. At which redshift value did $\Omega_\Lambda = \Omega_M$? At what time will $\Omega_\Lambda = 0.9$ ($\Omega_\Lambda = 0.99$) ?

2. In a flat spacetime, objects of a fixed physical size subtend smaller and smaller angles as they are further and further away; in an expanding universe this is not necessarily so. Consider the angular size $\theta(z)$ of an object of physical size L at redshift z. In a matter-dominated flat universe, at what redshift is $\theta(z)/L$ a minimum? If all galaxies are at least 10 kpc across (and always have been), what is the minimum angular size of a galaxy in such a universe? (Express your result both in terms of H_0, and plugging in $H_0 = 70$ km/s/Mpc.)

3. Define $R_{\rm eq}$ as the scale factor and $t_{\rm eq}$ as the time when the density of radiation $\rho_{\rm R}$ (including massless neutrinos) equals the density of matter $\rho_{\rm M}$. Assume a flat universe, $k = 0$, and ignore the effects of Λ.

1. Show that
$$\frac{t}{t_{\rm eq}} = \frac{1}{2-\sqrt{2}}\left[\left(\frac{R}{R_{\rm eq}} - 2\right)\left(\frac{R}{R_{\rm eq}} + 1\right)^{1/2} + 2\right] . \tag{1}$$
2. Show that
$$H_{\rm eq} = \frac{4}{3}(\sqrt{2}-1)t_{\rm eq}^{-1} . \tag{2}$$
3. Calculate $1 + z_{\rm eq} = R_0/R_{\rm eq}$ from
$$\rho_{\rm R0} = \frac{\pi^2}{30} g_{*0} T_0^4 . \tag{3}$$
Express your answer in terms of $\Omega_{\rm M0}$ and $h = H_0/(100$ km/sec/Mpc). That is, use your knowledge of T_0 and g_{*0}.
4. Calculate the photon temperature $T_{\rm eq}$ in terms of h.