Physics 445: Problem Set 2

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Due Friday 28 April, 2:30 p.m.

1. Consider the Standard Electroweak Gauge Theory, with gauge group

$$SU(2)_L \times U(1)_Y, \tag{1}$$

and take the covariant gauge fixing

$$\mathcal{F}_a(A_\mu) = \partial_\mu A^\mu_a + i g_a \xi \left(< \phi^\dagger > T_a \; (\phi - < \phi >) - (\phi - < \phi >)^\dagger T_a \phi \right) \tag{2}$$

with ξ the usual gauge fixing parameter, and ϕ is the Higgs doublet with hypercharge 1/2.

Compute the would-be Goldstone and ghost masses in this theory. Based on these results, compute the Goldstone and ghost propagators and derive the Feynman rules for the ghost and Goldstone couplings to the gauge fields.

Extra credit: Try to derive these relations for a general theory in which the gauge group is a product of simple unitary groups.

2. Consider an $SU(2)$ gauge theory, without Higgs fields, and add to it an explicit mass term

$$\begin{aligned} \mathcal{L}_{\rm mass} &= -\frac{\mu^2 g^2}{2} A_{\mu,a} A^{\mu,a} \\ &= -\mu^2 g^2 Tr[A_\mu A^\mu] \end{aligned} \tag{3}$$

Apply the usual Fadeev Popov gauge fixing method, and demonstrate that in this case, the gauge degrees of freedom cannot be decoupled. One obtains an effective Lagrangian

$$\mathcal{L} = -\frac{1}{2} Tr[F_{\mu\nu} F^{\mu\nu}] - \mu^2 Tr \left[\partial_\mu U \partial^\mu U^\dagger - i2 A_\mu g \left(\partial^\mu U^\dagger \right) U + g^2 A_\mu A^\mu \right], \tag{4}$$

where U is the original gauge degree of freedom, $U = \exp(i\alpha_a T_a)$. One can now interpret U as new degrees of freedom, which under a gauge transformation U' transform as $U'U$. One obtains the equivalent of a gauge invariant theory, where the original non-gauge invariant theory is obtained by going to the Unitary gauge $U'U = I$.

Compare the above to what happens if one would introduce a scalar Higgs doublet in the spectrum, in the limit that the quartic coupling tends to infinity. In this case, the Higgs fluctuations acquire infinite mass and therefore no physical Higgs particle appears in the spectrum. Such a theory violates tree-level unitarity and it is not renormalizable.

3. Consider the decay of a muon by the emission of a virtual charged boson W^- and a neutrino. The virtual gauge boson emits an electron and an anti-neutrino. Knowing that the W^- acquires mass via the Higgs mechanism, compute the amplitude of the decay and, using the fact that the characteristic masses and momenta in the process are much smaller than the W^- mass, demonstrate that this process provides information about the vacuum expectation value of the Higgs field.

Extra credit: Compute the decay width of the muon into an electron and a pair of massless neutrinos. How is it related to the Higgs vacuum expectation value and the muon mass ?