Inclusive Production of Heavy Quarkonium

Geoffrey Bodwin, Argonne

CERN Yellow Report on Heavy Quarkonium Physics
Chapter 5: Production
G.T. Bodwin, E. Braaten, M. Krämer, A.B. Meyer, V. Papadimitriou
with contributions from C.-H. Chang, J. Lee, R. Vogt

M. Krämer
Prog. Part. Nucl. Phys. **47** 141, (2001)
[arXiv:hep-ph/0106120]

Outline

General Considerations

- Standard Truncation in v
- Polarization
- Color-Singlet Model (CSM)
- Color-Evaporation Model (CEM)
- Multiple Gluon Emission
- Uncertainties in the Theory of Quarkonium Production

Comparisons Between Theory and Experiment

- Quarkonium Production at the Tevatron
  - Cross Sections
  - Polarized Quarkonium Production
- Quarkonium Production in Fixed-Target Experiments
  - Total Cross Sections
  - Polarization
- Quarkonium Production in B Decays
  - Branching Fractions of B mesons into Charmonium
  - Inclusive Rates into Charmonium
  - Polarization of J/ψ's at CLEO

- Quarkonium Production at LEP
  - $Z \to J/\psi + X$
  - $\gamma\gamma \to J/\psi + X$
  - $Z \to \Upsilon + X$
- Quarkonium Production at HERA
  - Inelastic Quarkonium Photoproduction
  - Quarkonium Production in DIS
- Quarkonium Production in e^+e^- Annihilation Near 10.6 GeV
  - J/ψ Cross Section
  - Angular Distribution of J/ψ's
  - Polarization of J/ψ's
  - Double $c\bar{c}$ Production at Belle
- Conclusions

General Considerations

Standard Truncation in v

- The heavy-quark spin symmetry allows us to relate matrix elements for quarkonium states that differ by a spin flip.
- Examples of color-singlet matrix elements:

$$
\begin{aligned}
\langle \mathcal{O}_1^{J/\psi}({}^3S_1) \rangle &= 3\ \langle \mathcal{O}_1^{\eta_c}({}^1S_0) \rangle, \\
\langle \mathcal{O}_1^{\chi_{cJ}}({}^3P_J) \rangle &= \tfrac{1}{3}(2J+1) \langle \mathcal{O}_1^{h_c}({}^1P_1) \rangle.
\end{aligned}
$$

- Examples of color-octet matrix elements:

$$
\begin{aligned}
\langle \mathcal{O}_8^{J/\psi}({}^3S_1) \rangle &= 3 \langle \mathcal{O}_8^{\eta_c}({}^1S_0) \rangle, \\
\langle \mathcal{O}_8^{J/\psi}({}^1S_0) \rangle &= 3\ \langle \mathcal{O}_8^{\eta_c}({}^3S_1) \rangle, \\
\langle \mathcal{O}_8^{J/\psi}({}^3P_J) \rangle &= 3\ \langle \mathcal{O}_8^{\eta_c}({}^1P_1) \rangle, \\
\langle \mathcal{O}_8^{\chi_{cJ}}({}^3S_1) \rangle &= \tfrac{1}{3}(2J+1) \langle \mathcal{O}_8^{h_c}({}^1S_0) \rangle.
\end{aligned}
$$

- These relations hold up to corrections of order v^2.
- Applying these results, we arrive at the simplest truncation of the v expansion that is
  - consistent through a given order in v,
  - phenomenologically viable.

- For J/ψ and η_c production the standard truncation is

$$\begin{aligned}\langle\mathcal{O}_1^{J/\psi}(^3S_1)\rangle &\sim v^0,\\ \langle\mathcal{O}_8^{J/\psi}(^1S_0)\rangle &\sim v^3,\\ \langle\mathcal{O}_8^{J/\psi}(^3S_1)\rangle &\sim v^4,\\ \langle\mathcal{O}_8^{J/\psi}(^3P_0)\rangle &\sim v^4.\end{aligned}$$

- Define a linear combination of matrix elements:

$$M_k^H = \langle\mathcal{O}_8^H(^1S_0)\rangle + \frac{k}{m_c^2}\langle\mathcal{O}_8^H(^3P_0)\rangle.$$

  Many observables are sensitive only to this linear combination with a specific value of k.

- For production of the P-wave states χ_{c0}, χ_{c1}, χ_{c2}, and h_c the standard truncation is

$$\begin{aligned}\langle\mathcal{O}_1^{\chi_{c0}}(^3P_0)\rangle &\sim v^2,\\ \langle\mathcal{O}_8^{\chi_{c0}}(^3S_1)\rangle &\sim v^2.\end{aligned}$$

Polarization

- For 1^{--} states, polarization can be measured from the angular distribution of the decay into lepton pairs.
- Let θ be the angle in the quarkonium rest frame between the positive lepton and a chosen polarization axis.

$$d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta,$$

$$-1 \leq \alpha \leq +1.$$

- $\alpha = 1$ corresponds transverse polarization;
  $\alpha = -1$ corresponds to longitudinal polarization.
- $\alpha = (1-3\xi)/(1+\xi)$, where ξ is the longitudinal-polarization fraction.
- The reference axis depends on the process.
  - At the Tevatron: the boost vector from the quarkonium rest frame to the CM frame of the colliding hadrons.
  - In fixed-target experiments: the boost vector from the quarkonium rest frame to the lab frame.
  - In e^+e^- colliders: the boost vector from the quarkonium rest frame to the e^+e^- CM frame.

Color-Singlet Model (CSM)

- Proposed for η_c and χ_c production via two gluon fusion shortly after the discovery of the J/ψ (1975–76) (Einhorn, S.D. Ellis; S.D. Ellis, Einhorn, Quigg; Carlson, Suaya).
- Applied to J/ψ and η_c production in B-meson decays in 1980 (Kühn; Degrand, Toussaint; Kühn, Nussinov, Rückl; Wise).
- Applied to production of J/ψ plus a gluon in 1980–81 (C.H. Chang; Baier, Rückl; Berger, Jones; W.Y. Keung).
- The CSM drops all of the color-octet terms in the NRQCD-factorization approach.
- The CSM keeps only the leading-in-v color-singlet term that has the same quantum numbers as the quarkonium.
- The vacuum-saturation approximation relates the CSM production and decay matrix elements.
  - Allows one to make absolutely normalized predictions for production rates.
- The heavy-quark spin symmetry relates CSM matrix elements within a given orbital-angular-momentum multiplet.
  - Leads to non-trivial predictions for quarkonium polarization.

Color-Evaporation Model (CEM)

(Fritsch; Halzen; Gluck, Owens, Reya; Barger, Keung, Phillips)

- The quarkonium production rate is assumed to be proportional to the perturbative rate for $Q\bar{Q}$ production below the open heavy-quark threshold, averaged over color and spin.
- Each quarkonium state has a constant of proportionality, which is assumed to be process independent.
- Can calculate the CEM cross sections using the perturbative NRQCD factorization expression for $c\bar{c}$ production.
- Leads to predictions for the relative sizes of the NRQCD matrix elements for compatibility with the CEM (GTB, Braaten, Lee):

$$\langle \mathcal{O}_n^H \rangle = \frac{3(2j_n+1)}{2l_n+3} C_n k_{\max}^{2l_n} \langle \mathcal{O}_1^H({}^1S_0) \rangle .$$

$C_n = 1$ for color-singlet m.e.;
$C_F = 4/3$ for color-octet m.e.
$k_{\max} = \sqrt{m_{\text{meson}}^2 - m^2} \sim mv$.

- The CEM gives an additional power of v^2 for each unit of orbital angular momentum, in agreement with NRQCD.
- However, in general, the CEM is inconsistent with the velocity-scaling and color factors of the NRQCD matrix elements.

- The CEM and NRQCD predict very different proportions for the various $c\bar{c}$ spin, orbital-angular-momentum, and color channels in quarkonium production.
- Nevertheless, the CEM agrees fairly well with much of the data.
- Some specific areas of disagreement with NRQCD factorization:
  - The CEM predicts zero polarization for quarkonium production.
  - The CEM predicts that $\sigma[\chi_{c1}]/\sigma[\chi_{c2}] = 3/5$.

Multiple Gluon Emission

- Effects of multiple gluon emission can be very important for
  - transverse-momentum distributions,
  - distributions near kinematic limits,
  - production near threshold.
- Several methods are used to incorporate effects of multiple gluon emission into theoretical predictions.
- Resummation
  - Sums logarithmically enhanced (soft and collinear) terms to all orders in α_s.
  - Typically carried out in leading log (LL) or next-to-leading log (NLL) approximations.
  - In principle, can be extended to arbitrarily high logarithmic accuracy.
  - Arbitrarily soft and collinear emissions enter, so non-perturbative functions appear.
    These are less important at large mass and transverse momentum.
  - Cannot reproduce effects of hard gluons at large angles (”Mercedes” events).
  - Use in conjunction with NLO calculations partially remedies this deficiency.

- Parton-Shower Monte Carlos
  - Calculate logarithmically enhanced terms in gluon emission.
  - A finite, but arbitrarily large, number of gluon emissions in calculated.
  - Early Monte Carlos, such as ISAJET, treat only leading collinear enhancements correctly.
  - PYTHIA and HERWIG treat leading collinear and soft enhancements correctly.
  - Perturbative showering may be supplemented by non-perturbative fragmentation.
  - Easily applied to any Born-level process.
  - Differential in all kinematic variables.
    Useful for applying experimental cuts.
  - Cannot reproduce effects of hard gluons at large angles.
  - Recent progress in matching shower Monte Carlos to NLO calculations.

- k_T Factorization
  - Takes into account initial-state radiation through parton distributions that depend on x and k_T.
  - Gives very different answers than standard collinear factorization.
  - k_T-dependent PDF's are not well known.
  - There are unresolved theoretical issues, such as the universality of the k_T-dependent PDF's.
- k_T Smearing
  - Phenomenological model for multiple initial-state radiation.
  - PDF's are given by standard collinear-factorization times a Gaussian distribution in k_T.
  - The width of the Gaussian distribution is a process-dependent phenomenological parameter.
  - Captures some crude features of multiple gluon emission, but probably incorrect in detail.
    * Shower Monte Carlos and resummation produce p_T distributions with longer tails.

Uncertainties in the Theory of Quarkonium Production

- Corrections to the factorization of the hard-scattering process
  - In the unpolarized case, errors of order Λ_{QCD}^2/p_T^2.
  - In the case of polarized quarkonium, errors of order Λ_{QCD}/p_T.
- Corrections to the NRQCD formula of higher order in v
  - Typically relative order v^2. ($v^2 \approx 0.3$ for charmonium; $v^2 \approx 0.1$ for bottomonium.)
  - For some processes, can be relative order v^1 (spin-flip matrix elements).
  - Systematically improvable to any accuracy, but at the cost of additional matrix elements.
  - In the case of quarkonium decays, some corrections are known to have large coefficients ($\sim 5v^2$).

– Near the edge of phase space
  * the momentum of accompanying gluons may be important or
  * the difference between $2m$ and the quarkonium mass may be important.
  * Then the corresponding class of corrections must be resummed to all orders in v (Beneke, Rothstein, Wise).

- NRQCD operator matrix elements.
  - The color-singlet production and decay matrix elements are equal in the vacuum-saturation approximation.
  - There is no simple relation between the color-octet production and decay matrix elements.
  - Decay matrix elements can be computed on the lattice (GTB, Kim, Sinclair).
  - It is not known how to formulate the calculation of the production matrix elements on a Euclidean lattice.
  - Production matrix elements must be extracted from the data.
  - Often several matrix elements contribute to a given process.
    - Difficult to disentangle the contributions by using their kinematic dependences.
    - The important linear combinations vary from process to process, making tests of universality difficult.

- Corrections to the perturbative expressions for the short-distance coefficients.
  - Nominally relative order α_s, but coefficients can be large.
    - $*$ May need to resum $\log(p_T^2/m^2)$, $\log(z)$, $\log(1-x)$, $\log(x)$.
    - $*$ If the p_T spectrum is steeply falling, multiple soft-gluon emission can greatly increase the cross section ($\sim$ factor 4 for J/ψ at the Tevatron) (Cano-Coloma, Sanchis-Lozano).
    - $*$ May need to resum corrections proportional to β_0 (Beneke, Braun; GTB, Y.-Q. Chen).
    - $*$ Some calculated corrections are large ($\sim 5\alpha_s$) for no known reason.

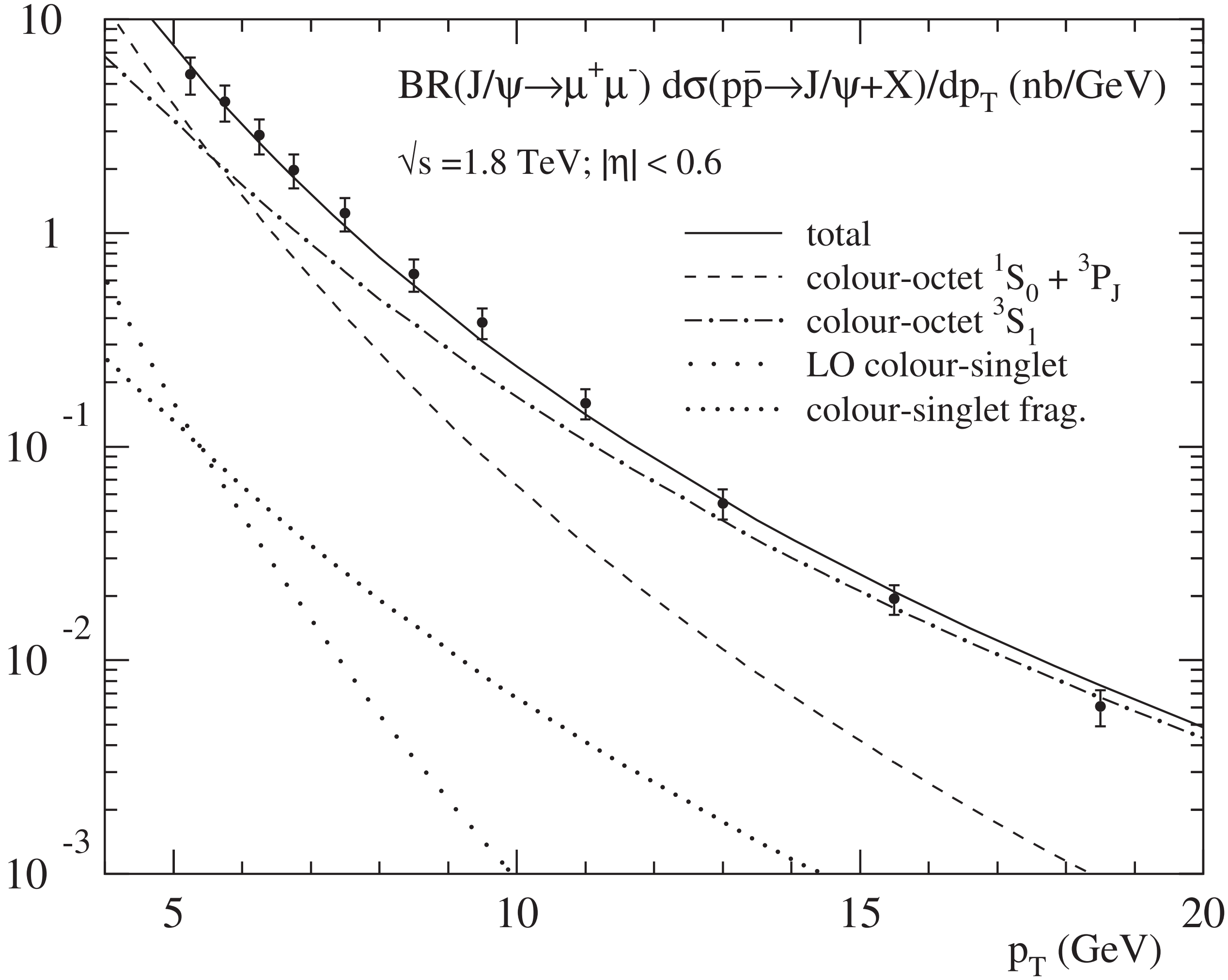
- Uncertainty in m.
  - $\sim 8\%$ for m_c.
  - $\sim 2.4\%$ for m_b.
  - Can be very significant for charmonium rates that are proportional to a large power of the mass.
- Uncertainties in PDF's
  - Uncertainties in distribution shapes may make it difficult to use kinematic dependences to disentangle contributions from various operator matrix elements.
- Uncertainty in α_s.

Comparisons Between Theory and Experiment

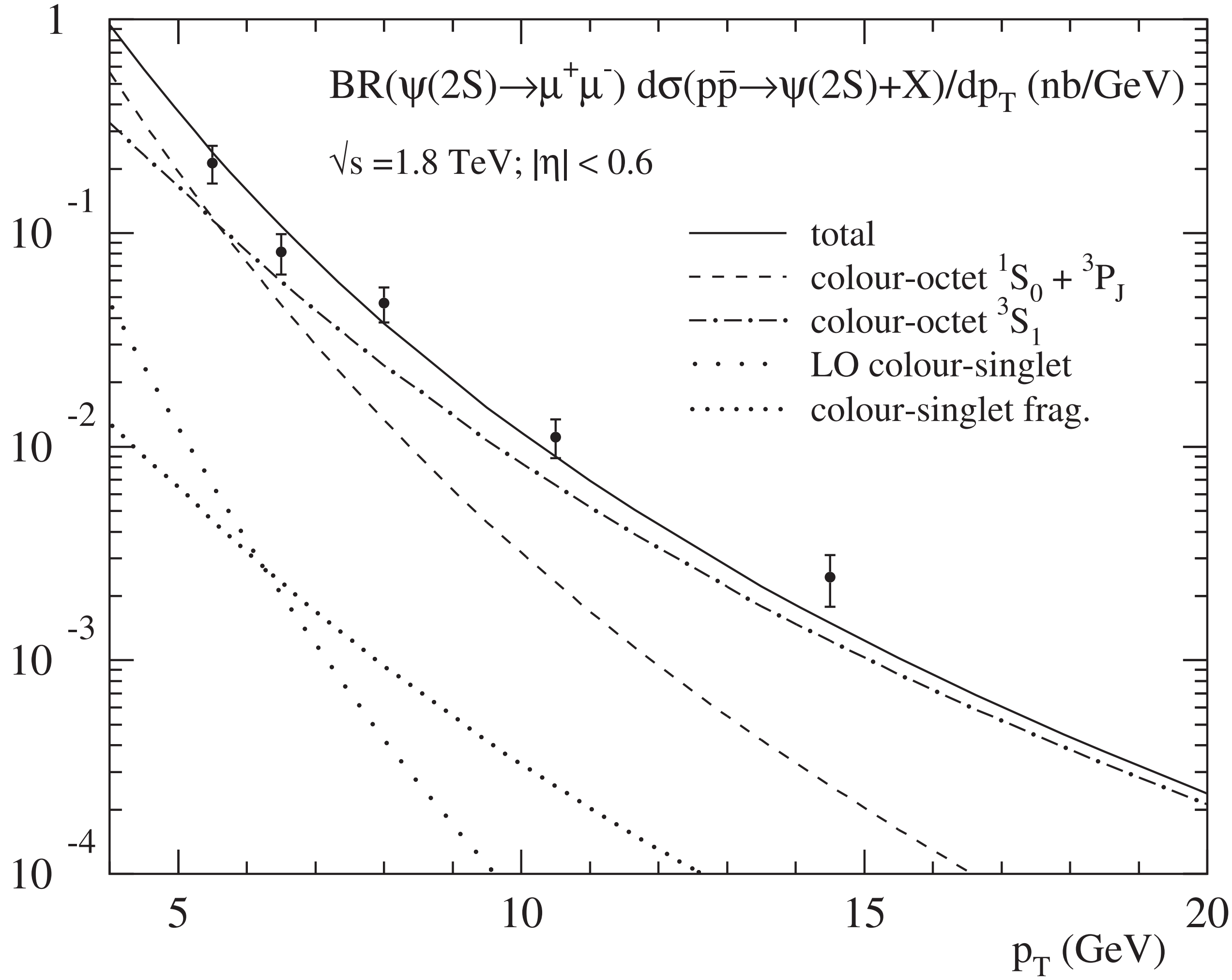
Quarkonium Production at the Tevatron

Cross Sections

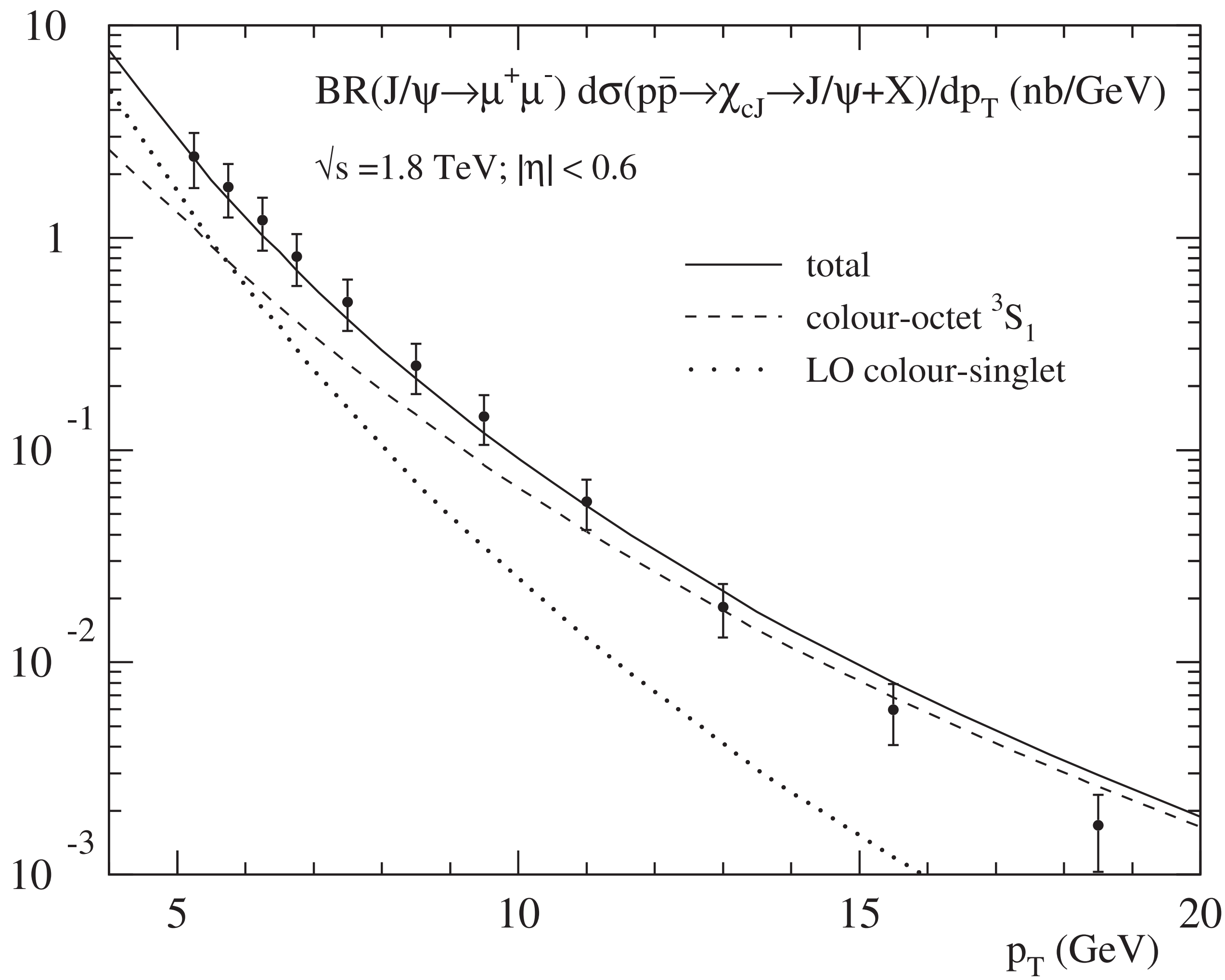
- Quarkonium production at the Tevatron is more than an order of magnitude larger than the prediction of the color-singlet model.
- Can fit the data for J/ψ, χ_c, $\psi(2S)$, Υ and $\Upsilon(2S)$ production by an appropriate choice of the color-octet NRQCD matrix elements.
- p_T distributions are consistent with NRQCD, but not with the color-singlet model.



BR(J/ψ→μ+μ-) dσ(pp̄→J/ψ+X)/dpT (nb/GeV)
√s =1.8 TeV; |η| < 0.6
total
colour-octet 1S0 + 3PJ
colour-octet 3S1
LO colour-singlet
colour-singlet frag.
10
1
10-1
10-2
10-3
5
10
15
20
pT (GeV)



BR(ψ(2S)→μ+μ-) dσ(pp̄→ψ(2S)+X)/dpT (nb/GeV)
√s =1.8 TeV; |η| < 0.6
total
colour-octet 1S0 + 3PJ
colour-octet 3S1
LO colour-singlet
colour-singlet frag.
1
10 -1
10 -2
10 -3
10 -4
5
10
15
20
pT (GeV)



BR(J/ψ→μ+μ-) dσ(pp̄→χcJ→J/ψ+X)/dpT (nb/GeV)
√s =1.8 TeV; |η| < 0.6
total
colour-octet 3S1
LO colour-singlet
10
1
10-1
10-2
10-3
5
10
15
20
pT (GeV)

- Matrix elements for charmonium production, statistical errors only (from M. Krämer).

H	$\langle \mathcal{O}_1^H \rangle$	$\langle \mathcal{O}_8^H(^3S_1) \rangle$	$M_{3.5}^H$
J/ψ	$1.16\ \mathrm{GeV}^3$	$(1.19 \pm 0.14) \times 10^{-2}\ \mathrm{GeV}^3$	$(4.54 \pm 1.11) \times 10^{-2}\ \mathrm{GeV}^3$
$\psi(2S)$	$0.76\ \mathrm{GeV}^3$	$(0.50 \pm 0.06) \times 10^{-2}\ \mathrm{GeV}^3$	$(1.89 \pm 0.46) \times 10^{-2}\ \mathrm{GeV}^3$
χ_{c0}	$0.11\ \mathrm{GeV}^5$	$(0.31 \pm 0.04) \times 10^{-2}\ \mathrm{GeV}^3$	

- Color-singlet matrix elements from potential models fitted to spectrum and decays (Buchmüller, Tye; Eichten, Quigg).
- For J/ψ production, the relevant linear combinations of color-octet matrix elements are
  $\langle \mathcal{O}_8(^3S_1) \rangle$,
  $M_k^{J/\psi} = (k/m^2)\langle \mathcal{O}_8^{J/\psi}(^3P_0) \rangle + \langle \mathcal{O}_8^{J/\psi}(^1S_0) \rangle$,
  with $k \approx 3.5$.
- For $p_T \gtrsim 3m_c$, J/ψ production is dominated by gluon fragmentation into quarkonium through $\mathcal{O}_8(^3S_1)$.
  - Goes as $1/p_T^4$, while other contributions go as $1/p_T^8$.
  - Smaller experimental error bars could help to resolve the different p_T dependences with greater precision.

- Velocity scaling in the production of the S-wave states
  - Expect
$$\langle \mathcal{O}_8 \rangle / \langle \mathcal{O}_1 \rangle \sim v^4.$$
  - Petrelli, Cacciari, Greco, Maltoni, Mangano advocate
$$\langle \mathcal{O}_8 \rangle / \langle \mathcal{O}_1 \rangle \sim v^4/(2N_c).$$
  - Based on normalization of operators in free-quark states.
  - The extracted color-octet matrix elements are roughly compatible with this. [$v^4/(2N_c) \approx 0.015$.]
  - However, a much more stringent test of the theory is to check the universality of the extracted matrix elements in another process.
- Velocity scaling in the production of the P-wave states
  - Expect
$$(\langle \mathcal{O}_8 \rangle / m_c^2) / \langle \mathcal{O}_1 \rangle \sim v^0$$
    or
$$(\langle \mathcal{O}_8 \rangle / m_c^2) / \langle \mathcal{O}_1 \rangle \sim v^0/(2N_c).$$
  - The extracted P-wave color-octet matrix element is somewhat smaller than the latter expectation.
  - Similar results are seen in matrix elements for P-wave quarkonium decay from phenomenology (Maltoni) and from lattice determinations (GTB, Kim, Sinclair).

Caveats

- The extracted values of the octet matrix elements (especially M_k) are very sensitive to the small p_T behavior of the cross section.
  - Leads to a sensitivity to the behavior of the gluon distribution at small x.
  - Effects of multiple soft-gluon emission are important—their omission leads to overestimates of the sizes of the matrix elements.
- Effects of corrections of higher order in α_s can be large.
  - Known to be large in some decays, e.g., $J/\psi \to \gamma\gamma\gamma$.
  - Dependence of the lowest-order result on the factorization and renormalization scales is large (Beneke, Krämer).
  - A new channel for color-singlet production involving t-channel gluon exchange first appears in relative-order α_s.
  - Real-gluon corrections to color-singlet $^3S_1$ production (Petrelli, Maltoni) give a large contribution.
  - Relative-order α_s corrections for χ_0 and χ_2 known (Petrelli, Cacciari, Greco, Maltoni, Mangano).
  - Relative-order α_s corrections for the fragmentation process known (Beneke, Rothstein).
- Resummation of logs of p_T^2/m^2 for the fragmentation process is important (Braaten, Doncheski, Fleming, Mangano; Beneke, Rothstein; Sanchis-Lozano).

- J/ψ production matrix elements in units of 10^{-2} GeV^3. First error bar is statistical. Second error bar (where present) is from variation of factorization and renormalization scales.

Reference	PDF		$\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle$	$M_k^{J/\psi}$	k
LO collinear factorization					
CL	MRS(D0)		0.66 ± 0.21	6.6 ± 1.5	3
BK	CTEQ4L		$1.06 \pm 0.14^{+1.05}_{-0.59}$	$4.38 \pm 1.15^{+1.52}_{-0.74}$	3.5
	GRV-LO(94)		$1.12 \pm 0.14^{+0.99}_{-0.56}$	$3.90 \pm 1.14^{+1.46}_{-1.07}$	
	MRS(R2)		$1.40 \pm 0.22^{+1.35}_{-0.79}$	$10.9 \pm 2.07^{+2.79}_{-1.26}$	
BKL	MRST-LO(98)		0.44 ± 0.07	8.7 ± 0.9	3.4
	CTEQ5L		0.39 ± 0.07	6.6 ± 0.7	
Parton shower radiation					
S	CTEQ2L		0.96 ± 0.15	1.32 ± 0.21	3
	MRS(D0)		0.68 ± 0.16	1.32 ± 0.21	
	GRV-HO(94)		0.92 ± 0.11	0.45 ± 0.09	
KK	CTEQ4M		0.27 ± 0.05	0.57 ± 0.18	3.5
k_t-smearing					
		$\langle k_t\rangle$[GeV]			
P	CTEQ4M	1	1.5 ± 0.22	8.6 ± 2.1	3.5
		1.5	1.7 ± 0.19	4.5 ± 1.5	
SMS	MRS(D$'_{-}$)	0.7	1.35 ± 0.30	8.46 ± 1.41	3
		1	1.5 ± 0.29	7.05 ± 1.17	
k_t-factorization					
HKSST1	KMS		$\approx 0.04 \pm 0.01$	$\approx 6.5 \pm 0.5$	5

- CL: P. Cho and A. K. Leibovich, Phys. Rev. **D53** (1996) 6203.
- BK: M. Beneke and M. Krämer, Phys. Rev. **D55** (1997) 5269.
- P: A. Petrelli, Nucl. Phys. Proc. Suppl. **86** (2000) 533.
- SMS: K. Sridhar, A. D. Martin and W. J. Stirling, Phys. Lett. **B438** (1998) 211.
- CS: B. Cano-Coloma and M. A. Sanchis-Lozano, Nucl. Phys. **B508** (1997) 753.
- S: M. A. Sanchis-Lozano, Nucl. Phys. Proc. Suppl. **86** (2000) 543.
- KK: B. A. Kniehl and G. Kramer, Eur. Phys. J. **C6** (1999) 493.
- HKSST1: P. Hagler, R. Kirschner, A. Schafer, L. Szymanowski and O. V. Teryaev, Phys. Rev. D **63** (2001) 077501.
- BKL: E. Braaten, B. A. Kniehl and J. Lee, Phys. Rev. D **62** (2000) 094005.
- HKSST2: P. Hagler, R. Kirschner, A. Schafer, L. Szymanowski and O. V. Teryaev, Phys. Rev. Lett. **86** (2001) 1446.

- Large dependence on choice of factorization, renormalization scales.
- Large dependence on choice of parton distributions.
- Effects of multiple soft-gluon emission taken into account by parton-shower Monte Carlos by Sanchis-Lozano and Kniehl and Krämer and by Gaussian k_T smearing by Petrelli and Sridhar, Martin, and Stirling.
- Sanchis-Lozano includes resummation of logs of p_T^2/m^2.
- HKSST use the k_T-factorization formalism to resum large logarithms in the limit $s >> 4m_c^2$.

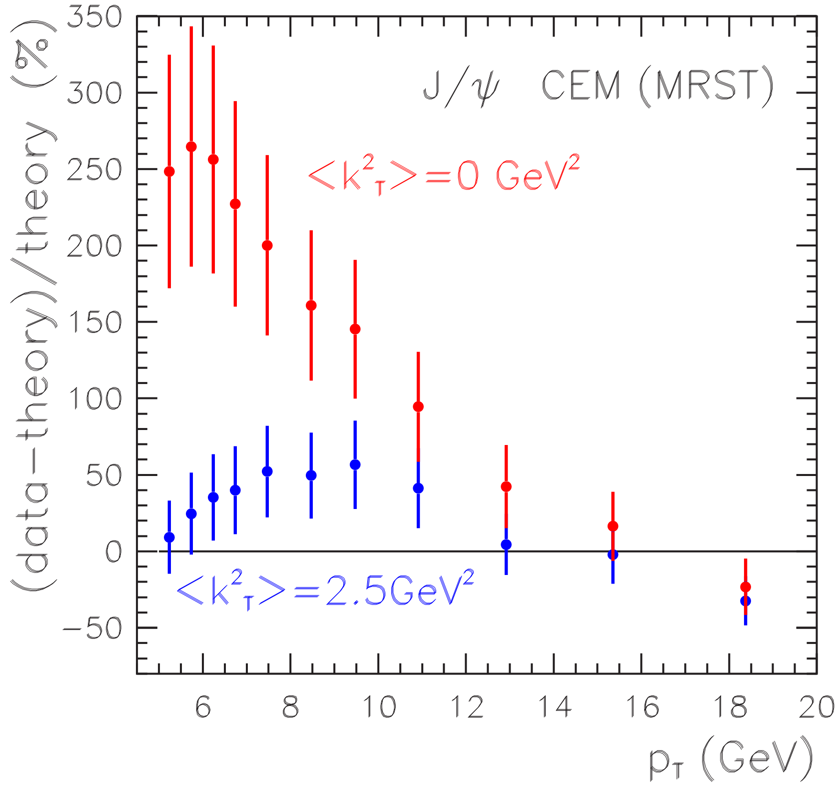
- $\psi(2S)$ production matrix elements in units of 10^{-2} GeV^3.

Reference	PDF	$\langle\mathcal{O}_8^{\psi(2S)}(^3S_1)\rangle$	$M_k^{\psi(2S)}$	k
LO collinear factorization				
CL	MRS(D0)	0.46 ± 0.1	1.77 ± 0.57	3
BK	CTEQ4L	$0.44 \pm 0.08^{+0.43}_{-0.24}$	$1.80 \pm 0.56^{+0.62}_{-0.30}$	3.5
	GRV-LO(94)	$0.46 \pm 0.08^{+0.41}_{-0.23}$	$1.60 \pm 0.51^{+0.60}_{-0.44}$	
	MRS(R2)	$0.56 \pm 0.11^{+0.54}_{-0.32}$	$4.36 \pm 0.96^{+1.11}_{-0.50}$	
BKL	MRST-LO(98)	0.42 ± 0.1	1.3 ± 0.5	3.4
	CTEQ5L	0.37 ± 0.09	0.78 ± 0.36	
Parton shower radiation				
CS	CTEQ2L	0.14 ± 0.03	0.33 ± 0.09	3
	MRS(D0)	0.11 ± 0.03	0.28 ± 0.07	
	GRV-HO(94)	0.13 ± 0.02	0.04 ± 0.05	

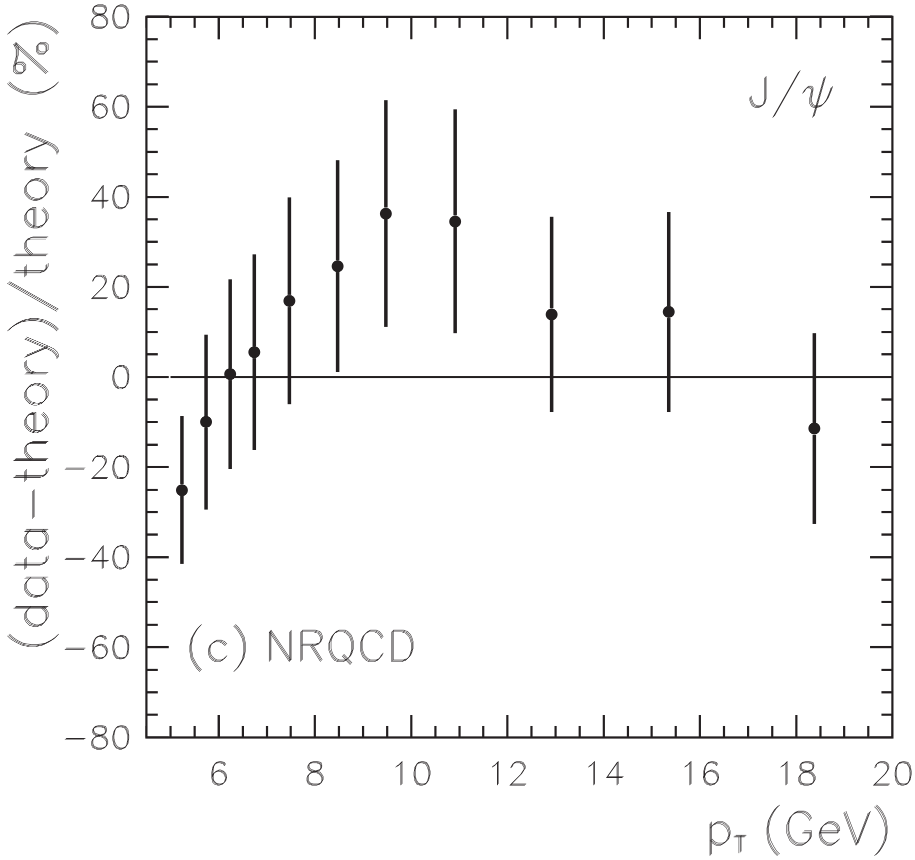
- χ_c production matrix elements in units of 10^{-2} GeV^3.

Reference	PDF	$\langle\mathcal{O}_1^{\chi_{c0}}(^3P_0)\rangle$ [GeV^5]	$\langle\mathcal{O}_8^{\chi_{c0}}(^3S_1)\rangle$ [10^{-2} GeV^3]
LO collinear factorization			
CL	MRS(D0)	0.11 (input)	0.33 ± 0.04
KK	CTEQ4L	0.23 ± 0.03	0.068 ± 0.018
BKL	MRST-LO(98)	0.09 ± 0.01 (input)	0.23 ± 0.03
	CTEQ5L	0.09 ± 0.01 (input)	0.19 ± 0.02
k_t-factorization			
HKSST2	KMS	0.11 (input)	0.03 ± 0.01

- The color-evaporation model also gives reasonable fits to the CDF data, but only if k_T smearing is included:



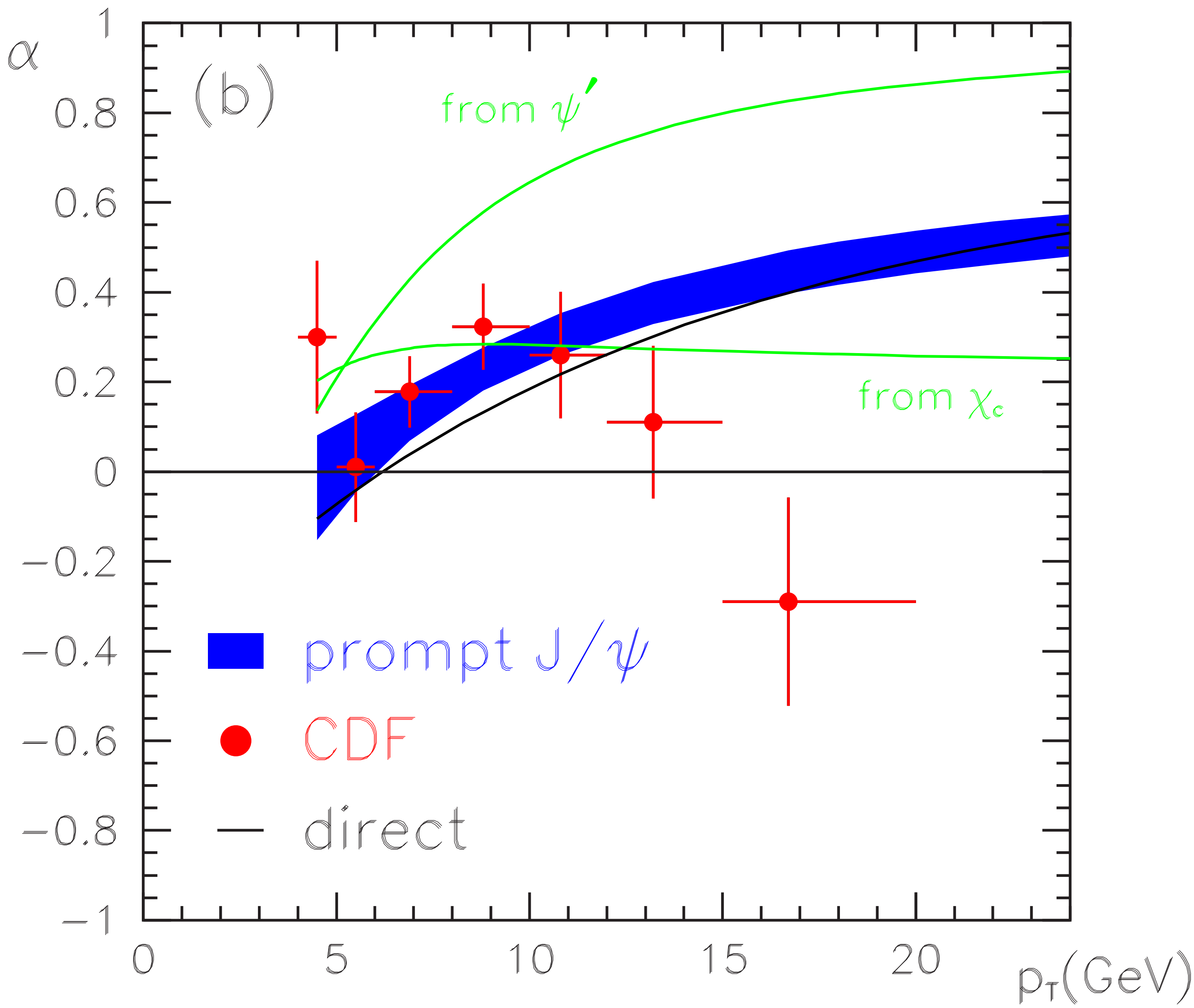
- The NRQCD-factorization predictions with no k_T smearing fit the CDF data:



- The ratio of total cross sections $R_{\chi_c} = \sigma[\chi_{c1}]/\sigma[\chi_{c2}]$
  - NRQCD (Maltoni): $R_{\chi_c} = 0.9 \pm 0.2$.
  - CEM: $R_{\chi_c} = 3/5$.
  - Expt. (CDF): $R_{\chi_c} = 1.04 \pm 0.29(\text{stat.}) \pm 0.12(\text{sys.})$.
  - The data slightly favor the NRQCD prediction.

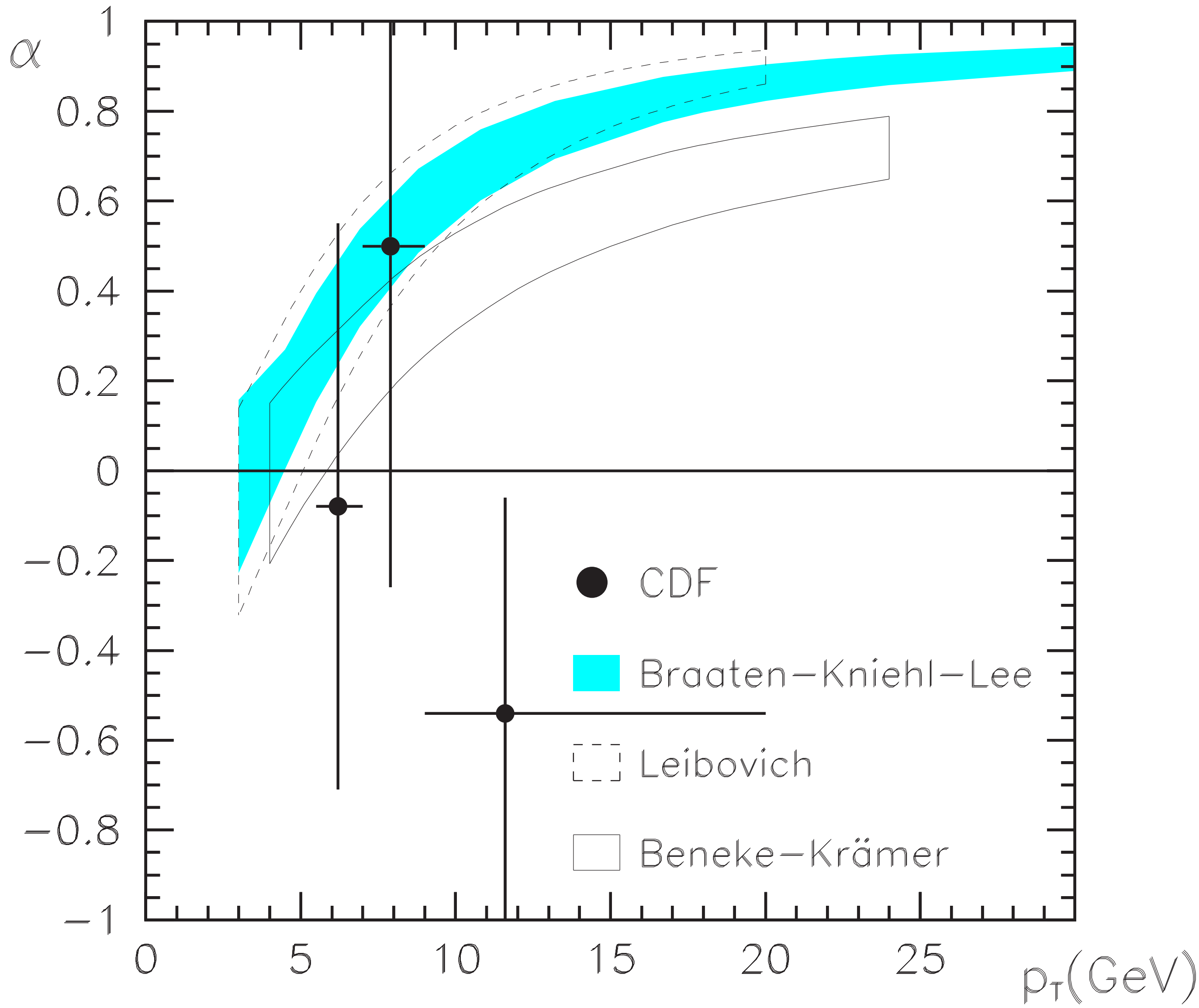
Polarized Quarkonium Production

- Potentially a "smoking gun" for the Color-Octet Mechanism.
- For large-p_T quarkonium production ($p_T \gtrsim 3m_c$ for J/ψ), gluon fragmentation via the color-octet mechanism dominates ($\langle \mathcal{O}_8(^3S_1) \rangle$).
- At large p_T, the gluon is nearly on mass shell, and, so, is transversely polarized.
- In color-octet gluon fragmentation, nearly all of the gluon's polarization is transferred to the J/ψ (Cho, Wise).
- Radiative corrections, color-singlet production dilute this (Beneke, Rothstein; Beneke, Krämer).
- In the J/ψ case, feeddown is important, but has now been taken into account (Braaten, Lee).
  - Feeddown from χ_c states is about 30% of the J/ψ sample and dilutes the polarization.
  - Feeddown from $\psi(2S)$ is about 10% of the J/ψ sample and is largely transversely polarized.



- $d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta$.
  - $\alpha = 1$ corresponds to transverse polarization;
  - $\alpha = -1$ corresponds to longitudinal polarization.

- In the $\psi(2S)$ case, feeddown is not important, but statistics are not as good.



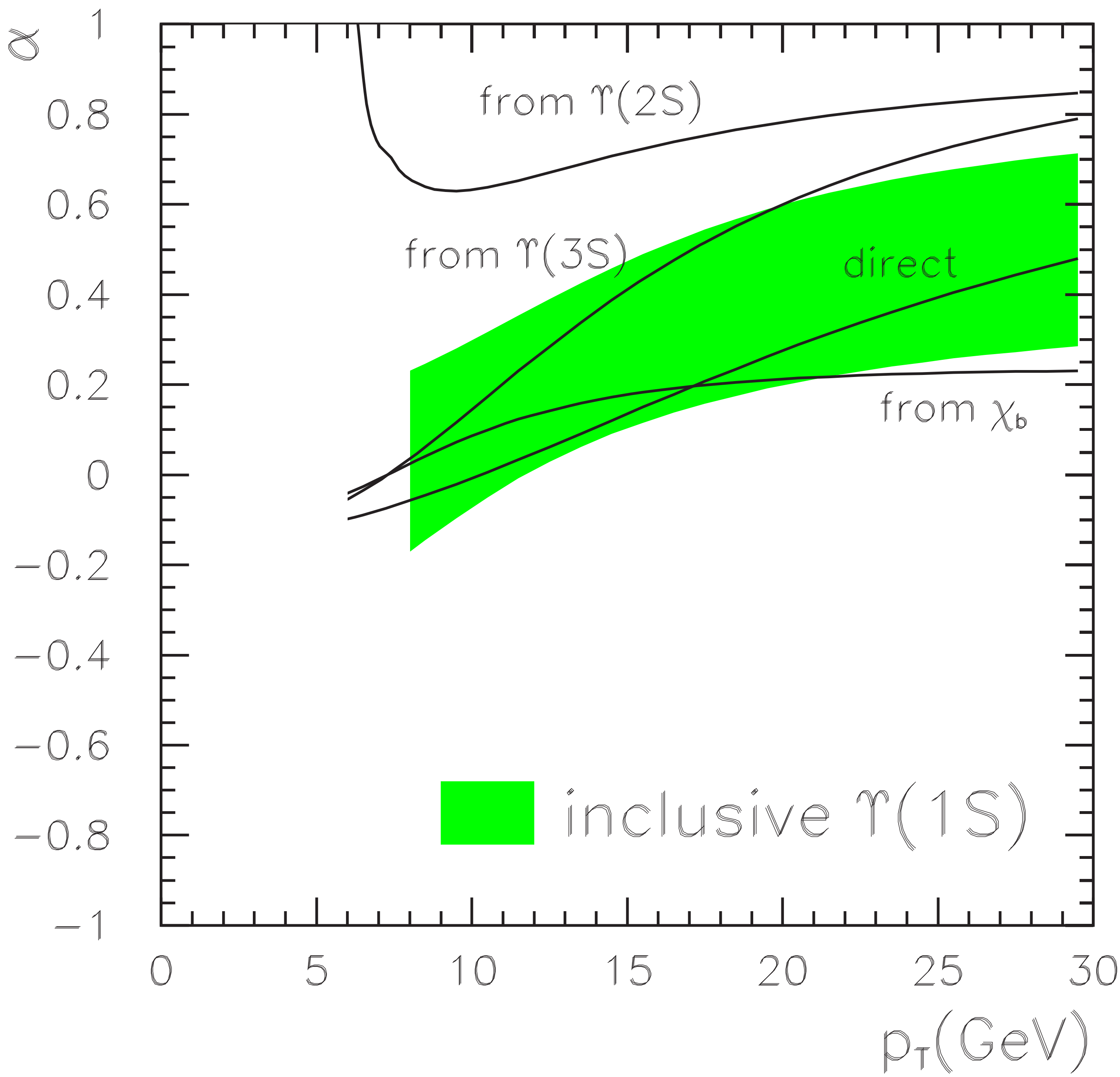
- The observed J/ψ and $\psi(2S)$ polarizations are much smaller than the prediction and seem to decrease with p_T.

There are many sources of theoretical uncertainty

- Uncertainties in matrix elements (shown in plots)
- Contributions of higher order in α_s
  - Calculated for ${}^3S_1$ color-octet fragmentation (Braaten, Lee), which gives the bulk of the polarization.
  - Corrections to the non-fragmentation process could conceivably increase the unpolarized contribution by a factor 2.
- Multiple soft-gluon emission
  - Polarization depends on a ratio of processes.
  - Effects of multiple soft-gluon emission tend to cancel.
- Large order-v^2 corrections to gluon fragmentation to quarkonium (GTB, Lee).
  - $+50\%$ for the color-singlet part.
    Yields a small correction to total the rate.
  - -40% for the color-octet part.
    Changes the normalization of the fitted matrix element, but not the rate.
  - Does the v expansion converge?

- Existing calculations assume that 100% of the $Q\bar{Q}$ polarization is transferred to the quarkonium.
  - Spin-flip corrections are suppressed only by v^2, not v^4, relative to the non-flip part. (GTB, Braaten, Lepage)
  - It could happen that the spin-flip corrections are anomalously large.
  - Do the velocity-scaling rules need to be modified? (Brambilla, Pineda, Soto, Vairo; Fleming, Rothstein, Leibovich)
  - A lattice calculation of color-octet decay matrix elements indicates that spin-flip processes are indeed suppressed by a factor v^2 or smaller (GTB, Lee, Sinclair).
- In spite of these uncertainties, it is difficult to see how there could not be substantial polarization in J/ψ or $\psi(2S)$ production for $p_T > 3m_c$.

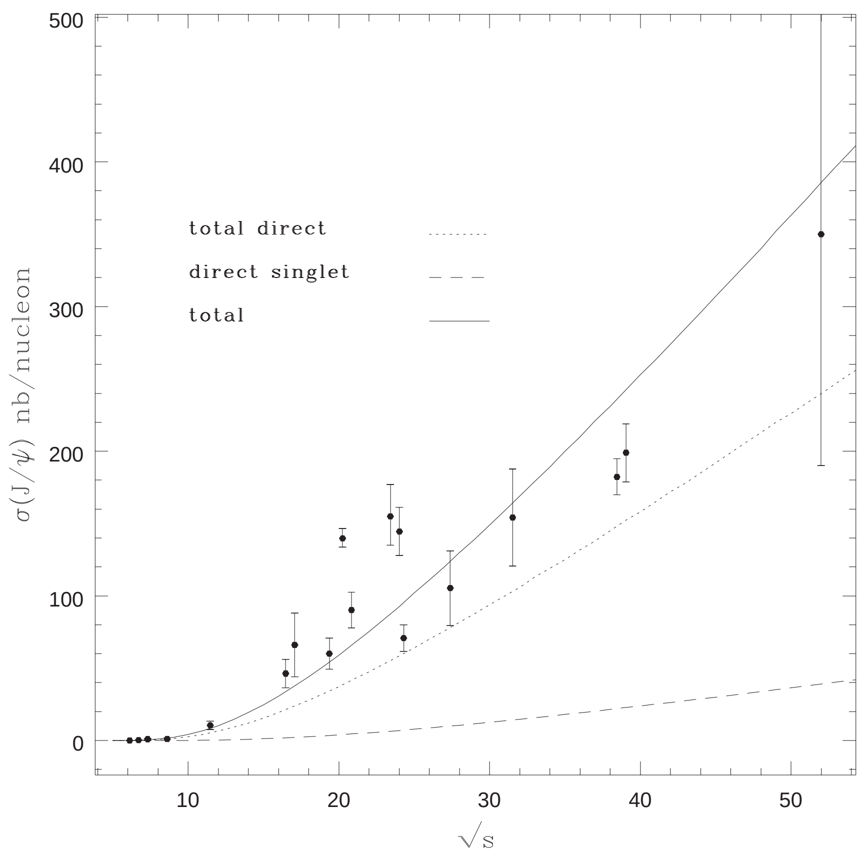
- Compared to J/ψ polarization, Υ polarization has smaller v-expansion uncertainties.
- But it is necessary to go to higher p_T to insure that the fragmentation mechanism dominates, so that there is substantial polarization.



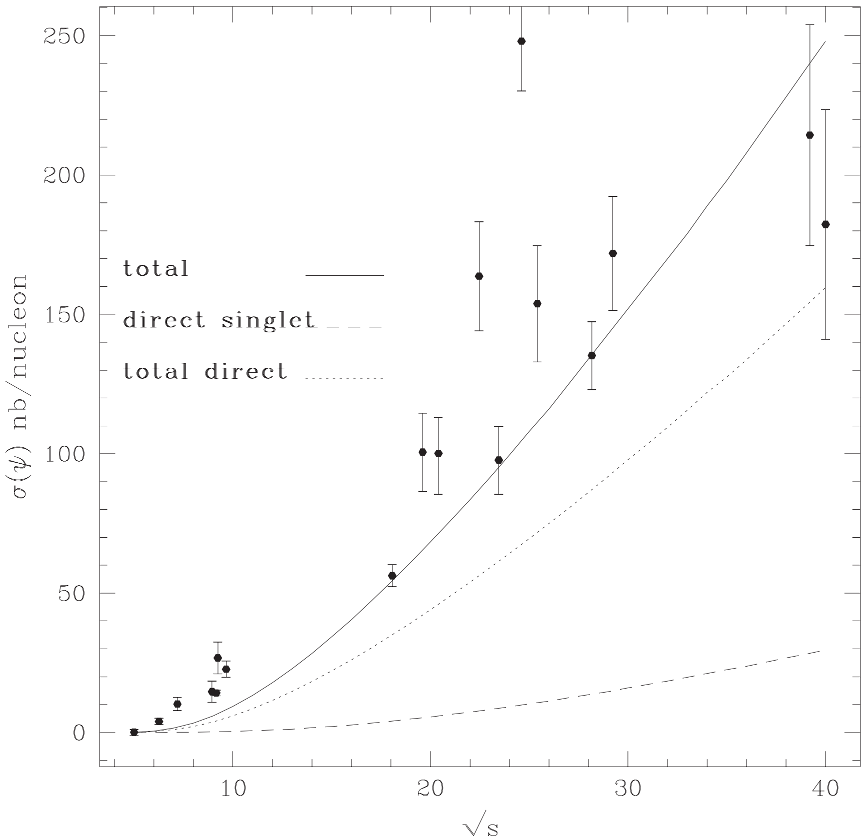
- CDF finds $\alpha = -0.06 \pm 0.20$ for 1 GeV $< p_T < 20$ GeV.

Total Cross Sections

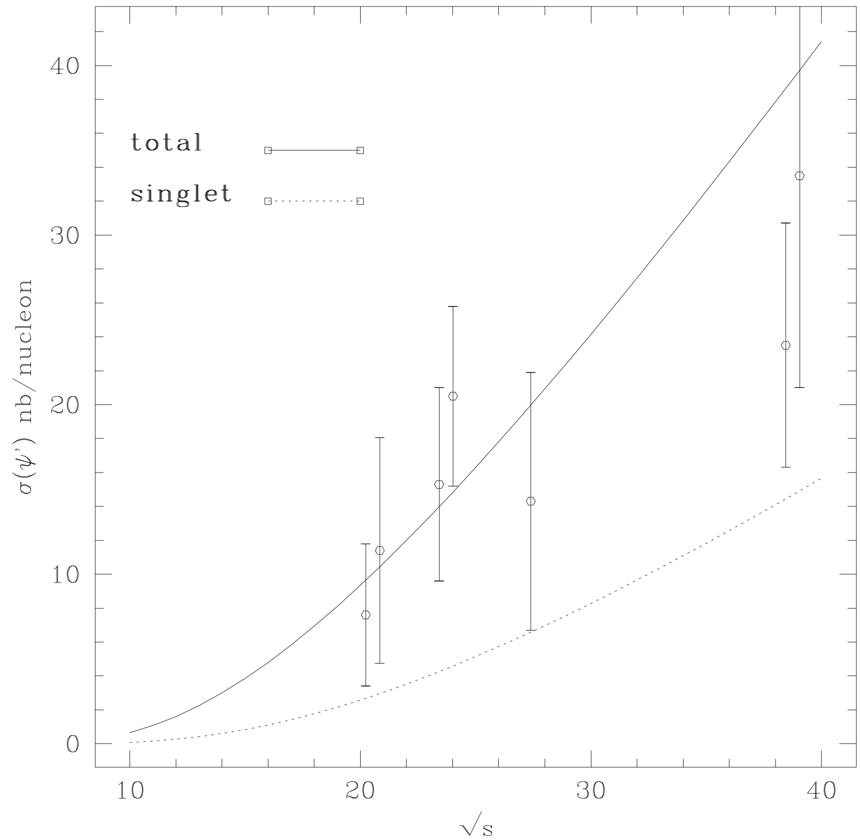
- The relevant linear combinations are now
  $\langle \mathcal{O}_8({}^3S_1) \rangle$,
  $M_k = (k/m^2)\langle \mathcal{O}_8({}^3P_0) \rangle + \langle \mathcal{O}_8({}^1S_0) \rangle$, with $k \approx 7$.
- Comparison involves total cross sections, so can fit only M_k.
  - Use Tevatron octet matrix elements in other cases.
- The leading-order result (Beneke, Rothstein), using CTEQ3L PDF's, gives



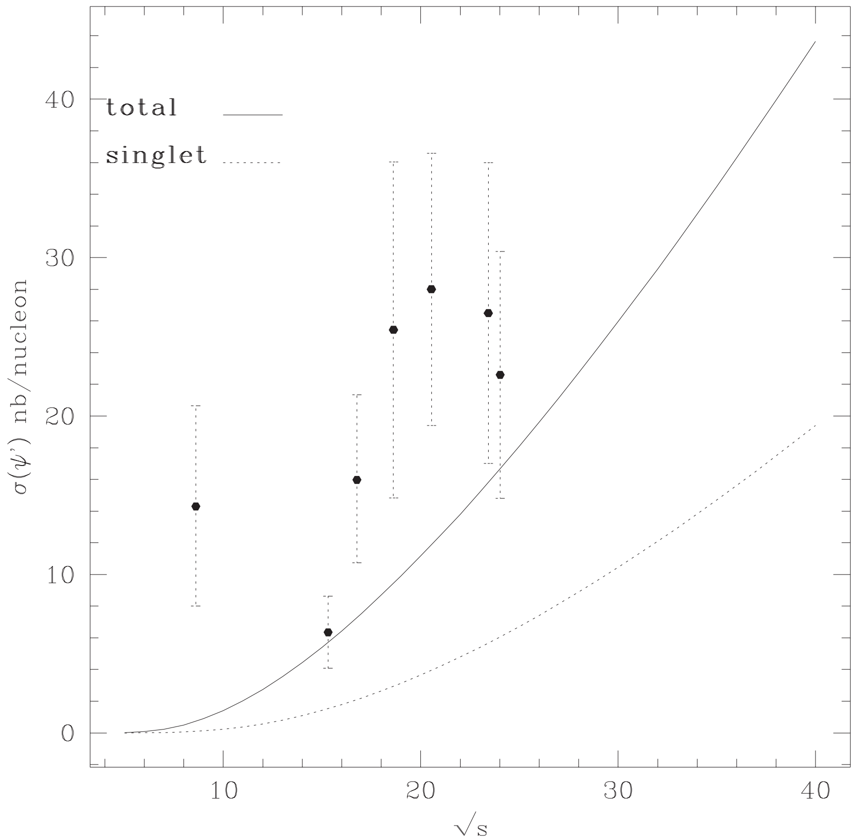
for J/ψ production with $x_F > 0$ in pN collisions,



for J/ψ production with $x_F > 0$ in πN collisions,



for $\psi(2S)$ production with $x_F > 0$ in pN collisions,



for $\psi(2S)$ production with $x_F > 0$ in πN collisions.

$$M_7 = \begin{cases} 3.0 \times 10^{-2}\ \text{GeV}^3 & (J/\psi); \\ 0.52 \times 10^{-2}\ \text{GeV}^3 & (\psi(2S)). \end{cases}$$

- Relative-order-α_s corrections give a large k-factor (Petrelli, Cacciari, Greco, Maltoni, Mangano).
- The NLO result, using CTEQ4M PDF's, gives

$$M_{6.4}^{J/\psi} = 1.8 \times 10^{-2}\ \text{GeV}^3$$
$$M_{6.4}^{\psi(2S)} = 0.26 \times 10^{-2}\ \text{GeV}^3$$

  – The NLO $M_{6.4}$ is about a factor 2 smaller than the LO M_7.

- Seems to be in disagreement with Tevatron results (2–6 times too small).
  - The Tevatron is sensitive to M_3, so comparisons are somewhat uncertain.
    - $\overline{\mathrm{MS}}$ matrix elements need not be positive.
    - Factorization-scheme dependence isn't resolved until relative-order α_s.
  - Does factorization hold for the total cross section?
  - Kinematic corrections from the difference between $2m$ and the quarkonium mass may be large.

- Theoretical and experimental uncertainties cancel in the ratios of cross sections.

$$R_\psi = \frac{\sigma[\psi(2S)]}{\sigma[J/\psi]},$$

$$R_{\chi c} = \frac{\sigma[\chi_{c1}]}{\sigma[\chi_{c2}]},$$

$$F_{\chi c} = \sum_{J=0}^{2} \mathrm{Br}[\chi_{cJ}(1P) \to J/\psi + X] \, \frac{\sigma[\chi_{cJ}(1P)]}{\sigma[J/\psi]}.$$

- Experimental results for R_ψ:

Experiment	beam/target	$\sqrt{s}$/GeV	R_ψ
E537	$\bar{p}$W	15.3	0.185 ± 0.0925
E705	pLi	23.7	$0.14 \pm 0.02 \pm 0.004 \pm 0.02$
E705	$\bar{p}$Li	23.7	$0.25 \pm 0.22 \pm 0.007 \pm 0.04$
E771	pSi	38.8	0.14 ± 0.02
HERA-B	p(C, W)	41.5	0.13 ± 0.02
E537	π^-W	15.3	0.2405 ± 0.0650
E673	πBe	20.6	0.20 ± 0.09
E705	π^+Li	23.7	$0.14 \pm 0.02 \pm 0.004 \pm 0.02$
E705	π^-Li	23.7	$0.12 \pm 0.03 \pm 0.03 \pm 0.02$
E672/706	π^-Be	31.1	$0.15 \pm 0.03 \pm 0.02$

- NRQCD (Beneke, Rothstein):
  $R_\psi = 0.16$ for pN and $\pi^- N$ collisions.
- The CSM (Beneke, Rothstein):
  $R_\psi = 0.14$ for pN collisions;
  $R_\psi = 0.16$ $\pi^- N$ collisions.
- The CEM uses this ratio as an input.
- R_ψ does not discriminate between these approaches.

- Experimental results for $F_{\chi c}$ and $R_{\chi c}$:

Experiment	beam/target	$\sqrt{s}$/GeV	$F_{\chi c}$	$R_{\chi c}$
E673	pBe	$19.4/21.7$	0.47 ± 0.23	0.24 ± 0.28
E705	pLi	23.7	—	$0.08^{+0.25}_{-0.15}$
E705	pLi	23.7	0.30 ± 0.04	—
E771	pSi	38.8	—	$0.53 \pm 0.20 \pm 0.07$
HERA-B	p(C, W)	41.5	$0.32 \pm 0.06 \pm 0.04$	—
WA11	πBe	18.6	0.305 ± 0.050	0.68 ± 0.28
E673	πBe	18.9	0.31 ± 0.10	0.96 ± 0.64
E673	πBe	20.6	0.37 ± 0.09	0.9 ± 0.4
E705	πLi	23.7	—	$0.52^{+0.57}_{-0.27}$
E705	π^+Li	23.7	0.40 ± 0.04	—
E705	π^-Li	23.7	0.37 ± 0.03	—
E672/706	π^-Be	31.1	$0.443 \pm 0.041 \pm 0.035$	$0.57 \pm 0.18 \pm 0.06$

- NRQCD (Beneke, Rothstein):
  $F_\chi = 0.27$ for pN collisions;
  $F_\chi = 0.28$ for $\pi^- N$ collisions.
- The CSM (Beneke, Rothstein):
  $F_\chi = 0.68$ for pN collisions;
  $F_\chi = 0.66$ for $\pi^- N$ collisions.
- The CEM uses this ratio as an input.
- The F_χ experimental results clearly favor NRQCD over the CSM.

- Large variations in the NRQCD predictions for R_χ.
  - NRQCD at LO with the standard truncation (Beneke, Rothstein):
    $R_\chi = 0.07$ for pN collisions;
    $R_\chi = 0.05$ $\pi^- N$ collisions.
  - NRQCD assuming that the ${}^3P_2$ and ${}^3P_0$ color-octet matrix elements dominate (Beneke):
    $R_\chi \approx 0.3$ for pN and πN collisions.
  - Consistent with a result of Gupta and Mathews once that is corrected to take into account the dominant color-singlet channel in χ_{c2} production.
  - Beneke and Rothstein suggest that higher-twist (m^2/p_T^2) corrections may be large.
  - NRQCD at LO in v but NLO in α_s (Maltoni):
    $R_\chi = 0.04$–0.1 as the beam energy ranges from 200 GeV to 800 GeV.
- Range of NRQCD predictions: $R_\chi = 0.04$–0.3.
- CSM (Beneke, Rothstein; Beneke):
  $R_\chi \approx 0.05$–0.07 for pN and πN collisions.
- CEM: $R_\chi = 3/5$.

- Some inconsistency in the data.
- Differences between πN and pN experiments contradict the CEM.
  Not expected in NRQCD unless the $q\bar{q}$ production channel is enhanced.
- The data are significantly larger than the CSM predictions.
- The pN data favor the NRQCD predictions.
- The πN data favor the CEM predictions.
- Does factorization hold for total cross section?

Polarization

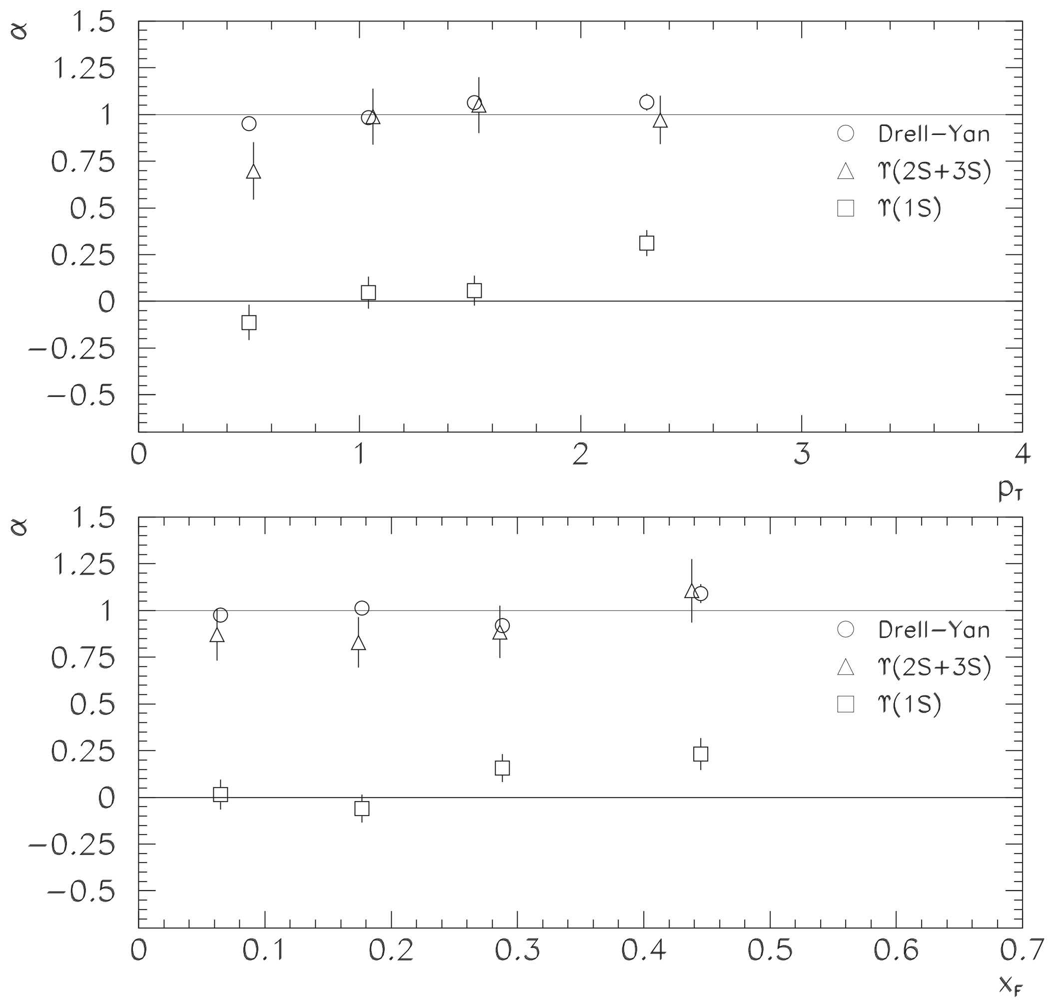
- Experimental results for J/ψ polarization:

Experiment	beam/target	Beam Energy/GeV	α
E537	(π, p)(Be, Cu, W)	125	0.024–0.032
E672/706	pBe	530	0.01 ± 0.15
E672/706	pBe	800	-0.11 ± 0.15
E771	pSi	800	-0.09 ± 0.12
E866	pCu	800	0.069 ± 0.08
HERA-B	p(C, W)	920	$(-0.5, +0.1) \pm 0.1$

- NRQCD (Beneke, Rothstein) gives $0.31 < \alpha < 0.63$
  - Includes feeddown from χ_c states.
- The CSM (Vanttinen, Hoyer, Brodsky, Tang) predicts substantial transverse polarization.
- The CEM predicts $\alpha = 0$.
- Specific predictions for HERA-B for $p_T = 1.5$–4 GeV (J. Lee):
  NRQCD: $\alpha = 0$–0.1,
  CSM: $\alpha = 0.2$–0.4.

- Conventional fixed-target results are consistent with $\alpha = 0$ and favor the CEM over NRQCD and the CSM.
  - Can question whether resummation is needed and whether NRQCD factorization holds at the smaller values of p_T.
- HERA-B results are consistent with $\alpha = 0$ and favor NRQCD and the CEM over the CSM.
- The E615 experiment measures $\psi(2S)$ polarization in πN collisions at 253 GeV.
  - Experiment: $-0.12 < \alpha < 0.16$.
  - NRQCD: $0.15 < \alpha < 0.44$.

- A small, but nonzero, transverse polarization of Υ's has been observed in p-Cu collisions (E866).



- Less than the NRQCD prediction $\alpha = 0.28 - 0.31$ (Kharchilava, Lohse, Somov, Tkabladze; Tkabladze).
- But disagrees with the CEM prediction $\alpha = 0$.
- Peculiar that polarization is seen in $\Upsilon(2S)$ and $\Upsilon(3S)$, but not in $\Upsilon(1S)$.

Quarkonium Production in B Decays

Branching Fractions of B Mesons into Charmonium

- Rates into J/ψ, $\psi(2S)$ χ_{c1}, and χ_{c2} have been measured at LEP in Z_0 decay and at CLEO.
- Larger than predictions of the color-singlet model by about a factor of 3.
- The color-octet contribution is suppressed by v^4, but enhanced by the Wilson coefficient in the effective electroweak action at the scale m_c.
- May involve an accidental cancellation of the color-singlet Wilson coefficient in leading order.
- Calculation of the color-octet term by Ko, Lee, Song gives a possible explanation of the factor of 3.

Inclusive Rates into Charmonium

- Beneke, Maltoni, Rothstein have calculated inclusive rates into J/ψ and $\psi(2S)$ at next-to-leading order in α_s.
- Extracted matrix elements:

$$M_{3.1}^{J/\psi} = (1.5^{+0.8}_{-1.1}) \times 10^{-2}\ \text{GeV}^3.$$
$$M_{3.1}^{\psi(2S)} = (0.5 \pm 0.5) \times 10^{-2}\ \text{GeV}^3.$$

  - Uncertainty from experiment, $\langle \mathcal{O}_8(^3S_1)\rangle$, and $\langle \mathcal{O}_1(^3S_1)\rangle$.
- A calculation by J.-P. Ma that takes into account initial-state hadronic corrections gives

$$M_{3.4}^{J/\psi} = 2.4 \times 10^{-2}\ \text{GeV}^3.$$
$$M_{3.4}^{\psi(2S)} = 1.0 \times 10^{-2}\ \text{GeV}^3.$$

- The octet matrix elements are considerably smaller than the Tevatron results, but the errors are large.
- NRQCD calculation of χ_c production in B-meson decay (GTB, Braaten, Yuan, Lepage):
  - Predicts a non-zero χ_{c2} rate.
  - The χ_{c2} rate vanishes in the color-singlet model.
  - A possible test of the color-octet mechanism.
  - Feeddown from $\psi(2S)$ in data.
  - Subtracted data is compatible with zero or with a small color-octet contribution.

Polarization of J/ψ's at CLEO

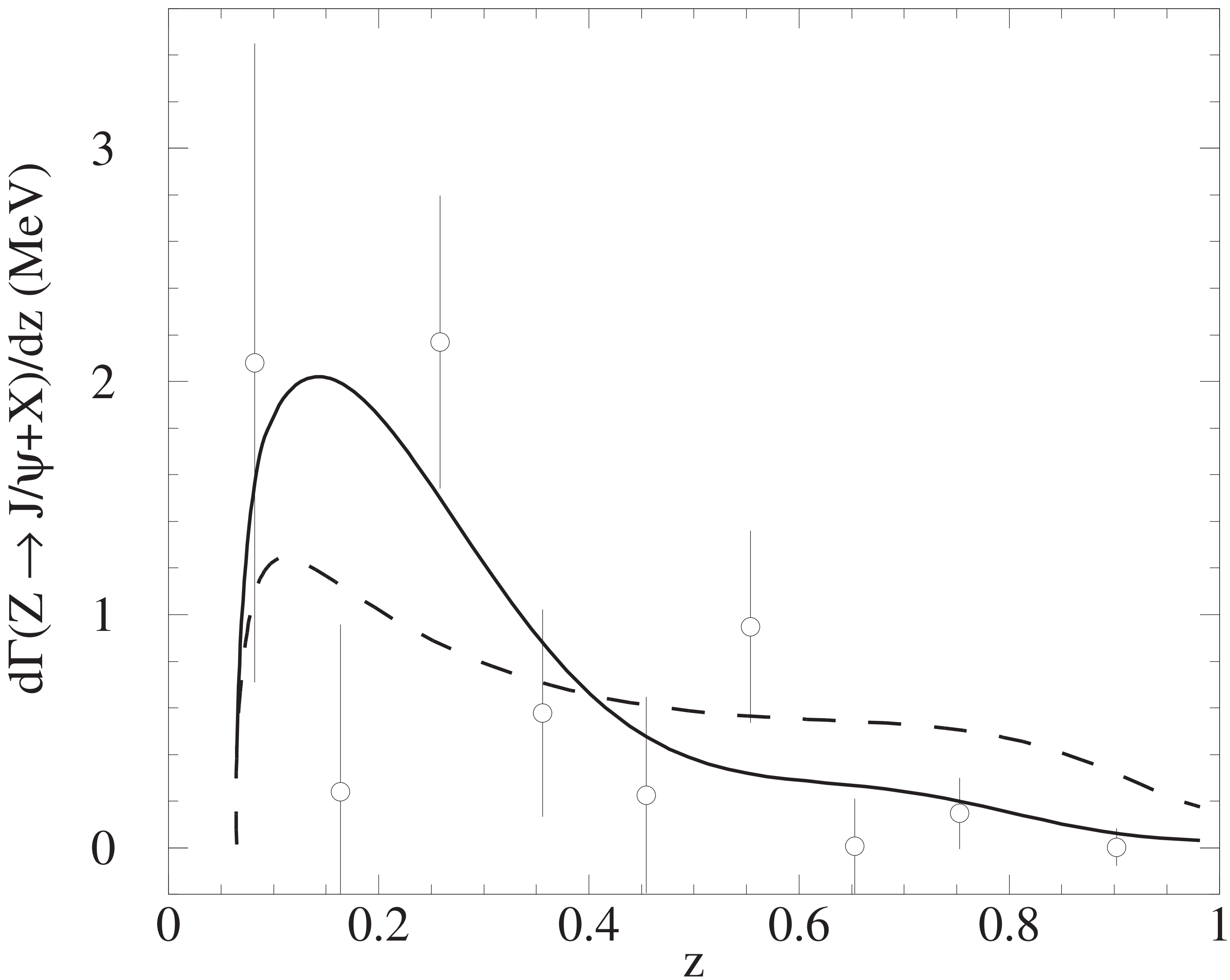
- Experiment: $\alpha = -0.30 \pm 0.08$.
- NRQCD (Fleming, Hernandez, Maksymyk, Nadeau): $\alpha = -0.33 \pm 0.08$
- CSM (Fleming, Hernandez, Maksymyk, Nadeau): $\alpha = -0.40 \pm 0.07$
- CEM: $\alpha = 0$.
- Rules out the CEM at the 5σ level.

$Z \rightarrow J/\psi + X$

- Boyd, Leibovich, and Rothstein resummed $\log(M_Z^2/M_\psi^2)$, $\log(z^2)$.
- $\langle \mathcal{O}_8(^3S_1) \rangle$ contributions dominate.
- Yields

$$\langle \mathcal{O}_8(^3S_1) \rangle = (1.9 \pm 0.5_{stat} \pm 1.0_{theory}) \times 10^{-2} \text{ GeV}.$$

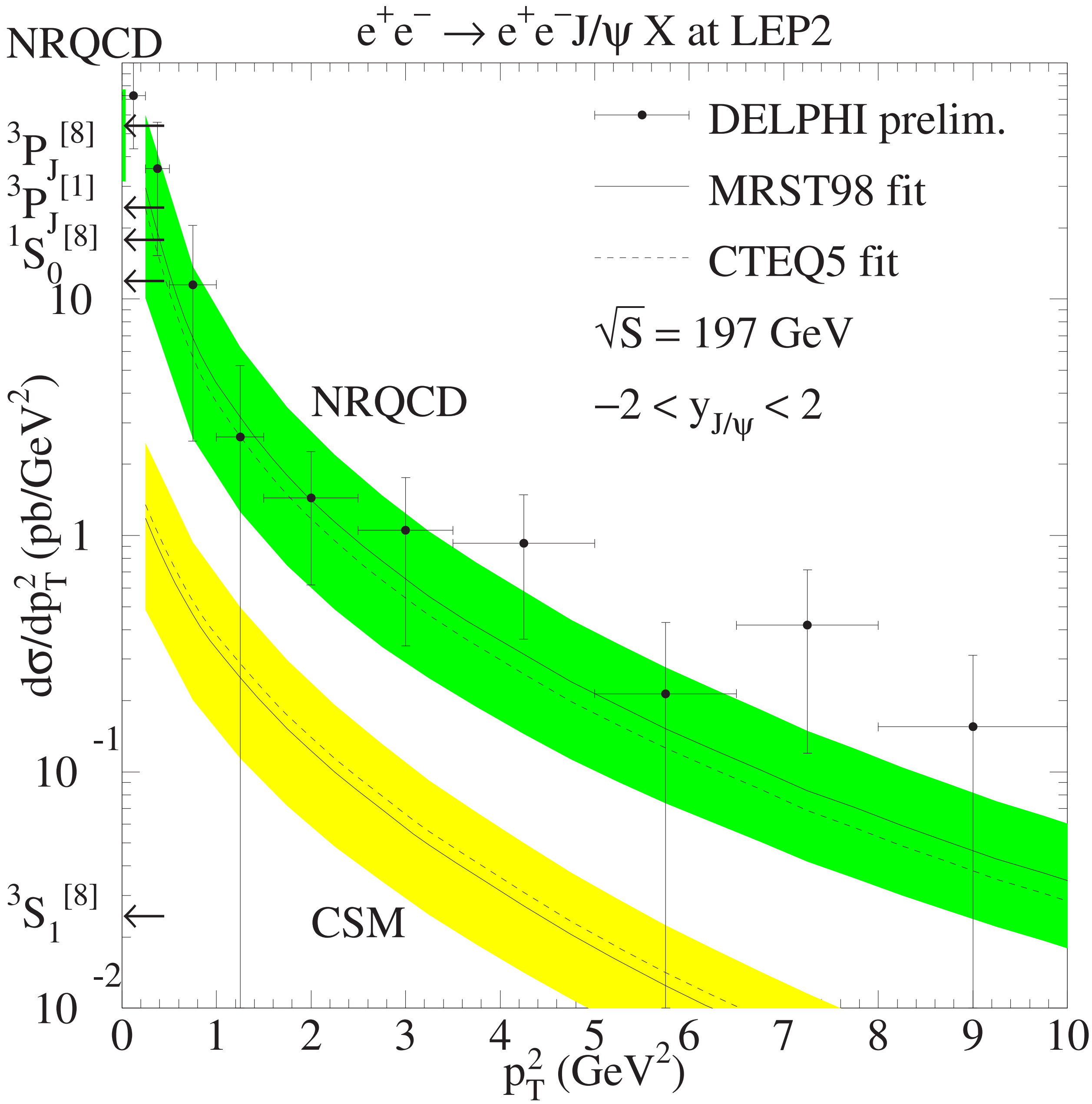
- A factor of 2 larger than the Tevatron value and has smaller theory errors, but includes feeddown from χ_c and $\psi(2S)$ states.
- A bump at small z is a signature for color-octet production, but errors in the data are too large to confirm this feature.



dΓ(Z → J/ψ+X)/dz (MeV)
3
2
1
0
0
0.2
0.4
0.6
0.8
1
z

$\gamma\gamma \to J/\psi + X$

- Comparison of theory (Klasen, Kniehl, Mihaila, Steinhauser) with Delphi data clearly favors NRQCD over the CSM.



- Theory uses Braaten-Kniehl-Lee matrix elements and MRST98LO (solid) and CTEQ5L (dashed) PDF's.
- Theoretical uncertainties from
  - Renormalization and factorization scales (varied by a factor 2),
  - Color-octet matrix elements.
    - Different linear combination of matrix elements than in Tevatron cross sections.

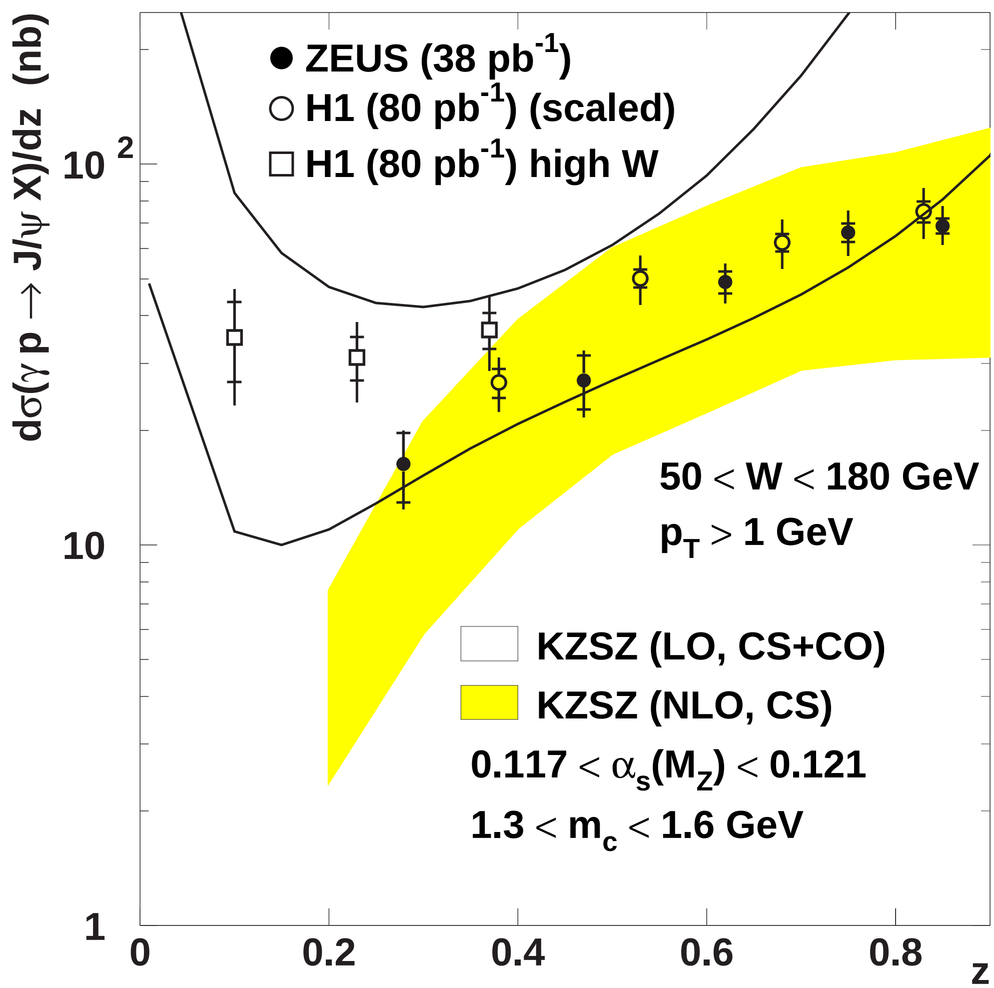
$Z \to \Upsilon + X$

- $Br(Z^0 \to \Upsilon + X)$
  - NRQCD: 5.9×10^{-5}.
  - CSM: 1.7×10^{-5}.
  - OPAL: $[1.0 \pm 0.4(\text{stat.}) \pm 0.1(\text{sys.}) \pm 0.2(\text{prod. mech.})] \times 10^{-4}$.
- Data are compatible with NRQCD factorization, but not with the CSM.

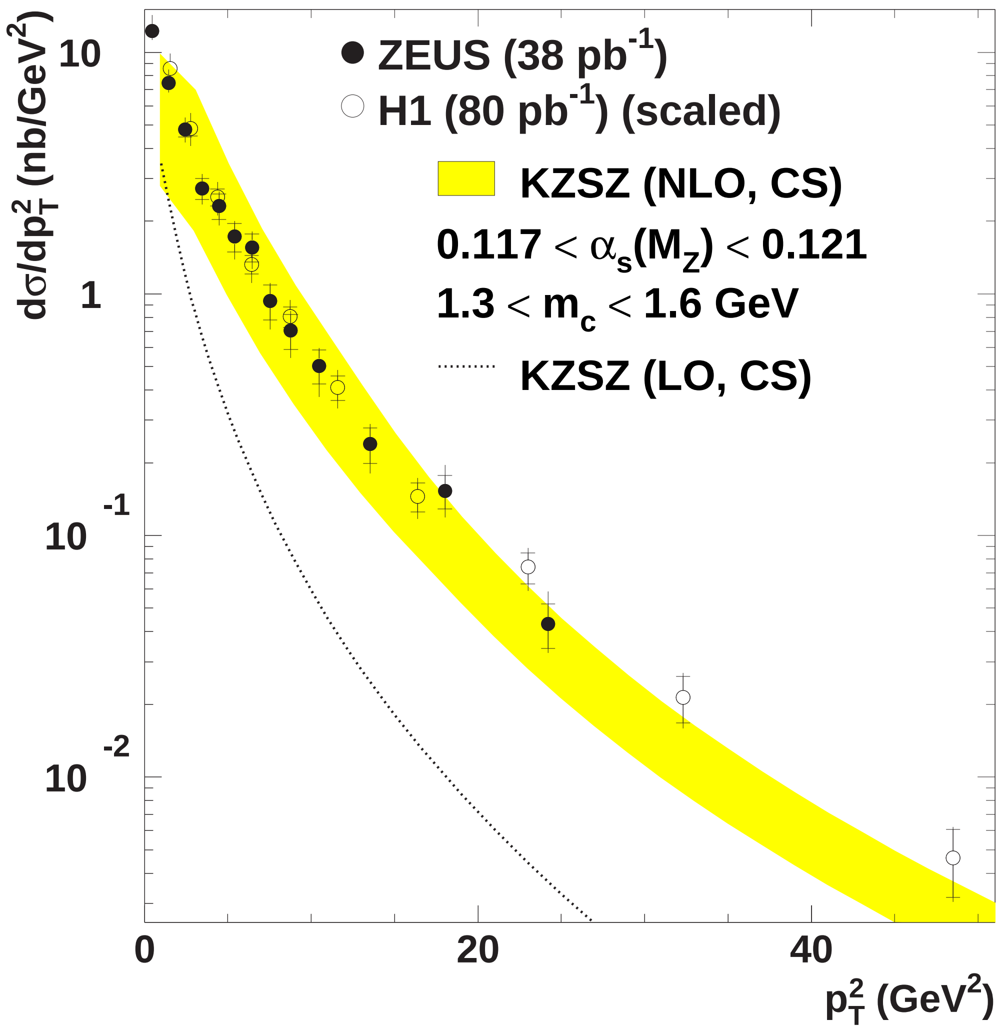
Quarkonium Production at HERA

Inelastic Quarkonium Photoproduction

- NRQCD calculations by Cacciari, Krämer; Amundson, Fleming, Maksymyk; Ko, Lee, Song; Kniehl, Krämer.
- NLO CSM calculations by Krämer; Krämer, Zunft, Steegborn, Zerwas.



- There seems to be little room for the color-octet contribution in the photoproduction data.
- $p_T > 1$GeV cut.
  Can question whether factorization is OK at such small p_T.
- However, the data differential in p_T are compatible with color-singlet production alone even large p_T.



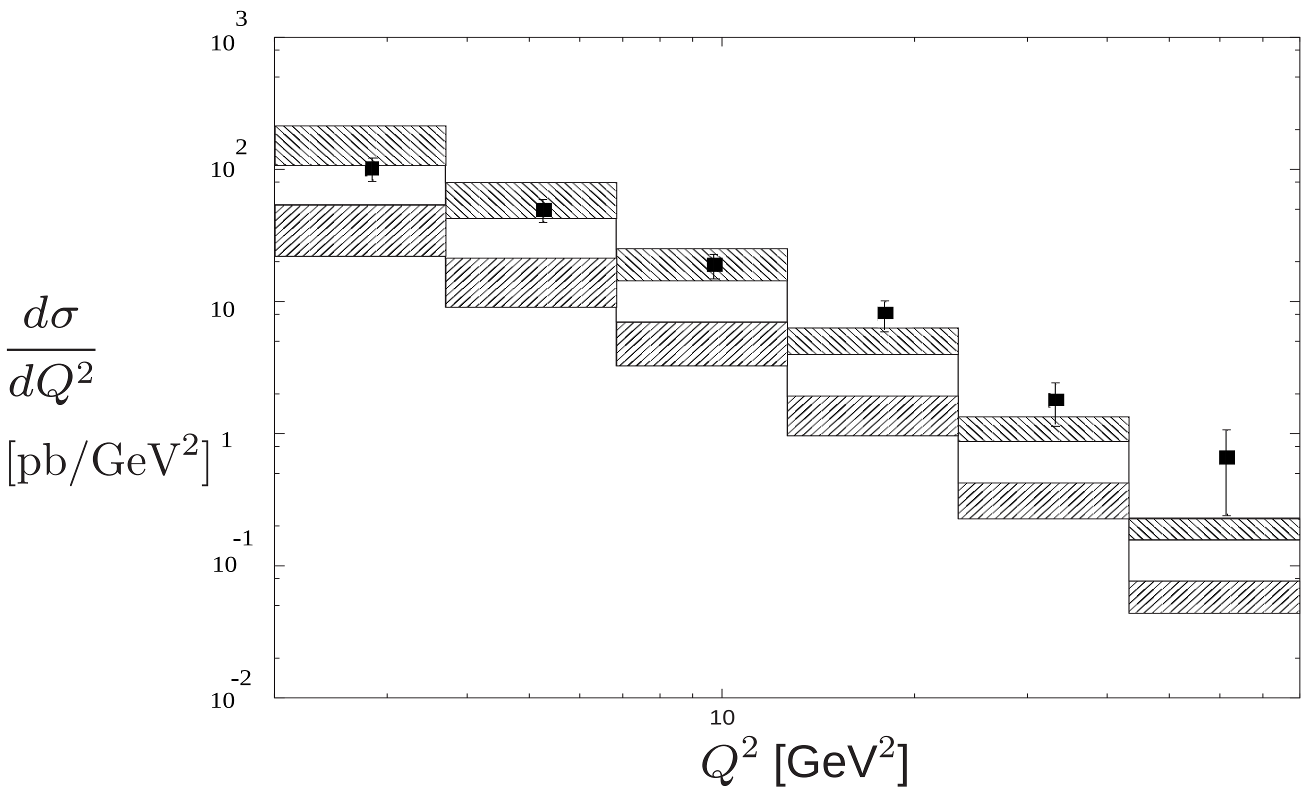
- NLO corrections increase the color-singlet piece substantially.
  - They include $\gamma + g \to (c\bar{c}) + gg$, which is dominated by t-channel gluon exchange.
  - For large p_T, this process goes as $\alpha_s^3 m_c^2/p_T^6$, instead of $\alpha_s^2 m_c^4/p_T^8$.

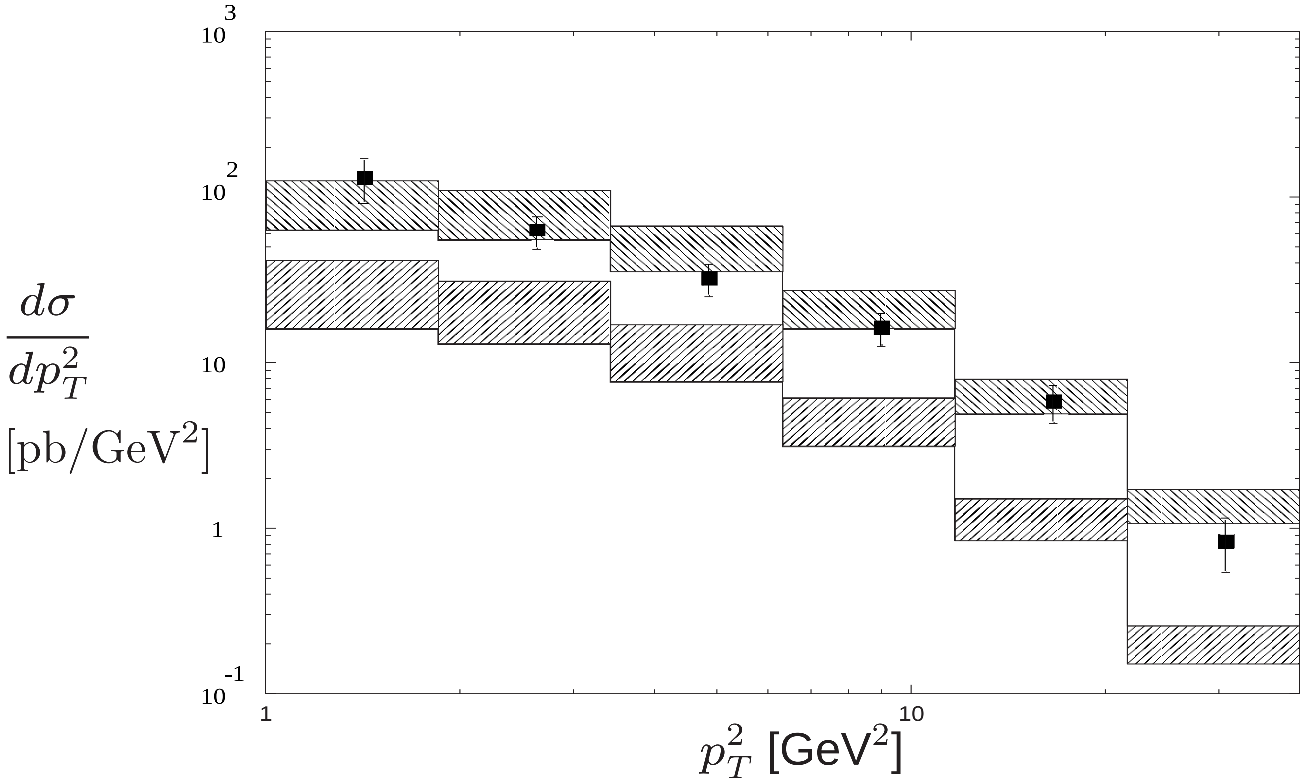
- The data are fit well with no color-octet contribution. But. . .
- Uncertainties in m_c could lower the color-singlet contribution by about a factor of 2, leaving more room for a color-octet contribution.
- There are large uncertainties in the color-octet matrix elements
  - Different linear combinations appear in photoproduction ($M_7^{J/\psi}$) than appear in hadroproduction at the Tevatron $M_3^{J/\psi}$).
- Soft-gluon resummation decreases the sizes of the matrix elements extracted from the Tevatron data.
- The color-octet contribution is calculated only at leading order in α_s for photoproduction.
- Near $z = 1$, resummation of multiple soft-gluon emission is needed (Beneke, Schuler, Wolf).

- The v expansion breaks down near $z = 1$.
  - Matrix elements of higher order in v correct for the difference between $Q\bar{Q}$ kinematics and quarkonium kinematics.
  - Near $z = 1$, these matrix elements become large and must be resummed.
  - Resummation of the v expansion leads to a nonperturbative shape function. (Beneke, Rothstein, Wise)
  - Future calculations could use shape functions extracted from e^+e^- data plus soft-gluon resummation to make a firm prediction.

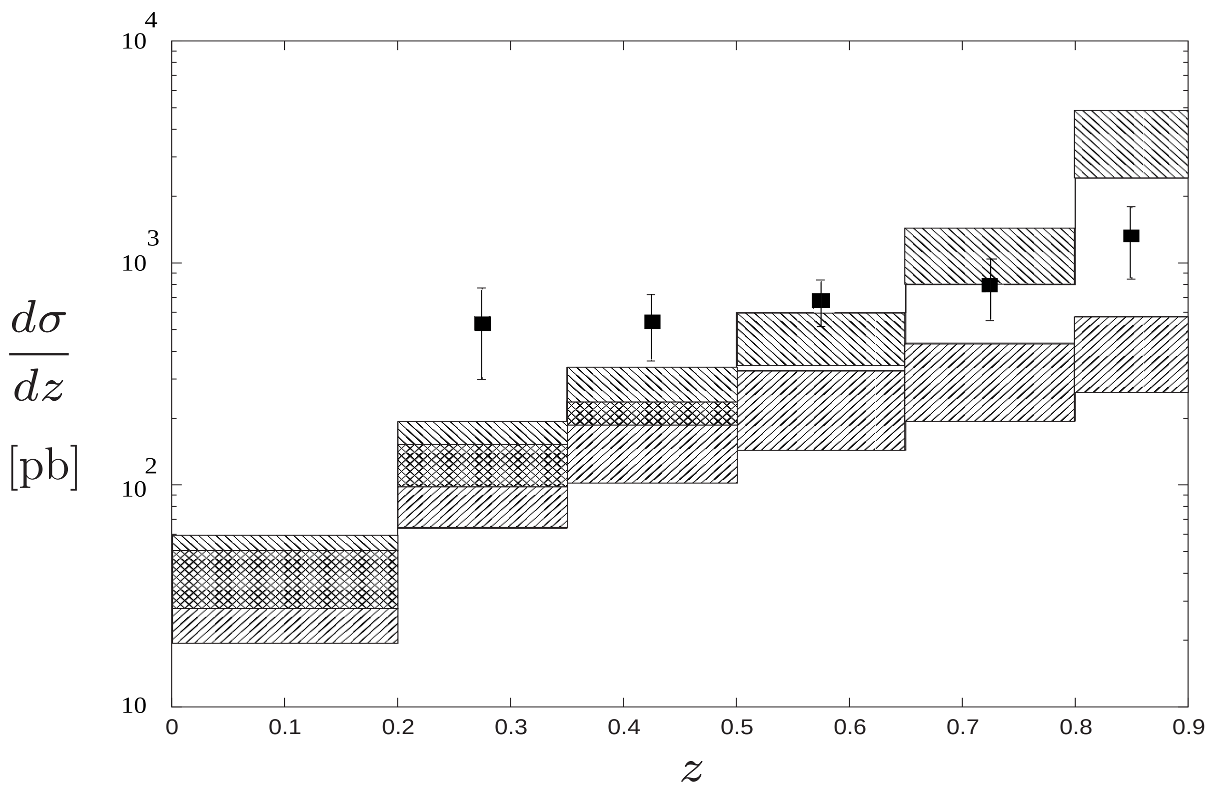
Quarkonium Production in DIS

- Calculations by Kniehl and Zwirner.
- H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).
- The data favor the NRQCD result when plotted vs. Q^2 and p_T^2, but not z.





$\frac{d\sigma}{dp_T^2}$ [pb/GeV^2]
p_T^2 [GeV^2]



$\frac{d\sigma}{dz}$ [pb]
z

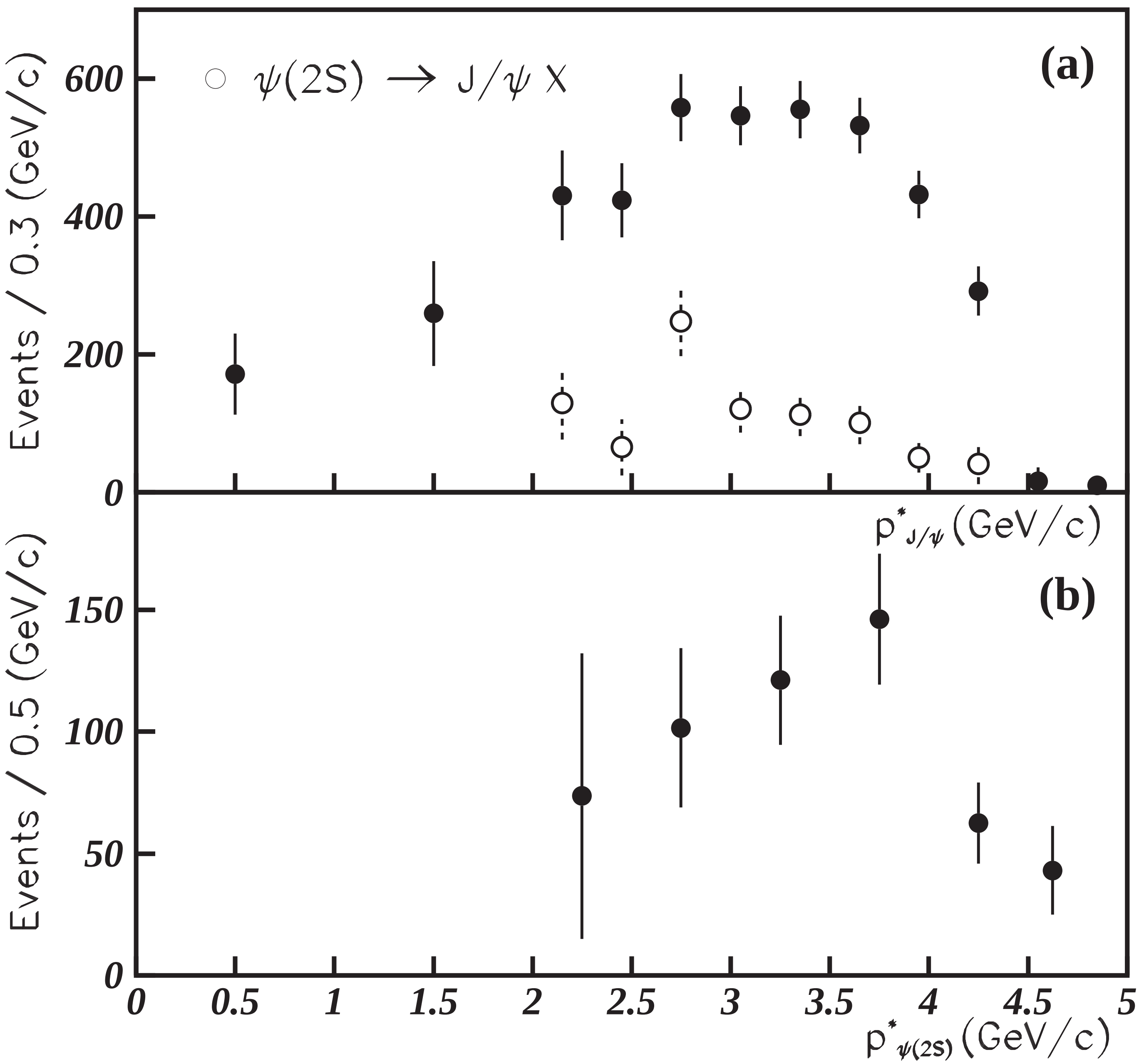
- Calculation uses Braaten-Kniehl-Lee matrix elements and MRST98LO and CTEQ5L PDF's.
- Theoretical uncertainties from
  - PDF's
  - Renormalization and factorization scales (varied by a factor 2),
  - Color-octet matrix elements.
    - Different linear combination of matrix elements than in Tevatron cross sections.
- The calculation of Kniehl and Zwirner disagrees with a number of previous results.
  These disagreements have not yet been resolved fully.

Quarkonium Production in e^+e^- Annihilation Near 10.6 GeV

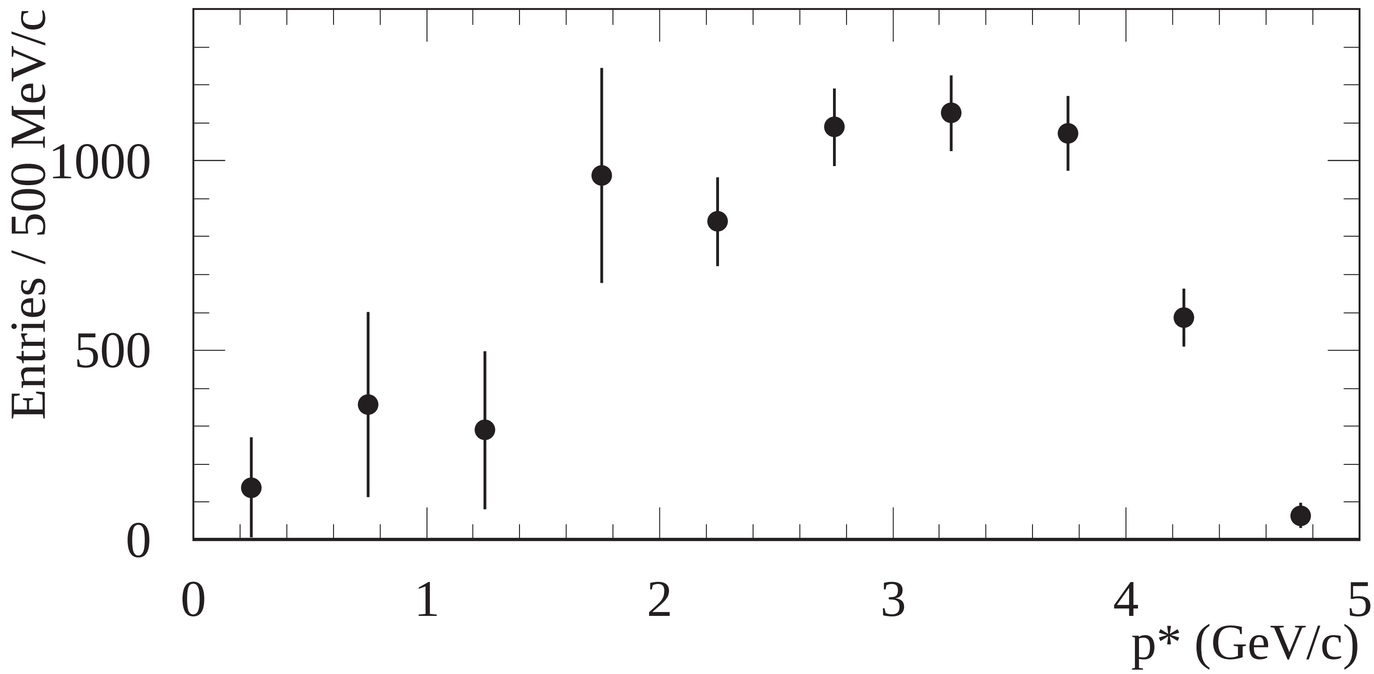
J/ψ Cross Section

- Belle: $\sigma(e^+e^- \to J/\psi\, X) = 2.52 \pm 0.21 \pm 0.21$ pb.
- BaBar: $\sigma(e^+e^- \to J/\psi\, X) = 1.47 \pm 0.10 \pm 0.13$ pb.
- CSM (F. Yuan, C.-F. Qiao, K.-T. Chao; G.A. Schuler; P. Cho, A.K. Leibovich): $\sigma(e^+e^- \to J/\psi\, X) = 0.45 - 0.81$ pb.
- NRQCD (F. Yuan, C.-F. Qiao, K.-T. Chao; G.A. Schuler): $\sigma(e^+e^- \to J/\psi\, X) = 1.1 - 1.6$ pb.
- 3σ discrepancy between experiments, but NRQCD seems to be favored.

- J/ψ center-of-mass momentum distribution at Belle:



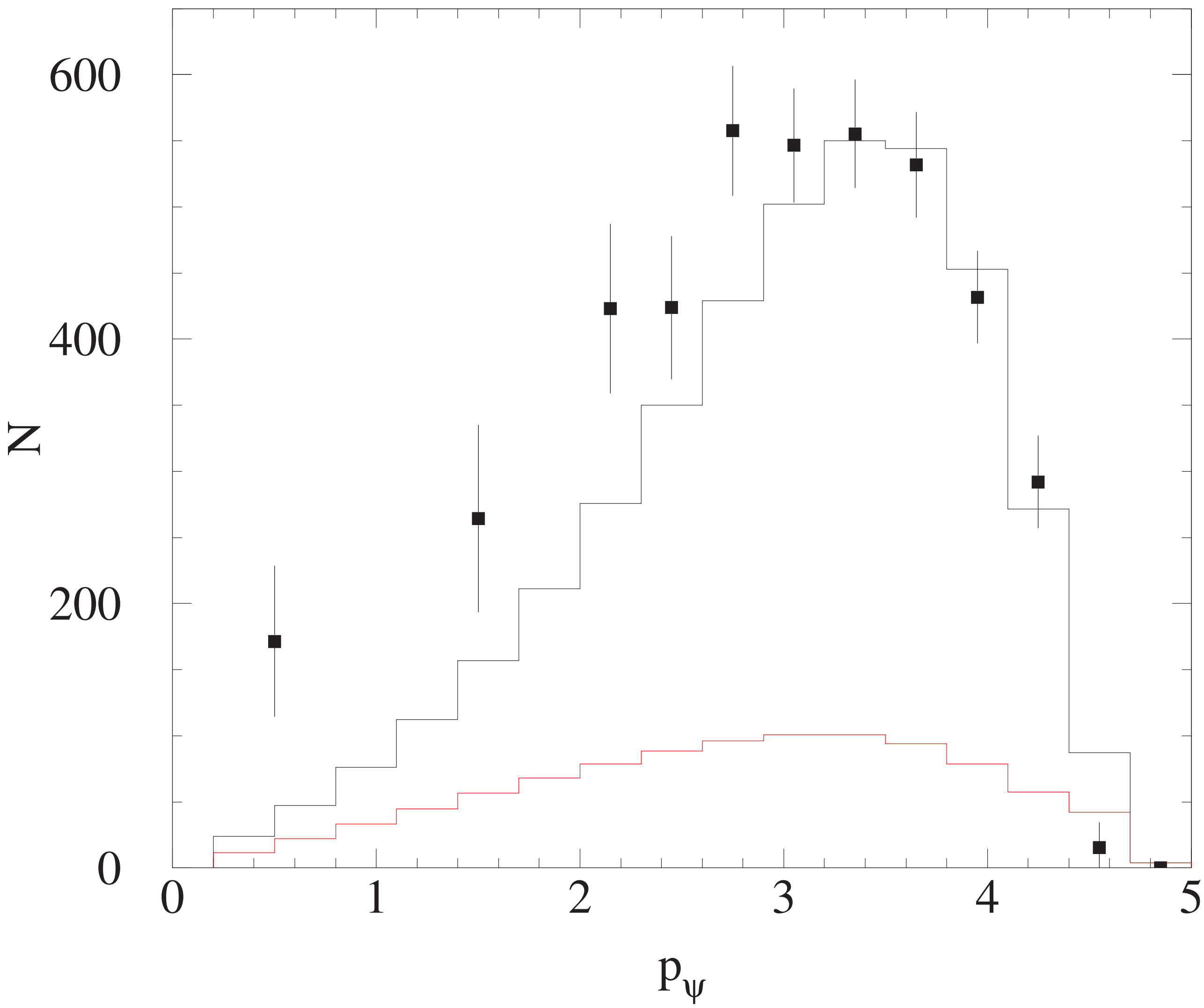
- J/ψ center-of-mass momentum distribution at BaBar:



- Neither experiment sees the enhancement at $z = 1$ that is expected from color-octet production at leading order in α_s.
- But resummation of multiple gluon emission and the v expansion are very important near $z = 1$.

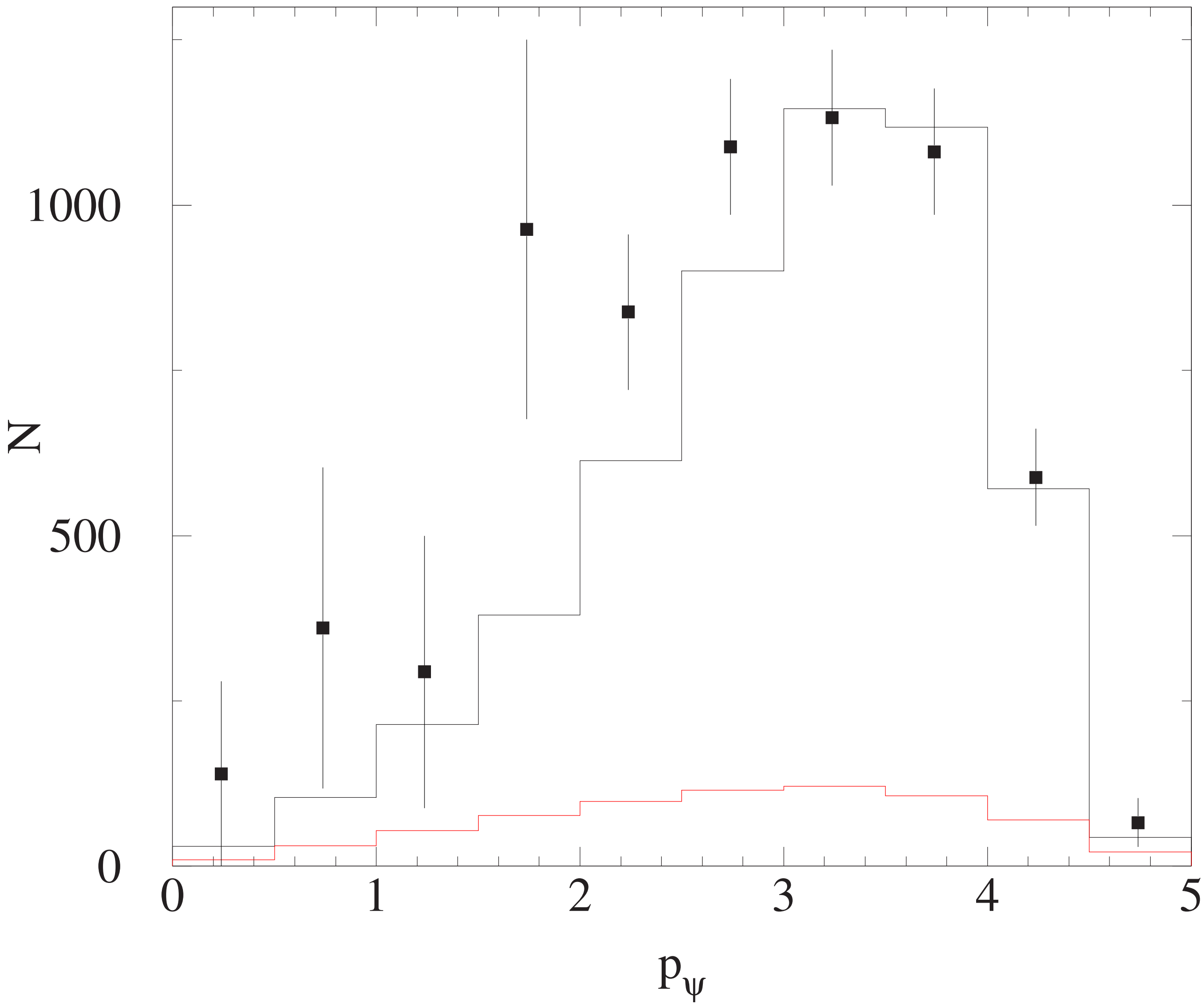
- Soft-gluon-resummation and shape-function effects have been calculated for $e^+e^- \to J/\psi + X$ by Fleming, Leibovich, and Mehen.

Belle data:

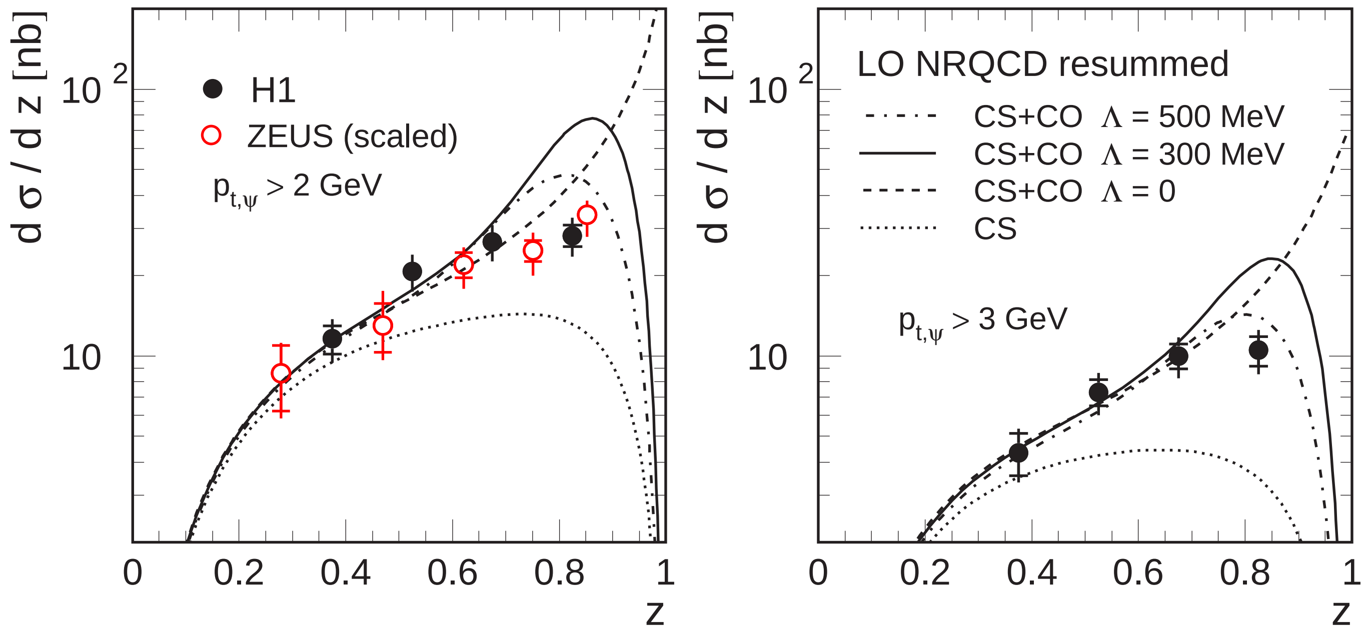


- Red is color singlet; black is color-octet plus color singlet.

BaBar data:



- Inclusion of a shape function with reasonable choices of parameters leads to an improved fit.



- New higher-p_T data are more compatible with a color-octet contribution.
- Could use a shape function fitted to e^+e^- data to make predictions for other processes, such as photoproduction at HERA.

Angular distribution of J/ψ's

- Fit to $1 + A\cos^2\theta^*$.
- Belle:
  $A = 0.3^{+0.5}_{-0.4}$ for 2.0 GeV $< p^* < 2.6$ GeV;
  $A = 1.1^{+0.4}_{-0.3}$ for 2.6 GeV $< p^* < 3.4$ GeV;
  $A = 1.1^{+0.4}_{-0.3}$ for 3.4 GeV $< p^* < 4.9$ GeV.
- BaBar:
  $A = 0.05 \pm 0.22$ for $p^* < 3.5$ GeV;
  $A = 1.5 \pm 0.6$ for $p^* > 3.5$ GeV.
- NRQCD (Braaten, Chen):
  $A \approx 0$ at small p^*;
  $A = 0.6$–1.0 at large p^*.
- CSM (Braaten, Chen):
  $A \approx 0$ at small p^*;
  $A \approx -0.8$ at large p^*.
- The Belle and BaBar data favor NRQCD, but the uncertainties are large.

Polarization of J/ψ's

- Belle:
  $\alpha = -0.4 \pm 0.2$ for 2.0 GeV $< p^* < 2.6$ GeV,
  $\alpha = -0.4 \pm 0.1$ for 2.6 GeV $< p^* < 3.4$ GeV,
  $\alpha = -0.2 \pm 0.2$ for 3.4 GeV $< p^* < 4.9$ GeV.
- BaBar:
  $\alpha = -0.46 \pm 0.21$ for $p^* < 3.5$ GeV,
  $\alpha = -0.80 \pm 0.09$ for $p^* > 3.5$ GeV.
- No indication of dominance of the color-octet mechanism.
- NLO color-octet calculation and analysis of feeddown from higher charmonium states is needed.
- Is p^* too small to obtain substantial transverse polarization?

Double $c\bar{c}$ Production at Belle

$e^+e^- \to J/\psi + \eta_c$

- Belle obtains

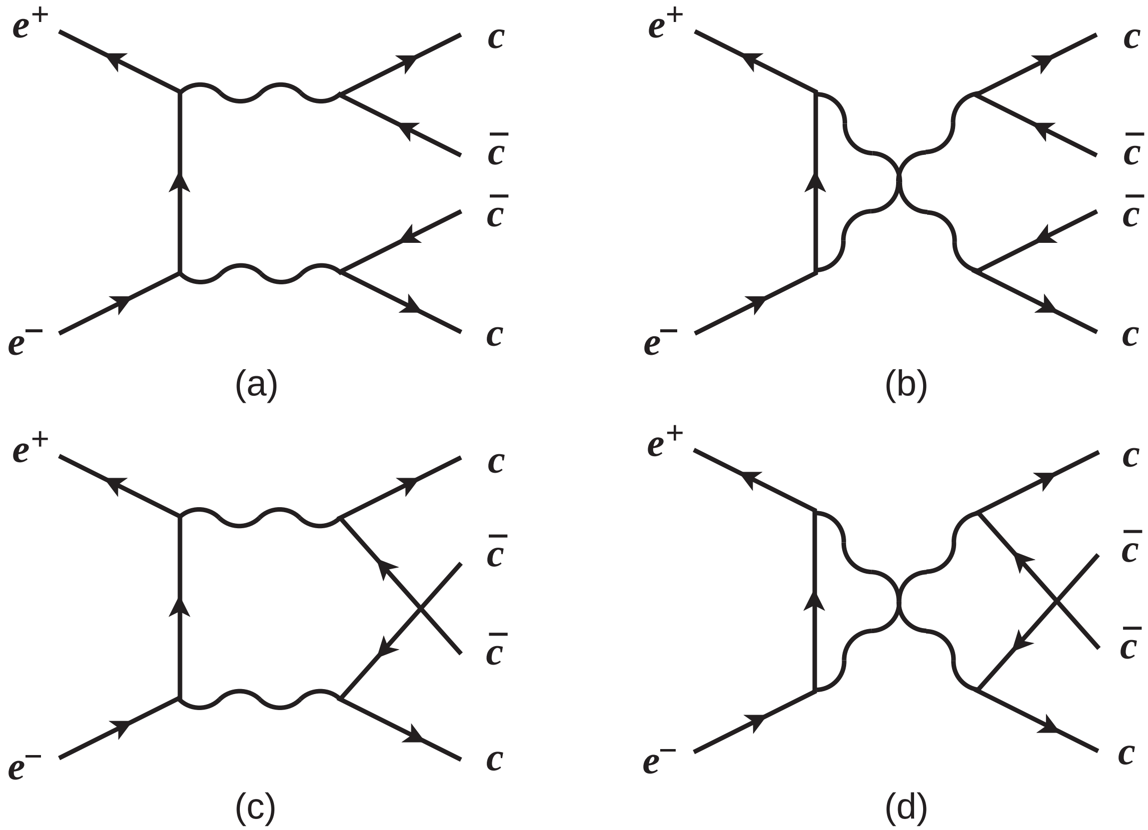
$$\sigma(e^+e^- \to J/\psi + \eta_c)B[\geq 4] = 33^{+7}_{-6} \pm 9 \text{ fb.}$$

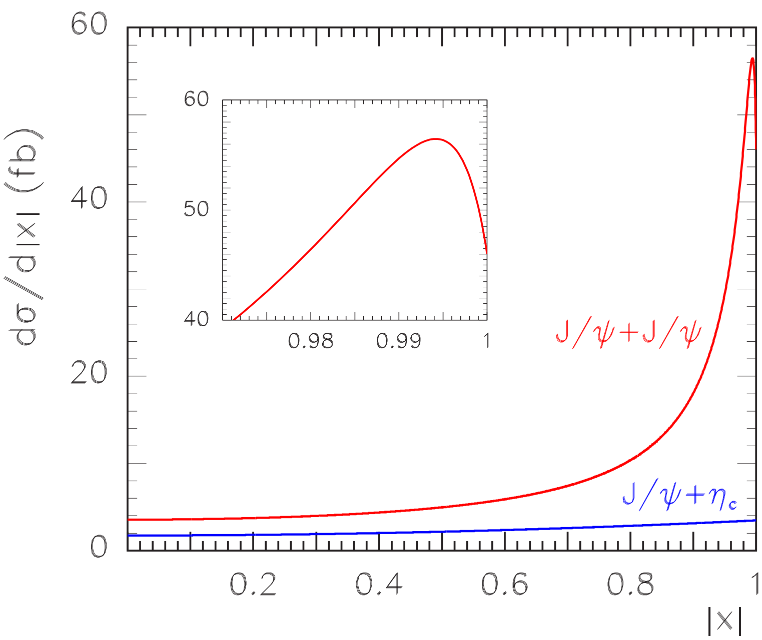
- NRQCD predicts

$$\sigma(e^+e^- \to J/\psi + \eta_c) = 2.31 \pm 1.09 \text{ fb.}$$

  - First calculation by Braaten, Lee.
  - Confirmed by Liu, He, Chao (NRQCD) and Brodsky, Ji, and Lee (light-front QCD).
  - Includes -21% QED interference correction.
  - Uncertainties from higher orders in α_s, v, matrix elements.
  - Exclusive process: color-singlet only.
  - Matrix elements are fairly well determined from $J/\psi \to e^+e^-$ and $\eta_c \to \gamma\gamma$.

- Some of the $J/\psi + \eta_c$ data sample may consist of $J/\psi + J/\psi$ events (GTB, Braaten, Lee).
  - The Belle resolution is 110 MeV, but $M_{J/\psi} - M_{\eta_c} = 120$ MeV.
  - $J/\psi + J/\psi$ is $C = +1$, so that state is produced in a two-photon process.
  - Suppressed by relative to $J/\psi + \eta_c$ by $(\alpha/\alpha_s)^2$
  - But fragmentation diagrams are enhanced by
    - $(E_{\rm beam}/2m_c)^4$ from gluon propagators,
    - $\log[8(E_{\rm beam}/2m_c)^4]$ from a would-be collinear divergence.


e^+
e^-
c
$\bar{c}$
(a)
(b)
(c)
(d)


$d\sigma/d|x|$ (fb)
$|x|$
$J/\psi+J/\psi$
$J/\psi+\eta_c$
0
20
40
60
0.2
0.4
0.6
0.8
1
0.98
0.99
50

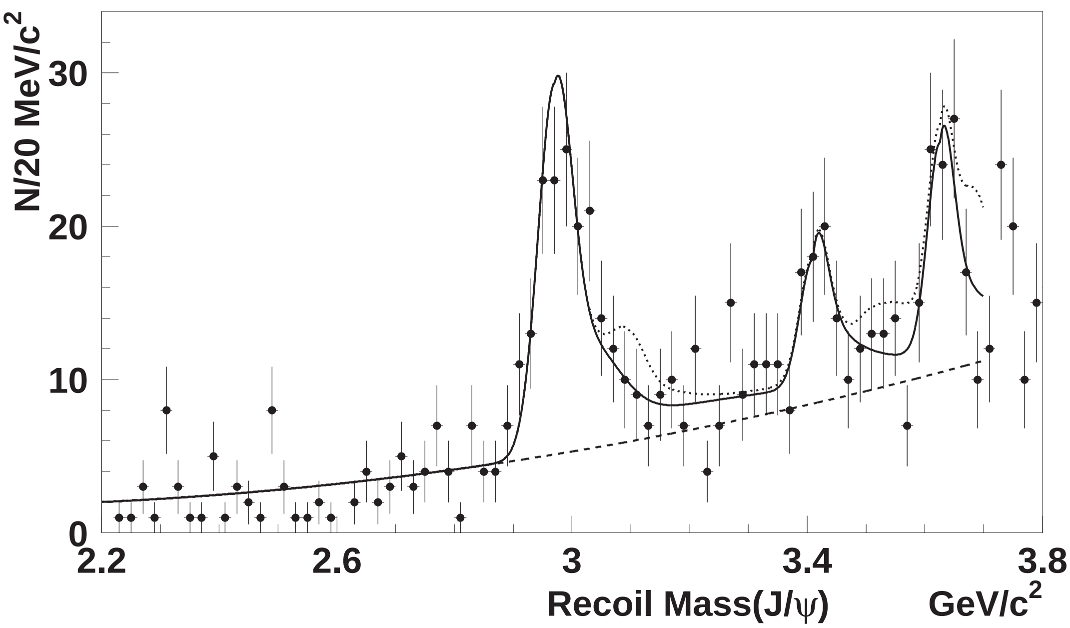
- Prediction:

$$\sigma(e^+e^- \to J/\psi + J/\psi) = 8.70 \pm 2.94 \text{ fb.}$$

  - Corrections of higher order in α and v may reduce this by a factor 3.
  - Comparable with the prediction

$$\sigma(e^+e^- \to J/\psi + \eta_c) = 2.31 \pm 1.09 \text{ fb.}$$

- New Belle result for spectrum recoiling against J/ψ:



- η_c, χ_{c0}, $\eta_c(2S)$ seen.
- No evidence for J/ψ, χ_{c1}, $\psi(2S)$ (dashed line).
- There is no significant $J/\psi + J/\psi$ signal observed:

$$\sigma(e^+e^- \to J/\psi + J/\psi) < 7 \text{ fb.}$$

$e^+e^- \to J/\psi + c\bar{c}$

- New Belle result:

$$\begin{aligned} &\sigma(e^+e^- \to J/\psi + c\bar{c})/\sigma(e^+e^- \to J/\psi + X) \\ &\quad = 0.82 \pm 0.15 \pm 0.14 \\ &\quad > 0.48 \text{ (90\% confidence level)} \end{aligned}$$

- pQCD plus color-singlet model (Cho, Leibovich; Baek, Ko, Lee, Song; Yuan, Qiao, Chao):

$$\sigma(e^+e^- \to J/\psi + c\bar{c})/\sigma(e^+e^- \to J/\psi + X) \approx 0.1.$$

- The experimental and theoretical double-$c\bar{c}$ cross sections also disagree.
  - Belle: $\sigma(e^+e^- \to J/\psi + c\bar{c}) \approx 0.9$ pb.
  - Theory: $\sigma(e^+e^- \to J/\psi + c\bar{c}) =$ 0.10–0.15 pb.
- The order-α_s^2 calculation is incomplete.
  - Lacks color-octet contributions, including those that produce $J/\psi\, c\bar{c}$.
  - But suppressed by $v^4 \approx 0.1$.
  - Could the short-distance coefficients could be large?
- No reason to expect the corrections of higher order in α_s and v to be large.

The discrepancies in the double $c\bar{c}$ cross sections are among the largest in the standard model.

- Theory and experiment differ by almost an order of magnitude—larger than any known “k-factor.”
- This is a problem not just for NRQCD factorization, but for pQCD in general.
- For $e^+e^- \to J/\psi + \eta_c$, one obtains the same result in the NRQCD and light-front approaches.
- It is difficult to see how any perturbative calculation of

$$\sigma(e^+e^- \to J/\psi + c\bar{c})/\sigma(e^+e^- \to J/\psi + X)$$

  could give a value as large as 80%.
- It is very important for BaBar to check the Belle double $c\bar{c}$ results.
- Other possibilities:
  - new production mechanisms,
  - inapplicability of perturbative QCD,
  - new physics.

Conclusions

- NRQCD factorization provides a formalism for computing quarkonium production in QCD.
  - NRQCD factorization is a consequence of QCD, not a model.
  - It holds in the limit m, $p_T >> \Lambda_{\rm QCD}$.
- Comparisons of theoretical predictions with experiment test both NRQCD and the hard-scattering-factorization machinery.
- The correct v scaling and the universality of the NRQCD production matrix elements are important tests of NRQCD factorization.
  - Matrix elements are fit to Tevatron data and agree with the v-scaling rules.
  - Other processes test universality.
- The NRQCD factorization approach has been successful in describing several processes for which the color-singlet model fails:
  - quarkonium production at the Tevatron,
  - $\gamma\gamma \to J/\psi + X$ at LEP,
  - quarkonium production in DIS at HERA.

- Other NRQCD predictions do not agree well with experiment:
  - polarized quarkonium production at the Tevatron,
  - inelastic quarkonium photoproduction at HERA,
  - $\sigma(e^+e^- \to J/\psi\, c\bar{c})/\sigma(e^+e^- \to J/\psi\, X)$ at Belle.
- Polarization in quarkonium production is a particularly important test.
  - Smoking gun for the color-octet mechanism.
  - So far, no evidence for the large transverse polarization associated with color-octet production.
  - Experimental uncertainties at high p_T are large.
  - NLO corrections and resummation of multiple gluon emission may reduce the size of the color-octet matrix elements extracted from the Tevatron data.
  - Lattice calculations indicate that de-polarizing spin-flip interactions are suppressed by a factor v^2, as expected.
- Inelastic Photoproduction at HERA is an important test of universality of the NRQCD matrix elements.
  - Experimental and theoretical uncertainties are large.
  - Resummation in v and α_s is needed near $z = 1$.

- The Belle results on inclusive and exclusive double-$c\bar{c}$ production present a severe challenge to pQCD.
  - One of the largest discrepancies in the standard model.
  - A check by BaBar would be very useful.
- Progress on reduction of experimental and theoretical uncertainties should allow for more meaningful tests.
- Many improvements in the theoretical predictions are needed:
  - calculations at NLO in α_s,
  - understanding and control of large non-logarithmic corrections,
  - calculations of corrections of higher order in v,
  - understanding and control of large corrections of higher order in v,
  - resummation of large corrections of higher order in α_s and v,
    * phenomenology of shape functions,
  - better determination of m_c,
  - reduction in the uncertainties in NRQCD matrix elements.
    * Lattice calculations can help the pin down the matrix elements.
    * It is not yet known how to formulate the calculation of production matrix elements on the lattice.

- Over the last decade, there has been a great deal of experimental and theoretical progress in heavy-quarkonium production.
- There are still many interesting and challenging problems in heavy-quarkonium production that remain to be solved.